CREEP TRANSITION STRESSES OF A THICK ISOTROPIC SPHERICAL SHELL BY FINITESIMAL DEFORMATION UNDER STEADY-STATE OF TEMPERATURE AND INTERNAL PRESSURE

by

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Creep stresses for a thick isotropic spherical shell by finitesimal deformation under steady-state temperature and internal pressure have been derived by using Seth's transition theory. Results are depicted graphically. It is seen that shell made of incompressible material require higher pressure to yield as compared to shell made of compressible material. For no thermal effects, the result are same as given by Gupta, Bhardwaj, Rana, Hulsurkar, Bhardwaj, and Bailey.

Key words: *spherical shell, pressure, temperature, creep, transition stresses*

Introduction

The problem of elastic-plastic and creep of spherical shells under internal pressure have been discussed by Bailey [1] and the effects of steady-state of temperature on the above problem has been discussed by Derrington [2]. These authors have analyses the problem after making some simplifying assumptions, like infinitesimal deformation and incompressibility of the material. Additionally, these works are based on the use of a yield condition and creep strain laws. Seth [3] transition theory does not require these assumptions. It introduces the concept of generalized strain measure and then finds a solution of governing differential equation near the transition points. It has been shown by Hulsurkar [4], Seth [3, 5], Gupta *et al*. [6-16] that the asymptotic solution through the principal stress-difference give the creep stresses.

Seth [5] has defined the generalized principal strain measure as:

$$e_{ij} = \int_0^{e_{ij}^{\mathrm{A}}} \left[1-2e_{ij}^{\mathrm{A}}\right]^{\frac{n}{2}-1} \mathrm{d}e_{ij}^{\mathrm{A}} = \frac{1}{n}\left[1-(1-2e_{ij}^{\mathrm{A}})^{\frac{n}{2}}\right], \quad (i, j = 1, 2, 3) \tag{1.1}$$

where n is the measure and e_{ij}^{A} are the Almansi finite strain components. In Cartesian framework we can rapidly write down the generalized measure in terms of any other measure.

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In terms of the principal Almansi strain components e_{ij}^{A}, the generalized principal strain components e_{ij}^{M} are:

$$e_{ij}^{\mathrm{M}} = \left\{ \frac{1}{n} \left[1 - (1 - 2e_{ij}^{\mathrm{A}})^{\frac{n}{2}} \right] \right\}^{m} \tag{1.2}$$

For uniaxial case it is given by:

$$e = \left\{ \frac{1}{n} \left[1 - \left(\frac{l_0}{l} \right)^{n} \right] \right\}^{m} \tag{1.3}$$

where m is the irreversibility index and l_0 and l are the initial and strained lengths, respectively.

In this paper, the problem of creep stresses for a thick isotropic spherical shell by finitesimal deformation under steady-state of temperature and internal pressure is investigated by using the concept of generalized strain measures and asymptotic solution through the principal stresses-difference. Results have been presented graphically and discussed.

Governing equations

Consider a spherical shell of internal and external radii a and b, respectively, subjected to internal to uniform internal pressure p and a steady-state temperature Θ_0 applied at the internal surface of the shell. Due to spherical symmetry of the structure, the components of displacements in spherical co-ordinates (r, ϕ, z) are given by Seth [5]:

$$u = r(1-\beta); \quad v = 0; \quad w = 0 \tag{2.1}$$

where β is the position function, depending on $r = (x^2 + y^2 + z^2)^{1/2}$ only.

The generalized components of strain by finitesimal deformation from equation (1.2) are:

$$\begin{aligned} e_{rr} &= \frac{1}{n^m} \left[1 - (\beta + r\beta')^n \right]^m \\ e_{\phi\phi} &= \frac{1}{n^m} \left[1 - \beta^n \right]^m = e_{zz} \\ e_{r\phi} &= e_{\phi z} = e_{zr} = 0 \end{aligned} \tag{2.2}$$

where $\beta' = \mathrm{d}\beta/\mathrm{d}r$.

