THERMAL AND AERODYNAMIC PERFORMANCES OF THE SUPERSONIC MOTION

by

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Generally speaking, Mach number of 4 can be taken as a boundary value for transition from conditions for supersonic, into the area of hypersonic flow, distinguishing two areas: area of supersonic in which the effects of the aerodynamic heating can be neglected and the area of hypersonic, in which the thermal effects become dominant.

This paper presents the effects in static and dynamic areas, as well as presentation of G.R.O.M. software for determination of the values of aerodynamic derivatives, which was developed on the basis of linearized theory of supersonic flow.

Validation of developed software was carried out through different types of testing, proving its usefulness for engineering practice in the area of supersonic wing aerodynamic loading calculations, even at high Mach numbers, with dominant thermal effects.

Keywords*: distribution of circulation, perturbation velocity potential, aerodynamic derivatives*

Introduction

Most common division of the aerodynamics is [1, 2]: a) classical aerodynamics (comprised of incompressible, subsonic compressible, trans-sonic and supersonic aerodynamics), b) hypersonic aerodynamics, c) aerodynamics of free molecules, d) magneto-aerodynamics and e) Newtonian flows.

This paper considers both of the effects within the areas of super- and hypersonic aerodynamics. Weather the body is moving in supersonic or greater speeds, usually is defined by Mach number (M), which is, by definition, the ratio between velocity of the moving body and the speed of the sound in the environment in which the body is moving. However, the most difficult areas for distinguishing are the area of super- and hypersonics. The reason lies within the fact, that from Mach 3.2 upward, with the increase of Mach number, the influence of the aerodynamic heating increases. One of the possible divisions is in accordance with the methodology of Naval Surface Weapons Center (NSWC) [3]: value of Mach number between 1.02 (1.5) and 4 defines Low Supersonic Flow (LSF), while Mach number equal or greater than 4 (5) defines High Supersonic Flow (HSF).

Generally speaking, Mach number of 4 can be taken as a boundary value for transition from conditions for supersonic, into the area of hypersonic flow, although, very often other factors must be taken into account (not only the characteristic of the flow itself, but also the characteristics of boundary layer, body surface characteristics including heat resistance, local material thickness, etc.)

In general, aerodynamic heating wise, it can be said that in the process of determination of the aerodynamic derivative values, for missiles, and especially isolated wings of the missiles, in supersonic flow, the problem can be seen as divided into two areas, Mach number dependant: in the range of Mach number values of 1.4-3.2 the problem can be reflected upon as "cold", since those values are the same ones on which the influence of aerodynamic heating is becoming apparent, in the range of Mach numbers 3.2-4, the problem can be still depicted as "cold" since the influence of aerodynamic heating is present, and increasing, but still less than other influences, and determination of aerodynamic derivatives using the models that do not take it into account provide accurate enough estimations for engineering practice, while for the Mach number values equal or greater than 4, aerodynamic heating must be taken into account, through utilization of appropriate models, making this "hot" zone.

One of the most common methods that enables calculation of transient aerodynamic heating and surface temperatures at supersonic and hypersonic speeds, for complete flight trajectories, are NASA models [4, 5]. Semi-empirical theories are used to calculate laminar and turbulent heat transfer coefficients and a procedure for estimating boundary-layer transition is included.

Basic equations used:

- for calculating surface temperatures and heat flux for three-dimensional stagnation points and two-dimensional stagnation points without sweep, as well as for two-dimensional stagnation points with sweep:

$$q=(\rho_{\omega} C_{p,_{\omega}} \tau)\dot{T}_{\omega} = F(h_l)(H_{\mathrm{R}} - H_{\mathrm{W}}) - \beta T_{\mathrm{W}}^4 + S \tag{1}$$

The term S in equations is for solar and nocturnal radiation input if required. This term is negligible, except for low-speed flow and is normally set equal to zero. The term βT_W^4 is the heat lost by radiation from the surface of the aircraft to the atmosphere.

To obtain good surface temperatures and accurate heat flux, proper engineering judgment must be exercised in determining the heat capacity $(\rho_W C_p \tau)$ for the surface. Since the values of the specific heat $C_{p,\mathrm{W}}$and density ρ_{W} are the thermal properties of the material, only way to significantly vary the heat capacity is to change the material thickness τ.

