THE INFLUENCE OF THE MAGNETIC FIELD ON THE IONIZED GAS FLOW ADJACENT TO THE POROUS WALL

by

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This paper studies the influence of the magnetic field on the planar laminar steady flow of the ionized gas in the boundary layer. The present outer magnetic field is homogeneous and perpendicular to the body within the fluid. The gas of the same physical characteristics as the gas in the main flow is injected (ejected) through the contour of the body. The governing boundary layer equations for different forms of the electroconductivity variation law are transformed, brought to a generalized form and solved numerically in a four-parametric approximation. It has been determined that the magnetic field, through the magnetic parameter, has a great influence on certain quantities and characteristics of the boundary layer. It has also been shown that this parameter has an especially significant influence on the non-dimensional friction function, and hence the boundary layer separation point.

Key words: *ionized gas, boundary layer, porous wall, magnetic field, magnetic parameter, electroconductivity*

Introduction

The main objective of this investigation is to obtain generalized equations of the ionized gas flow adjacent the porous wall by application of the general similarity method, and to perceive the influence of the magnetic field on certain physical quantities and characteristic functions of the boundary layer.

The general similarity method was first used by Loitsianskii [1, 2]. In its original version, it was successfully used for problems of planar flow in the dissociated gas boundary layer [3, 4]. The method was then modified by Saljnikov [5]. Saljnikov's version of this method was applied in the MHD boundary layer theory for solution of different problems of bodies within electroconductive incompressible fluids. In the papers [6-8] flow problems in the case when the outer magnetic field is normal to the body contour were solved. In these

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q-flow problems, the fluid electroconductivity was either constant [6, 7] or different forms of electroconductivity variation laws were used [8]. The importance of electroconductivity variation was pointed out both for a practical application and theoretical and methodological aspects.

The paper [1] deals with the ionized gas flow in the boundary layer adjacent a flat plate, in the presence of magnetic field. The paper [9] studies the ionized gas flow in the boundary layer adjacent a non-porous body of arbitrary shape, while the papers [10, 11] study the same flow but adjacent a porous body of arbitrary shape. For both cases, different electroconductivity variation laws are used. The influence of electroconductivity variation on ionized gas flow adjacent the porous wall was also investigated [12].

The general similarity method in its both versions is based on the usage of a momentum equation and introduction of corresponding sets of parameters [4] that represent the so-called similarity parameters. In this paper, Saljnikov's version of the general similarity method is applied.

Model formulation

The results obtained in the boundary layer theory have a wide technical application. Boundary layer management in the technical practice can be achieved in different ways [13]. A body of porous contour, through which a fluid of the same characteristics as in the main current is injected ($v_w(x) > 0$) or ejected ($v_w(x) < 0$) with the velocity v_w (figure 1), is often used. One of successful ways of boundary layer management is by using a transversal magnetic field.

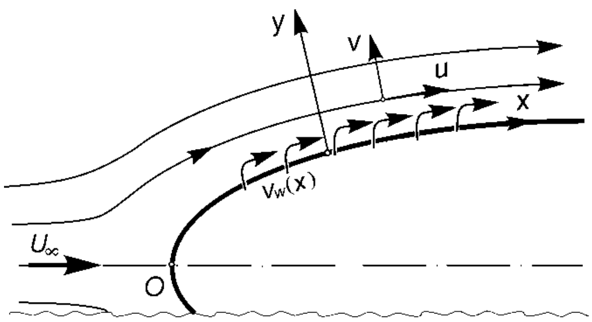

Figure 1. The gas flow adjacent to the body

If the ionized gas (air) flows in the magnetic field of the power $B_m = B_m(x)$, an electric flow is formed in the gas. The electric flow generates Lorentz force and Joule's heat. Due to these to effects, new terms that cannot be found in the equations for homogeneous unionized gas [1] appear in the ionized gas boundary layer equations.

One of the most important properties of the ionized gas is its electroconductivity σ. In order to perceive a full influence of the transversal magnetic field on the boundary layer characteristics, in the papers [9, 14, 15] three characteristic variation laws of σ were used:

$$\sigma = \sigma(x) \tag{1}$$

$$\sigma = \sigma_0 \left(1 - \frac{u}{u_\mathrm{e}}\right) \tag{2}$$

$$\sigma = \sigma_0 \frac{\nu_0}{u_\mathrm{e}^2} \frac{\partial u}{\partial y}, \qquad \sigma_0,\, \nu_0 = \mathrm{const.} \tag{3}$$

Therefore, the electroconductivity depends only on the longitudinal coordinate x (1), it is a function of the velocity ratio (2) and it is a function of the longitudinal velocity gradient (3).