The thermo-elastic stress-strain relations for isotropic material are given by Parkus [17] and Fung [18]:

$$T_{ij} = \lambda \delta_{ij} I_1 + 2\mu e_{ij} - \xi \Theta \delta_{ij} \tag{2.3}$$

where T_{ij} are the stress components, λ and μ – the Lame's constants, $I_1 = e_{\mathrm{kk}}$ is the first strain invariant, δ_{ij} – the Kronecker's delta, $\xi = \alpha(3\lambda + 3\mu)$, α being the coefficient of thermal expansion, and Θ – the temperature. Further, Θ has to satisfy:

$$\nabla^2 \Theta = 0 \tag{2.4}$$

Using eq. (2.2) in eq. (2.3), one gets:

$$\begin{aligned} T_{rr} &= \frac{\lambda+2\mu}{n^m}\left[1-(r\beta'+\beta)^n\right]^m + \frac{2\lambda}{n^m}\left[1-\beta^n\right]^m - \xi\Theta \\ T_{\phi\phi} &= T_{zz} = \frac{\lambda}{n^m}\left[1-(r\beta'+\beta)^n\right]^m + \frac{2(\lambda+\mu)}{n^m}\left[1-\beta^n\right]^m - \xi\Theta \\ T_{r\phi} &= T_{\phi z} = T_{zr} = 0 \end{aligned} \tag{2.5}$$

The equations of motion are all satisfied except:

$$\frac{\mathrm{d}}{\mathrm{d}r}\; rT_{rr} \;- \frac{2(T_{rr}-T_{\phi\phi})}{r} = 0 \tag{2.6}$$

The temperature satisfying Laplace eq. (2.4) with boundary condition:

$$\begin{aligned} \Theta &= \Theta_0 \quad \text{at} \;\; r = \mathrm{a} \\ \Theta &= 0 \quad \text{at} \;\; r = \mathrm{b} \end{aligned} \tag{2.7}$$

where Θ_0 is constant, is given by:

$$\Theta = \frac{\Theta_0 a}{b-a}\left(\frac{b}{r}-1\right) \tag{2.8}$$

Using eqs. (2.5) and (2.8) in eq. (2.6), we get a non-linear differential equation in β as:

$$\begin{aligned} &P(P+1)^{n-1}\beta\frac{\mathrm{d}P}{\mathrm{d}\beta}\left[1-\beta^n(P+1)^n\right]^{m-1} + P(P+1)^n\left[1-\beta^n(P+1)^n\right]^{m-1} + \\ &+ 2(1-c)P(1-\beta^n)^{m-1} + \frac{n^m c\xi\bar{\Theta}_0}{2\mu r\beta^n mn} - \frac{2c}{\beta^n mn}\left\{\left[1-\beta^n(P+1)^n\right]^m - (1-\beta^n)^m\right\} = 0 \end{aligned} \tag{2.9}$$

where $r\beta' = \beta P$ (P is function of β and β is function of r) and $c = 2\mu/(\lambda + 2\mu)$ is the compressibility factor.

For $m = 1$, which holds good for secondary stage of creep [9]. Equation (2.9) reduces to:

$$\left[\left(P+\frac{2c}{n}\right)(P+1)^n + 2P(1-c) - \frac{2c}{n} + \frac{c\xi\bar{\Theta}_0}{2\mu r\beta^n}\right]\frac{\mathrm{d}\beta}{\mathrm{d}P} + \beta P(P+1)^{n-1} = 0 \tag{2.10}$$

The transition points of β in eq. (2.9) are $P \to 0$, $P \to -1$, and $P \to \pm\infty$.The only critical point of interest is $P \to -1$ and $P \to \pm\infty$. The case of transition point $P \to \pm\infty$ is discussed by Gupta *et al.* [6] which gives the plastic stresses.