Heat transfer coefficient (h_1) determination (as given by Fay and Riddell for a Lewis number of 1.0 – no dissociation, and a Prandtl number of 0.71):

$$h_1 = 0.94(\rho_{\mathrm{st}}\mu_{1,\mathrm{st}})^{0.4}(\rho_{\mathrm{w}}\mu_{1,\mathrm{w}})^{0.1}\sqrt{\left(\frac{\mathrm{d}V}{\mathrm{d}x}\right)_{x=0}} \tag{2}$$

The velocity gradient $(du/dx)_{x=0}$ is given by:

$$\left(\frac{du}{dx}\right)_{x=0} = \frac{1}{\mathrm{R}}\sqrt{\frac{2(P_{\mathrm{st}} - P_1)g}{\rho_{\mathrm{st}}}} \tag{3}$$

Stagnation enthalpy Hst for three-dimensional flow and two-dimensional flow with no sweep:

$$H_{st} = H_2 + \frac{v_2^2}{2gJ} \tag{4}$$

Recovery enthalpy for two-dimensional flow with sweep:

$$H_R = H_2 + \frac{v_2^2}{2gJ} + 0.855\frac{v_1^2 \sin^2 \Lambda}{2gJ} \tag{5}$$

Other flow conditions:

$$P_{st} = P_2\left(1 + \frac{\gamma_2 - 1}{2}\mathrm{M}_2^2\right)^{\frac{\gamma_2}{\gamma_2 - 1}} \tag{6}$$

$$T_{st} = T_2\left(1 + \frac{\gamma_2 - 1}{2}\mathrm{M}_2^2\right) \tag{7}$$

The values for T_2 and M_2, are determined from the real gas tables.
Solving of eq. (4) and (5), implied two approaches:

1. Constant entropy solutions - constant entropy flow will only occur on a surface with a sharp leading edge or nose, many aircraft surfaces can be approximated by shapes where constant entropy solutions can be used with good result:
   1.1. Laminar heat transfer:

$$h_l = (F)\frac{0.332}{\sqrt{\mathrm{Re}_{,L}}}\sqrt{\frac{\rho^* \mu_l^*}{\rho_L \mu_{l,L}}}\left(\mathrm{Pr}_{,w}\right)^{-0.6}\left(\rho_L v_L\right) \tag{8}$$

$$H^* = 0.5\left(H_w + H_L\right) + 0.22\left(H_R + H_L\right) \tag{9}$$

   where H_w and H_L were obtained from real gas table.
   1.2. Turbulent heat transfer:

$$h_l = F\frac{0.185}{\left(\log\mathrm{Re}^*\right)^{2.584}}\left(\mathrm{Pr}_{,w}\right)^{0.4}\left(\rho_L^* V_L\right) \tag{10}$$

$$H_R = H_L + \left(\mathrm{Pr}_{,w}\right)^{1/3}\frac{V_L^2}{2gJ} \tag{11}$$

2. Variable entropy solutions - All surfaces with a blunt leading edge or blunt nose will have variable entropy flow
   2.1. Laminar heat transfer:

$$h_l = 0.22\left(\mathrm{R}\theta_{,L}\right)^{-1}\left(\frac{\mu_l^*}{\mu_{l,L}}\right)\left(\mathrm{Pr}_{,w}\right)^{-0.6}\left(\rho_L V_L\right) \tag{12}$$

where the momentum thickness "θ" is calculated as:

$$\theta = 0.664 \frac{\left[\int_0^{x_{\mathrm{Re}}} \rho^* \mu_1^* V_{\mathrm{L}} \mathrm{r}^2 \, \mathrm{d} x_{\mathrm{Re}} \right]^{1/2}}{\rho^* V_{\mathrm{L}} r} \tag{13}$$

$$H^* = 0.5 \left(H_{\mathrm{L}} + H_{\mathrm{w}} \right) + 0.22 \left(H_{\mathrm{R}} - H_{\mathrm{L}} \right) \tag{14}$$

2.2. Turbulent heat transfer:

$$h_1 = c_1 \left(\mathrm{R}\theta,_{\mathrm{L}} \right)^{-m} \left(\frac{\mu_1^*}{\mu_{1,\mathrm{L}}} \right)^m \left(\frac{\rho^*}{\rho_{\mathrm{L}}} \right)^{(1-m)} \left(\mathrm{Pr},_{\mathrm{w}} \right)^{-0.4} \left(\rho_{\mathrm{L}} V_{\mathrm{L}} \right) \tag{15}$$

with "θ" for flat plate being:

$$\theta = \frac{\left[c_2 V_{\mathrm{L}} \rho^* \left(\mu_1^* \right)^m x \right]^{c_4}}{\rho^* V_{\mathrm{L}}} \tag{16}$$

Boundary layer transition criteria - Two of the primary parameters that affect boundary layer transition are the local Reynolds number and local Mach number. Developed method uses the following equation that incorporates these parameters to predict transition:

$$\log \mathrm{Re},_{\mathrm{L}} > \left[\log \mathrm{Re},_{\mathrm{t}} + C_{\mathrm{M}} \left(\mathrm{M}_{\mathrm{L}} \right) \right] \tag{17}$$