In the case of the ionized gas flow in the magnetic field adjacent the porous wall under the conditions of equilibrium ionization, the governing equation system of the steady laminar planar boundary layer [1, 13] is:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{4}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_\mathrm{e} u_\mathrm{e} \frac{\mathrm{d}u_\mathrm{e}}{\mathrm{d}x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \underline{- \sigma B_\mathrm{m}^2 u} \tag{5}$$

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = -u\, \rho_\mathrm{e} u_\mathrm{e} \frac{\mathrm{d}u_\mathrm{e}}{\mathrm{d}x} + \mu \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(\frac{\mu}{\mathrm{Pr}} \frac{\partial h}{\partial y}\right) \underline{+ \sigma B_\mathrm{m}^2 u^2} \tag{6}$$

where (4) is the continuity equation, (5) dynamic and (6) energy equation.

The boundary conditions are as follows:

$$\begin{aligned} &u = 0, \quad v = v_\mathrm{w}(x), \quad h = h_\mathrm{w} \quad &\text{for} \quad y = 0 \\ &u \to u_\mathrm{e}(x), \quad h \to h_\mathrm{e}(x) \quad &\text{for} \quad y \to \infty \end{aligned} \tag{7}$$

The subscript ″e″ stands for physical quantities at the outer edge of the boundary layer, and the subscript ″w″ denotes the quantities on the wall of the body within the fluid.

The terms $-\sigma B_\mathrm{m}^2 u$ and $\sigma B_\mathrm{m}^2 u^2$ are determined with the order of Lorentz force and Joule's heat. The system (4) – (6) applies for the laws (2) and (3). If the electroconductivity changes in accordance with the law (1), the underlined terms in (5) and (6) are:

$$\underline{+ \sigma B_\mathrm{m}^2 (u_\mathrm{e} - u)} \tag{5'}$$

$$\underline{+ \sigma B_\mathrm{m}^2 (u^2 - u u_\mathrm{e})} \tag{6'}$$

Transformations

The general similarity method is based on the usage of a momentum equation. In order for this equation to have the simplest form instead of physical coordinates, new variables are introduced:

$$s(x) = \frac{1}{\rho_0\,\mu_0}\int_0^x \rho_w \mu_w \,\mathrm{d}x, \qquad z(x,y) = \frac{1}{\rho_0}\int_0^y \rho \, dy \tag{8}$$

as well as the stream function $\psi(s,\ z)$ by the relations:

$$u = \frac{\partial \psi}{\partial z}, \qquad \tilde{v} = \frac{\rho_0 \mu_0}{\rho_w \mu_w}\left(u\frac{\partial z}{\partial x} + v\frac{\rho}{\rho_0}\right) = -\frac{\partial \psi}{\partial s} \tag{9}$$

The quantities ρ_0 and μ_0 denote the known values of density and dynamic viscosity at a certain point of the boundary layer.

The boundary condition for the transversal velocity in (7) is characteristic for the porous wall. In order for the governing equations (4)-(6) to become generalized, the boundary conditions (7) should be brought to a form which applies to the non-porous structure.

Therefore, a new stream function $\psi^*(s,z)$ is introduced as:

$$\psi(s,z) = \psi_w(s) + \psi^*(s,z), \qquad \psi^*(s,0) = 0 \tag{10}$$

where $\psi(s,0) = \psi_w(s)$ is a function of the flow adjacent to the body within the fluid.

Another transformation of variables is needed. A new non-dimensional stream function Φ and non-dimensional enthalpy $\bar{h}$ are introduced by the following transformations:

$$s = s, \qquad \eta(s,z) = \frac{\sqrt{u_e^b}}{K(s)}z, \qquad \psi^*(s,z) = \frac{u_e}{\sqrt{u_e^b}}K(s)\Phi\left(\eta,\ \kappa,\ (f_k),\ (g_k),\ (\Lambda_k)\right)$$
$$h(s,z) = h_1 \cdot \bar{h}\left(\eta,\ \kappa,\ (f_k),\ (g_k),\ (\Lambda_k)\right) \tag{11}$$

$$h_1 = h_e + \frac{u_e^2}{2} = \text{const.}, \quad K(s) = \sqrt{a\nu_0 \int_0^s u_e^{b-1} ds}, \quad a, b = \text{const.} \tag{12}$$