The boundary conditions are:

$$T_{\mathrm{rr}} = -p \quad \text{at} \quad r = a \quad \text{and} \quad T_{\mathrm{rr}} = 0 \quad \text{at} \quad r = b \tag{2.11}$$

Asymptotic solution through $P \to -1$

For creep stresses, we define the transition function R through the principal stress difference (see Seth [3, 5], Hulsurkar [4], Gupta *et al*. [6-16]) as:

$$R = T_{rr} - T_{\theta\theta} \equiv \frac{2\mu}{nm}\left\{\left[1-\beta^n(P+1)^n\right]^m - (1-\beta^n)^m\right\} \tag{3.1}$$

Taking the logarithmic differentiating of eq. (3.1) with respect to β, one get:

$$\frac{\mathrm{d}}{\mathrm{d}\beta}(\log R) = mn\beta^{n-1}\frac{(1-\beta^n)^{m-1} - [1-\beta^n(P+1)^n]^{m-1}\left[(P-1)^n + (P+1)^{n-1}\beta\frac{\mathrm{d}P}{\mathrm{d}\beta}\right]}{[1-\beta^n(P+1)^n]^m - (1-\beta^n)^m} \tag{3.2}$$

Substituting the value of dP/dβ from eq. (2.9) in eq. (3.2), one get:

$$\frac{\mathrm{d}}{\mathrm{d}\beta}(\log R) = mn\beta^{n-1}\frac{\left\{\begin{array}{l}(1-\beta^n)^{m-1} + 2(1-c)(1-\beta^n)^{m-1} + \dfrac{c\xi\bar{\theta}_0 n^m}{2\mu r\beta^n mn} - \\ -\dfrac{2c}{mn\beta^n P}\{[(1-\beta^n(P+1)^n]^m - (1-\beta^n)^m]\}\end{array}\right\}}{[1-\beta^n(P+1)^n]^m - (1-\beta^n)^m} \tag{3.3}$$

The asymptotic value of eq. (3.3) as, $P \to -1$, is:

$$\frac{\mathrm{d}}{\mathrm{d}\beta}(\log R) = \frac{mn\beta^{n-1}(3-2c)(1-\beta^n)^{m-1}}{[1-(1-\beta^n)^m]} - \frac{c\xi\bar{\theta}_0 n^m}{2\mu r\beta[1-(1-\beta^n)^m]} + \frac{2c}{\beta} \tag{3.4}$$

Integrating of eqn. (3.4) gives:

$$R = A_0 r^{-2c}[1-(1-\beta^n)^m]^{3-2c}\exp(F_1) \tag{3.5}$$

where A_0 is a constant of integration and $F_1 = (c\xi\bar{\Theta}_0/2\mu)\int \mathrm{d}r/r^2[1-(1-\beta^n)^m]$

The asymptotic value of β as $P \to -1$ is D/r; D being a constant, therefore eq. (3.5) becomes:

$$R = T_{rr} - T_{\phi\phi} \equiv A_0 r^{-2c}[1-(1-D^n r^{-n})^m]^{3-2c}\exp(F_1) \tag{3.6}$$

Using eq. (3.6) in eq. (2.6), and integrating, one gets:

$$T_{rr} = -2A_0\int r^{-2c-1}[1-(1-D^n r^{-n})^m]^{3-2c}\exp(F_1)\mathrm{d}r + A_1 \tag{3.7}$$

where A_1 is a constant of integration, which can be determine by boundary condition.

Using boundary conditions (2.11) in eq. (3.7), one get:

$$A_1 = \{2A_0\int r^{-2c-1}[1-(1-D^n r^{-n})^m]^{3-2c}\exp(F_1)\mathrm{d}r\}_{r=b}$$

$$A_0 = \frac{-p}{2\int_a^b r^{-2c-1}[1-(1-D^n r^{-n})^m]^{3-2c}\exp(F_1)\mathrm{d}r}$$