G.R.O.M. software

Model, upon which the software was developed, is based on the linearized theory of supersonic flow, with the following assumptions [6, 7]:

1. Wing is thin.
2. Values of the aerodynamic derivatives are calculated for isolated wing.
3. Taper ratio is arbitrary.
4. End of the wing is parallel to free-stream.
5. Leading and trailing edge are strait lined, and their angles are constant.
6. Angle of the leading edge is positive i.e. leading edge is sweptback.
7. Angle of the trailing edge can be: zero, positive (sweptback trailing edge), and negative (swept-forward trailing edge), which is taken into account by the sign of factor k. Factor k represents the ratio of cotangent of the angle of the trailing edge with cotangent of the angle of the leading edge, and it can be positive (for both leading and trailing edge being positive-both edges being sweptback), negative (trailing edge being swept-forward) and zero (unswept trailing edge).

8. Theory is valid for the range of Mach numbers for which both leading and trailing edges are supersonic, or leading edge is subsonic and trailing edge is supersonic, and free-stream Mach number being equal or greater than 1.4.
9. Mach lines emanating from one wing do not influence the other, i.e. there is no wing-to-wing interference

In respect to calculated distribution of the spanwise loading depending on the angle of attack, constant rate of roll and constant rate of pitch, values of the following aerodynamic derivatives were calculated [7]:

- aerodynamic derivative of the lift force in respect to angle of attack

$$c_L^{\alpha} = \sum_{i=1}^{n} 2A \int_{dg}^{gg} \frac{\Gamma_{zone}}{V \cdot \alpha \cdot (b/2)} \mathrm{d}\left(\frac{y}{b/2}\right) \tag{18}$$

- aerodynamic derivative of the roll-dumping moment in respect to rate of roll

$$c_l^{w_x} = \sum_{i=1}^{n} -\frac{1}{2} \int_{dg}^{gg} \frac{\Gamma_{zone}}{w_x \cdot (b/2)^2} \mathrm{d}\left(\frac{y}{b/2}\right)^2 \tag{19}$$

- aerodynamic derivative of the pitch-dumping moment in respect to rate of pitch

$$c_m^{w_y} = \sum_{i=1}^{n} -\frac{1}{2} \int_{dg}^{gg} \frac{\Gamma_{zone}}{w_y \cdot B \cdot (b/2)^2} \mathrm{d}\left(\frac{y}{b/2}\right)^2 \tag{20}$$

- aerodynamic derivative of the drag force in respect to angle of attack on the square

$$\frac{D}{S_K q_p} = \frac{L^{\alpha}}{S_K q_p} \alpha^2 \Rightarrow c_D^{\alpha^2} = c_L^{\alpha} \tag{21}$$

Two families of wings were considered, as presented on fig. 1, number and markings of the zones on the wings included. These zones are determined by Mach cones positions and intersection points of the cones with the trailing edge. Case of the wings with sweptforward trailing edge (family of PNKI wings) was not considered because the form of functional expressions for circulation Γ is the same as for the case when trailing edge is sweptback (family of PSNI wings). However the possibility of determination of named aerodynamic derivatives for this family of wings exists, by changing the sign of factor *k*.

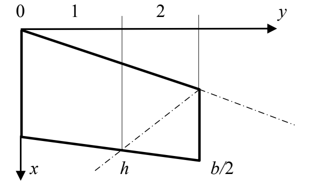
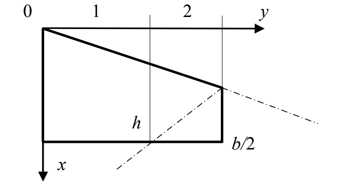
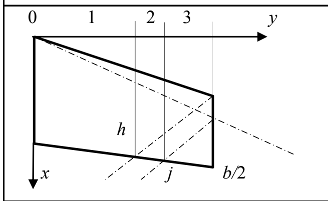
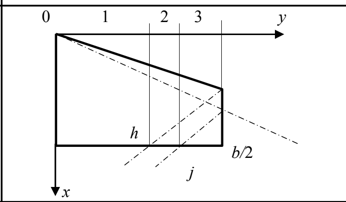
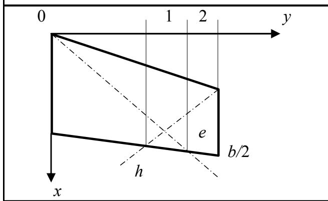
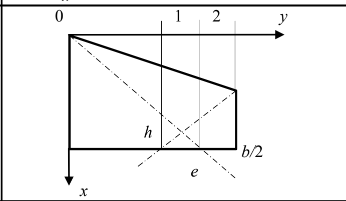
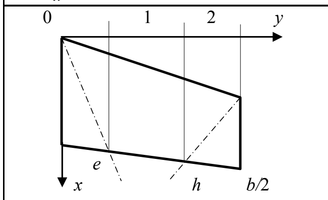
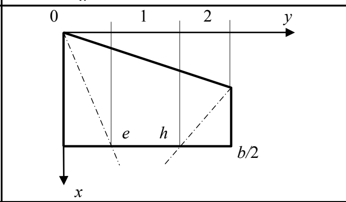
PSNI family of wings	Subcase tag	PNNI family of wings	Subcase tag
	PSNI 2		PNNI 2
	PSNI 5		PNNI 5
	PSNI 4		PNNI 4
	PSNI 3		PNNI 3