Substituting the value of A_0 and A_1 in eqs. (3.6) and (3.7), one get:

$$T_{rr} = -p\frac{\int_r^b r^{-2c-1}[1-(1-D^n r^{-n})^m]^{3-2c}\exp(F_1)\mathrm{d}r}{2\int_a^b r^{-2c-1}[1-(1-D^n r^{-n})^m]^{3-2c}\exp(F_1)\mathrm{d}r}$$

$$T_{\phi\phi} = T_{zz} = T_{rr} + \frac{pr^{-2c}[1-(1-D^n r^{-n})^m]^{3-2c}\exp(F_1)}{2\int_a^b r^{-2c-1}[1-(1-D^n r^{-n})^m]^{3-2c}\exp(F_1)\mathrm{d}r} \tag{3.8}$$

Equation (3.7) corresponds to only one stage of creep. If all the three stages of creep to be taken in account, we shall add the incremental values [3, 4, 19] of $T_{rr} - T_{\phi\phi}$. Thus from eq. (3.7), we have:

$$T_{rr} - T_{\phi\phi} = A_0 r^{-6c-3} \prod_{m,n} [1-(1-D^n r^{-n})^m]^{3-2c} \exp(F_1) \tag{3.9}$$

where m and n having three sets of values each corresponding to one stage of creep and the transitional creep stresses given by:

$$T_{rr} = -p \frac{\int_r^b r^{-6c-3} \prod_{m,n} [1-(1-D^n r^{-n})^m]^{3-2c} \exp(F_1) \mathrm{d}r}{\int_a^b r^{-6c-3} \prod_{m,n} [1-(1-D^n r^{-n})^m]^{3-2c} \exp(F_1) \mathrm{d}r}$$

$$T_{\phi\phi} = T_{zz} = T_{rr} + \frac{p r^{-6c-3} \prod_{m,n} [1-(1-D^n r^{-n})^m]^{3-2c} \exp(F_1) \mathrm{d}r}{\int_a^b r^{-6c-3} \prod_{m,n} [1-(1-D^n r^{-n})^m]^{3-2c} \exp(F_1) \mathrm{d}r} \tag{3.10}$$

where $F_1 = \dfrac{c\xi \bar{\Theta}_0}{2\mu} \displaystyle\int \frac{\mathrm{d}r}{\prod_{m,n} r^2 [1-(1-D^n r^{-n})^m]}$

Shell under steady-state of creep

Transitional creep stresses for secondary state of creep are obtained by putting $m = 1$ in eq. (3.8), one gets:

$$T_{rr} = -p \frac{\int_r^b r^{-3n+2c(n-1)-1} \exp\ F_1\ \mathrm{d}r}{\int_a^b r^{-3n+2c(n-1)-1} \exp(F_1) \mathrm{d}r}$$

$$T_{\phi\phi} = T_{zz} = T_{rr} + \frac{p r^{-3n+2c(n-1)-1} \exp(F_1)}{2\int_a^b r^{-3n+2c(n-1)-1} \exp(F_1) \mathrm{d}r} \tag{4.1}$$

where $F_1 = \alpha E(3 - 2c)\bar{\Theta}_0 r^{n-1}/Y(n-1)D^n$, α is the coefficient of thermal expansion, E – the Young's modulus, and Y – the yield in tension.

It is found that the value $|T_{rr} - T_{\phi\phi}|$ is maximum at $r = a$, therefore yielding of the shell starts at the internal surface and eq. (4.1) reduces to:

$$\left|T_{rr} - T_{\phi\phi}\right| = \frac{p a^{-3n+2c(n-1)} \exp(F_1)}{2\int_a^b r^{-3n+2c(n-1)-1} \exp(F_1) \mathrm{d}r} \equiv Y_1 \tag{4.2}$$

where Y_1 is the yields stress and $F_1 = \alpha E(3 - 2c)\bar{\Theta}_0 a^{n-1}/Y(n-1)D^n$.

For incompressible material *i. e.* ($c \to 0$), eqs. (4.1) and (4.2) reduces to :

$$T_{rr} = -p\frac{\int_r^b r^{-3n-1}\exp(F_1)\mathrm{d}r}{\int_a^b r^{-3n-1}\exp(F_1)\mathrm{d}r}$$

$$T_{\phi\phi} = T_{zz} = T_{rr} + p\frac{r^{-3n-1}\exp(F_1)}{2\int_a^b r^{-3n-1}\exp(F_1)\mathrm{d}r} \tag{4.3}$$

and

$$Y_1 = p\frac{a^{-3n}\exp(F_1)}{2\int_a^b r^{-3n-1}\exp(F_1)\mathrm{d}r}$$

where $F_1 = 3\alpha E\bar{\Theta}_0 a^{n-1}/Y(n-1)D^n$

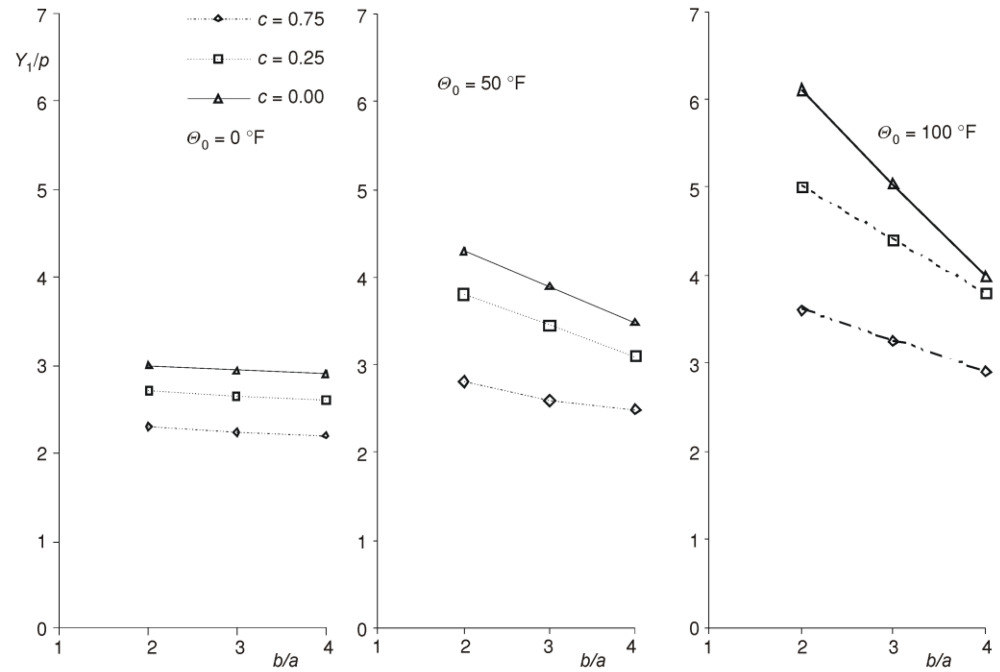

Figure 1. Yielding ratios Y_1/p for various shell thickness ratios at different temperature for n = 2

As a particular case, transitional creep stresses for a spherical shell under internal pressure are obtained by putting $\Theta_0 = 0$ in eqs. (4.1) and (4.2), one gets:

$$T_{rr} = -p\frac{\left(\frac{b}{r}\right)^{3n-2c(n-1)} - 1}{\left(\frac{b}{a}\right)^{3n-2c(n-1)} - 1}$$

$$T_{\phi\phi} = T_{zz} = p\frac{\frac{1}{2}[n(3-2c)-2(1-c)]\left(\frac{b}{r}\right)^{3n-2c(n-1)}-1}{\left(\frac{b}{a}\right)^{-3n-2c(n-1)}-1} \tag{4.4}$$