Figure 1: Wing shape and position of the Mach cones and integration borders for wings with sweptback leading and trailing edges (PSNI family of wings), and sweptback leading and unswept trailing edges (PNNI family of wings).

Main obstacle during the application of eq. in [7] for distribution of circulation was the appearance of discontinuities and interruptions within them, which presented the violation of basic principle that those functions must be continuous, thus causing the necessity for modifications, in order to ensure the continuity. In order to execute the necessary modifications, certain postulates had to be adopted [8, 9]:

1. Intersection points of Mach cones and trailing edge represent zone boundaries, and, in the same time represent the boundary values – value for distribution of circulation in the zone before must be, in the boundary (intersection) point, equal to the value for distribution of circulation in the next zone.
2. Equations for distribution of circulation in the first and second zone for the wing with four panels (subcases PNNI4 and PSNI4) are the same as for the wing with five panels (subcases PNNI5 and PSNI5).
3. Equation for distribution of circulation in the first zone for the wing with three panels (subcases PNNI3 and PSNI3) is the same as for the wing with five panels (subcases PNNI5 and PSNI5).
4. Equations for distribution of circulation in the third zone for the wing with three panels (subcases PNNI3 and PSNI3) are the same as for the wing with four panels (subcases PNNI4 and PSNI4).

5. Excluding articles that have factor k in themselves, which takes leading and trailing edge sweeps into account, out of functional expressions for PSNI family of wings, expressions for PNNI family of wings are obtained. For the case of unswept trailing edge $k \to \infty$ (family of the PNNI wings). For any other value of trailing edge sweep angle factor k has a final value.
6. Distribution of circulation values when the leading edge is sonic (Mach line coincident with leading edge – subcases PNNI2 and PSNI2), and when the leading edge is supersonic and there are 5 panels on the wing (subcases PNNI5 and PSNI5) should be very close and the diagram representations of distributions of circulation mutually very similar, provided that Mach numbers for sonic and supersonic leading edge are close to each other (meaning that angles of Mach cones differ in small amount).
7. For the small value of trailing edge sweep (around 1 degree), while the leading edge sweep remains the same, and aspect ratio and area of the wing are slightly changed, distribution of circulation values for PNNI and PSNI family of wings should be very close to each other.

Within two sets of equations the mentioned modifications were executed, based on above mentioned postulates [8, 9]:

a) equations for distribution of circulation resulting from a constant rate of roll - Since only the diagram for distribution of circulation for subcase PNNI5 was without discontinuities, it presented a starting point
b) equations for distribution of circulation resulting from a constant rate of pitch - Problem with this sets of equations was the one that for every wing in both families of wings discontinuities of functions in particular zones existed, thus making one unable to start modifications from any particular wing. The only solution was to start with the comparison of two wings (using postulates no. 1., 5. and 7.) and to obtain, through the iteration method, equations with no discontinuities.

On the Diagrams 1 through 4, for illustrative subcases, distribution of circulation is presented, before and after the modification, while in literature [8, 9] diagrams for all of the subcases are presented. Two things, while looking upon diagrams, should be noted: 1) For easier analysis the diagrams based upon unmodified equations, were drawn only through characteristic points of the wings, and 2) tag “μ” represents the Mach line angle.

a) Distribution of circulation resulting from a constant rate of roll – subcase PNNI2 –Comparison started in accordance with postulates no. 1 and 6 (Diagram 1). Based on the same postulates, the modification itself was conducted (Diagram 2).

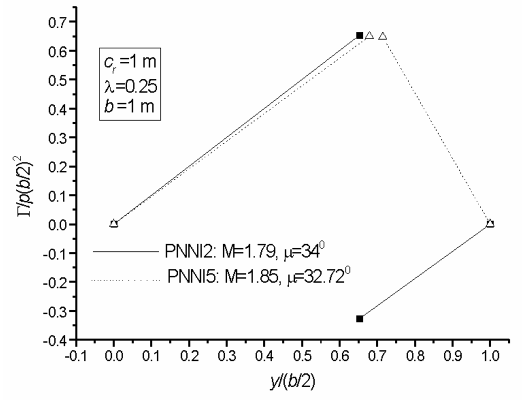


Diagram 1 – Distribution of circulation, for subcases marked as PNNI2 and PNNI5, drawn through characteristic points of the zones on the wing; unmodified equations

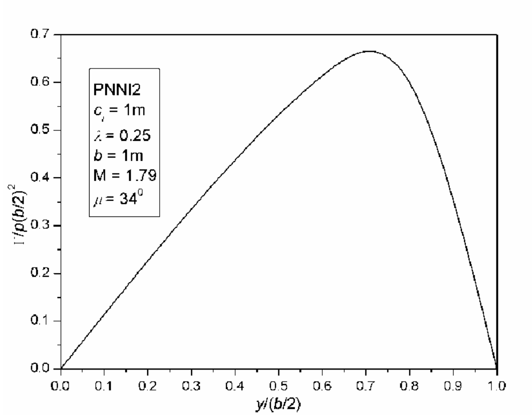


Diagram 2 – Distribution of circulation, subcase marked as PNNI2, modified equations

b) Distribution of circulation resulting from a constant rate of pitch –subcase PSNI2 - Since the equations for subcases PNNI2 and PSNI2 are the simplest, comparison and iteration process began with those two subcases.

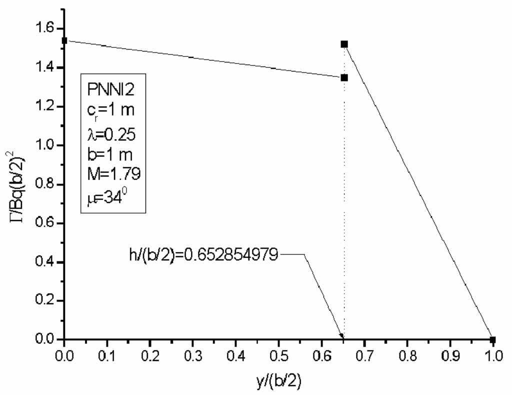


Diagram 3 – Distribution of circulation, for subcase marked as PNNI2, drawn through characteristic points of the zones on the wing; unmodified equations

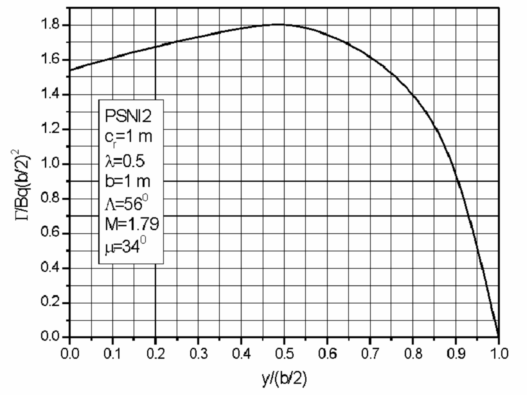


Diagram 4 – Distribution of circulation, subcase marked as PSNI2, modified equations

In order to validate performed modifications set of testing was necessary. This validation was executed through two types of "internal" testing [8, 10]:

- Variation of geometry
- Variation of Mach Number

Data of the executed "internal" testing [8, 10] will be, within this paper, presented only for aerodynamic derivative of the lift force in respect to angle of attack and aerodynamic derivative of the drag force in respect to angle of attack on the square for PNNI2 and PSNI2 subcases, as an illustration.

A) Variation of geometry: Excluding articles that have factor k in themselves, which takes leading and trailing edge sweeps into account, out of functional expressions for PSNI family of wings, expressions for PNNI family of wings are obtained. For the case of unswept trailing edge $k \to \infty$ (family of the PNNI wings). For any other value of trailing edge sweep angle factor k has a finite value.

That means that for small value of trailing edge sweep angle (1 degree) while sweep angle of the leading edge rests unchanged and aspect ratio and wing surface are slightly changed (as presented on fig. 2), results obtained for the PNNI family of wings and PSNI family of wings should be very close (tab. 1).

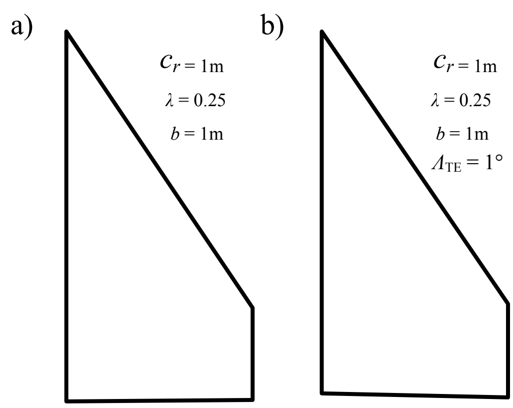


Figure 2: "Internal" testing – change of geometry: a) family of the PNNI wings; b) family of the PSNI wings