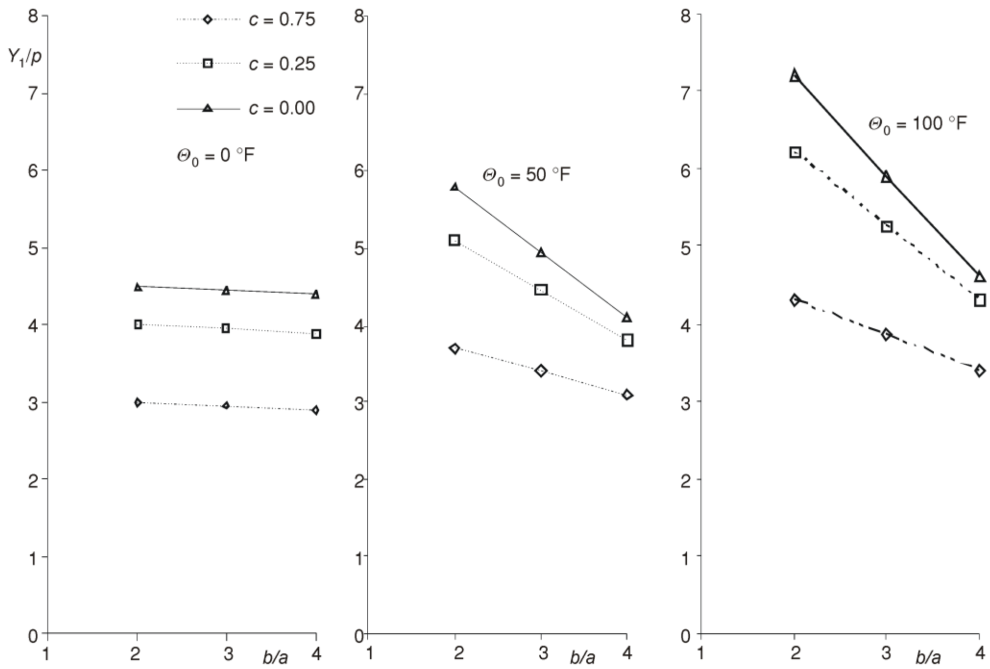

Figure 2. Yielding ratios Y_1/p for various shell thickness ratios at different temperature for n =3

For incompressible material $c \to 0$, eq. (4.4) become:

$$T_{rr} = -p\frac{\left(\frac{b}{r}\right)^{3n}-1}{\left(\frac{b}{a}\right)^{3n}-1}$$

$$T_{\phi\phi} = T_{zz} = p\frac{\frac{1}{2}(3n-2)\left(\frac{b}{r}\right)^{3n}-1}{\left(\frac{b}{a}\right)^{-3n}-1} \tag{4.5}$$

Equations (4.4) and (4.5) are same as obtained by Bailey [1], Hulsurkar [4], Gupta, *et al.* [20], and Bhardwaj [21].

Results and discussion

To show the effect of combined pressure and temperature on a shell, this problem has been solved by using Simpson's rule for integration in eqs. (4.1), (4.2), and (4.3). For mild steel we take in various values as given by [22]; $Y = 2.1 \cdot 10^7$ kg/ms^2, $E = 2.1 \cdot 10^{10}$ kg/ms^2, $\alpha = 2.93 \cdot 10^{-10}$ K, and $c = 0.0$, 0.25, and 0.75, and $\Theta_0 = 0$ K, 283.15 K, and 310.92 K.

In figs. 1 and 2, curves have been drawn between yield Y_1/p and different shell thickness ratios for $n = 2$ and 3, respectively. When heating effects are absent, it is seen that yielding of the thinner as well as thicker shells occurs generally at the same pressure, but with increasing temperature a thinner shell yields at higher pressure as compared to thicker shell. This yielding pressure goes on increasing with the increases in temperature and measure n. Shells made of incompressible material require higher pressure to yield as compared to shell made of compressible material.

Conclusions

It is seen that shell made of incompressible material require higher pressure to yield as compared to shell made of compressible material without thermal effects. The result are same as given by Gupta, Bhardwaj, Rana, and Hulsurkar [1, 3], Bhardwaj [4], Bailey [2], and Johnson *et al.* [23].

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Nomenclature

A_1 – constants of integration, [–]
a, b – internal and external radii of the circular cylinder, [m]
c – compressibility factor, [–]
p – internal pressure, [Pa]
T_{ij}, e_{ij} – stress strain rate tensors, [kgm^{-1}s^{-2}]
u, v, w – displacement components, [m]
Y – yield stress, [kgm^{-1}s^{-2}]

Greek symbols

Θ – temperature, [K]

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