Table 1: Review of comparison for PNNI2 and PSNI2 subcases:

M=1.6	PNNI2	PSNI2	deviation [%]
zone 1	2.574	2.581	0.26
zone 2	2.679	2.664	0.53
total	5.253	5.245	0.14

B) Variation of Mach number: This set of testing refers to the leading edge. In first case Mach cone angle is such that leading edge is subsonic in that way that Mach cone lies just in front of the leading edge (0.85 degrees). Second case is such that leading edge is supersonic in the way that Mach cone lies just behind (1.28 degrees) the leading edge (as presented on fig. 3). Small difference in Mach number value, in the cases of subsonic and supersonic leading edge, makes the aerodynamic derivative values mutually very similar (tab. 2).

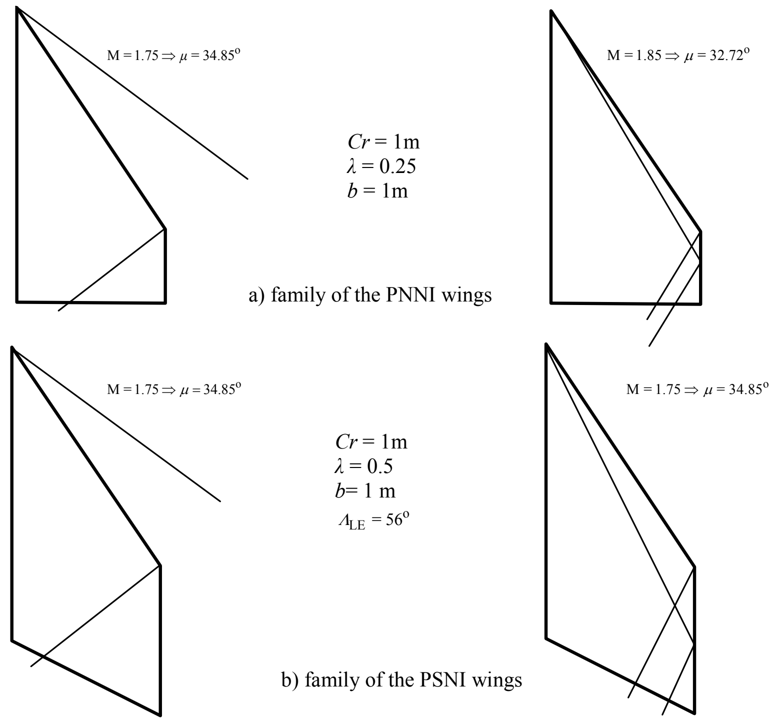


Figure 3: "Internal" testing by changing the Mach number

Table 2: Comparative review of derivative value for PNNI family of wings:

<table>
<tr><th>TAG</th><th>M=1.75</th><th>value</th><th></th><th>TAG</th><th>M=1.85</th><th>value</th></tr>
<tr><td rowspan="4">PNNI2</td><td>zone 1</td><td>3.334</td><td rowspan="4"></td><td rowspan="4">PNNI5</td><td>zone 1</td><td>3.431</td></tr>
<tr><td>zone 2</td><td>1.387</td><td>zone 2</td><td>0.162</td></tr>
<tr><td rowspan="2">total</td><td rowspan="2">4.721</td><td>zone 3</td><td>0.851</td></tr>
<tr><td>total</td><td>4.443</td></tr>
<tr><td colspan="7">Deviation: 6.25%</td></tr>
</table>

In order to determine the quality of obtained results, i.e. values of afore mentioned aerodynamic derivatives, comparative testing was conducted [8, 10]. For comparison, software Missile Datcom 97 was used. Within this paper graphical presentation of comparative testing for PSNI family of wings, along with appropriate comments, will be presented (Diagrams 5 through 7).

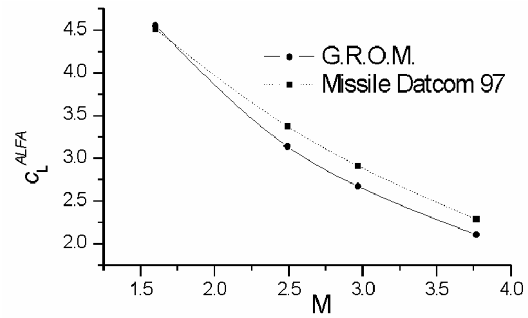


Diagram 5 – Comparative review of change of the aerodynamic derivative of the lift force in respect to angle of attack for the PSNI family of wings

It should be noted that diagram for the aerodynamic derivative of the drag force in respect to angle of attack on the squire, is the same as Diagram 5., only difference being that instead of tag c_L^{ALFA} should stand $c_D^{\alpha^2}$.

General conclusion is that that both methods show very similar results, with small deviations and the same gradient in the range of Mach numbers 2.2-3.8, i.e. for the flow conditions for which the leading edge is supersonic. In the range of the Mach numbers for which the leading edge is subsonic the values are very close to each other, but gradient characters are somewhat different.

This character difference in the zone that corresponds, or is very near, to the conditions for sonic leading edge is in correspondence with transitional regime of projectile acceleration and its durance is very short.

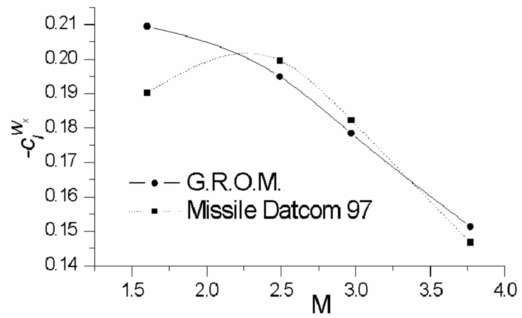


Diagram 6 – Comparative review of change of the aerodynamic derivative of the roll-dumping moment in respect to rate of roll for the PSNI family of wings

Generally, it can be said that deviation of the results obtained by two methods are small for all of the flow conditions. Any significant difference in the value change occurs only for the low supersonic Mach numbers, and it is linked with flight velocities which are achieved and exceeded very quickly (practically buster phase). This deviation is also the consequence of the control program. Within Missile Datcom software interpolation of the data obtained by experimental testing in aero tunnel for different Mach number values and different wing geometry is used for calculation of this derivative, resulting with the influences of: the existing vortex emanating from the nose of the projectile, body-wing interference, deviations of the productional and constructive sweep angle of the leading edge, wing build-in deviations of the sweep angle, unsymmetrical position of the wings on the body of the projectile, nonuniform mass distribution, wing roughness, possible existence of the aerodynamic impurities, leading edge is never sharp, wing is not of vanishingly small thickness. All of these influences decrease with the Mach number increase.

Problem in comparison for the value of the aerodynamic derivative of the pitch-dumping moment in respect to rate of pitch was impossibility for Missile Datcom 97 to provide data for this dumping moment for isolated wing. For the reason of completeness of this work values for aerodynamic derivatives of the pitch-dumping moment in respect to rate of pitch, for PSNI families of wings, will be presented without comparative data (Diagram 7). As an orientation control can be used diagram of value change of this derivative in respect to Mach number change presented in ref. [6].

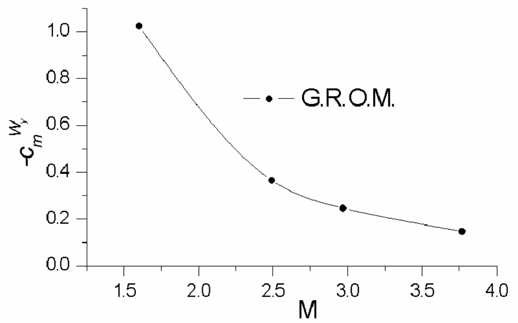


Diagram 7 – Review of change of the aerodynamic derivative of the pitch-dumping moment in respect to rate of pitch for the PSNI family of wings

Comparing Diagram 7 with diagrams in [6], it can be concluded that the shapes and trends of diagrams are similar.

Change in position of the point about which the wing rotates does not change the shape of the curve, but it only translates it vertically. Any significant changes in derivative gradient with the change of the point about which the wing rotates should not be expected.

Main observation of the comparative test results is the following: testing of G.R.O.M. software was conducted for Mach number value of 3.768, which is, in connection with the boundary of differentiation of super- and hyper sonic flow, very high value, almost at the borderline. Within developed software the influence of aerodynamic heating was not taken into account, since the purpose of it, is utilization in the area of supersonic flow, while control program does take it into account, up to certain level. Still, the values obtained by utilization of both programs and the difference between them, or better said, level of correspondence, prove that developed software can successfully be utilized for assessment of aerodynamic derivative values on, these, high Mach numbers, on which the effects of the aerodynamic heating occurs.

Conclusion

Basic equations for determination of distribution of circulation, whose modification was necessary, due to appearance discontinuities, which violated the basic principle of functions continuity, are the equations presented in NACA TN 2643 [7] paper, based on which software G.R.O.M. [8] was developed. Results of corrections [8, 9] are presented through the set of diagrams for subcases for which the modifications were executed.

The quality of corrections is checked through the sets of conducted “internal” testing, for presented families of wings [8, 10], which shoved that: while changing the geometry, the differences between two tested families of wings, remained in the boundary of one percent; while changing the Mach number the differences between two tested families of wings, remained in the boundary of five percent, excluding the values of aerodynamic derivative of the pitch-dumping moment in respect to rate of pitch for which the differences, as expected, are slightly higher, proving the method of equations modification.

This software [8] is used for calculation of the aerodynamic derivatives values - 1. the lift force in respect to angle of attack; 2. drag force in respect to angle of attack on the square, as static derivatives; 3. roll-dumping moment in respect to rate of roll; 4. pitch-dumping moment in respect to rate of pitch, as dynamic derivatives [8-10].

Results of comparative tests [8, 10] show high level of correspondence for calculated values of the aerodynamic derivatives values with the results obtained by the use of control program Missile Datcom 97, proving use value of G.R.O.M. software for engineering practice when fast assessment of aerodynamic derivatives and load of isolated the wing in supersonic flow is needed, even on very high Mach numbers, bearing on mind that with the increase of Mach number, especially form Mach number 3.2 upward, the influence of the aerodynamic heating increases, becoming more and more dominant.

It would be of interest to prove the usefulness of the developed program for mechanical parts devoted to high speed motion, which can be associated to high Mach numbers.

Nomenclature

A	- aspect ratio, [-]
B	- coefficient $(=(M^2-1)^{1/2})$, [-]
Btu	- British thermal units
b	- wing span, [m]
$c_D^{\alpha^2}$	- aerodynamic derivative of the drag force in respect to angle of attack on the square, [-]
$c_l^{w_x}$	- aerodynamic derivative of the roll dumping moment in respect to rate of roll, [-]
c_L^{α}	- aerodynamic derivative of the lifting force in respect to angle of attack, [-]
C_M	- transition Mach number coefficient, [-]
$c_m^{w_y}$	- aerodynamic derivative of the pitch dumping moment in respect to rate of pitch, [-]
C_M	- transition Mach number coefficient
Cp	- specific heat, [Btu lbm^{-1} °R^{-1}]
c_r	- root chord, [m]
c_1, c_2, c_4	- computational values, [-]
dg	- value of upper integration boundary, [m]
e	- y-wise coordinate of intersection of Mach cone from the root tip of the wing and trailing edge, [m]
F	- empirical factor in transient heating and heat transfer coefficient equations, [1.0]
gg	- value of lower integration boundary, [m]
H	- entropy, [Btu lbm^{-1}]
h_1	- heat transfer coefficient, [lbm (ft^2)$^{-1}$s^{-1}]
h	- y-wise coordinate of intersection of Mach cone from the tip of the wing and trailing edge, [m]
J	- mechanical equivalent of heat, [778 ft lb Btu^{-1}]

j	- y-wise coordinate of intersection of Mach cone emanating from the root tip of the wing, reflected an the end of the wing, and trailing edge, [m]
K	- coefficient $(=\text{ctg}\,\Lambda_{TE}/\,\text{ctg}\,\Lambda_{LE})$, [-]
m	- exponent in friction law, [-]
M	- Mach number, [-]
P	- pressure, [Pa]
Pr	- Prandtl number (= assumed to be 0.7), [-]
q	- heat flux
q_p	- dynamic pressure, [Pa]
r	- radius of body of revolution, [ft]
R	- radius of body, nose or leading edge, [ft]
Re	- Reynolds number $(=\rho V x_{Re}/\mu_1)$, [-]
$Re_{,t}$	- transition Reynolds number $(=\rho V x_{Re}/\mu_1)$, [-]
Rθ	- Reynolds number based on momentum thickness $(=\rho V\theta/\mu_1)$, [-]
S_1	- area of integration, [m^2]
S_K	- wing surface, [m^2]
T	- temperature, [°R]
$\dot{T}_w$	- rate of change of wall temperature, [°R s^{-1}]
V	- velocity, [ft s^{-1}]
w_x	- angular roll velocity, [s^{-1}]
w_y	- angular pitch velocity, [s^{-1}]
x,y,z	- rectangular coordinates
x_1, x_2	- auxiliary rectangular coordinates
x_{Re}	- flow distance, [ft]

Greek Letters

α	- angle of attack, [°]
γ	- ratio of specific heats, [-]
Γ	- spanwise distribution of circulation, [s^{-1}]
Δc_p	- pressure difference coefficient, [-]
θ	- boundary-layer momentum thickness, [ft]
Λ	- sweep of leading edge, [°]
μ_1	- dynamic viscosity, [lbm ft^{-1} s^{-1}]
μ	- Mach cone angle, [°]
ρ	- density, [lbm $(fr^3)^{-1}$]
τ	- wall or skin thickness, [ft]

Subscripts

2	- conditions behind normal shock
L	- local flow conditions in the inviscid shear layer or at the edge of the boundary layer
LE	- leading edge

R	- boundary-layer recovery
st	- stagnation
TE	- trailing edge
w	- wall

Superscripts

*	- evaluate at the reference enthalpy

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