In the relations (11) $\kappa = f_0$ is a local parameter of the ionized gas compressibility, (f_k) stands for a set of parameters of Loitsianskii's type [1], (g_k) is a set of magnetic parameters, and (Λ_k) denotes a set of porous wall parameters [10]. The introduced sets of parameters represent new independent variables (instead of the variable *s*) and for electroconductivity variation laws (1) and (2), they are defined as:

$$\kappa = f_0(s) = \frac{u_e^2}{2h_1} \tag{13}$$

$$f_k(s) = u_e^{k-1} u_e^{(k)} Z^{**k}, \quad k = 1, 2, 3, \ldots \tag{14}$$

$$g_k(s) = u_e^{k-1} N_\sigma^{(k-1)} Z^{**k} \tag{15}$$

$$\Lambda_{\mathrm{k}}(s) = -u_{\mathrm{e}}^{\mathrm{k}-1}\left(\frac{V_w}{\sqrt{\nu_0}}\right)^{(\mathrm{k}-1)} \cdot Z^{**\,\mathrm{k}-1/2} \tag{16}$$

where:

$$Z^{**} = \frac{\Delta^{**2}}{\nu_0}, \quad N_\sigma = \frac{\rho_0\,\mu_0}{\rho_\mathrm{w}\,\mu_\mathrm{w}}\overline{N}$$

$$\Delta^*(s) = \int_0^\infty \left(\frac{\rho_\mathrm{e}}{\rho} - \frac{u}{u_\mathrm{e}}\right)\mathrm{d}z, \quad \Delta^{**}(s) = \int_0^\infty \frac{u}{u_\mathrm{e}}\left(1 - \frac{u}{u_\mathrm{e}}\right)\mathrm{d}z$$

$$\tau_\mathrm{w}(s) = \left(\mu\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\rho_\mathrm{w}\,\mu_\mathrm{w}}{\rho_0}\frac{u_\mathrm{e}}{\Delta^{**}}\zeta; \qquad \zeta(s) = \left[\frac{\partial(u/u_\mathrm{e})}{\partial(z/\Delta^{**})}\right]_{z=0}, \qquad H = \frac{\Delta^*}{\Delta^{**}}$$

$$(1) \quad H_1 = \frac{\Delta^*_1}{\Delta^{**}}, \quad \overline{N} = \frac{\sigma B_\mathrm{m}^2}{\rho_\mathrm{e}}, \quad \Delta_1^*(s) = \int_0^\infty \frac{\rho_\mathrm{e}}{\rho}\left(1 - \frac{u}{u_e}\right)\mathrm{d}z \tag{17}$$

$$F_\mathrm{mp} = 2\left[\zeta - (2+H)f_1\right] - 2g_1H_1 - 2\Lambda_1$$

$$(2) \quad H_1 = \frac{\Delta^{**}_1}{\Delta^{**}}, \quad \overline{N} = \frac{\sigma_0 B_\mathrm{m}^2}{\rho_\mathrm{e}}, \quad \Delta_1^{**}(s) = \int_0^\infty \frac{u}{u_\mathrm{e}}\left(1 - \frac{u}{u_\mathrm{e}}\right)\frac{\rho_\mathrm{e}}{\rho}\mathrm{d}z$$

$$F_\mathrm{mp} = 2\left[\zeta - (2+H)f_1\right] + 2g_1H_1 - 2\Lambda_1$$

For the law (3), only the set of magnetic parameters (15) is different:

$$g_\mathrm{k}(s) = u_\mathrm{e}^{\mathrm{k}-2}\,N_\sigma^{(\mathrm{k}-1)}\nu_0^{1/2}\,Z^{**\,\mathrm{k}-1/2}, \qquad \mathrm{k} = 1, 2, 3, \ldots \tag{15'}$$

here:

$$N_\sigma = \frac{\rho_0\,\mu_0}{\rho_\mathrm{w}\,\mu_\mathrm{w}}\frac{\sigma_0 B_\mathrm{m}^2}{\rho_0}, \quad F_\mathrm{mp} = 2\left[\zeta - (2+H)f_1\right] + g_1 - 2\Lambda_1 \tag{17'}$$

Each of the sets of parameters satisfies the corresponding recurrent simple differential equation:

$$\frac{u_\mathrm{e}}{u'_\mathrm{e}}f_1\frac{\mathrm{d}\kappa}{\mathrm{d}s} = 2\,\kappa\,f_1 = \theta_0 \tag{18}$$

$$\frac{u_\mathrm{e}}{u'_\mathrm{e}}f_1\frac{\mathrm{d}f_\mathrm{k}}{\mathrm{d}s} = \left[(\mathrm{k}-1)f_1 + \mathrm{k}F_\mathrm{mp}\right]f_\mathrm{k} + f_{\mathrm{k}+1} = \theta_\mathrm{k}, \quad \mathrm{k} = 1, 2, 3, \ldots \tag{19}$$

$$\frac{u_\mathrm{e}}{u'_\mathrm{e}}f_1\frac{\mathrm{d}g_\mathrm{k}}{\mathrm{d}s} = \left[(\mathrm{k}-2)f_1 + \left(\mathrm{k} - \frac{1}{2}\right)F_\mathrm{mp}\right]g_\mathrm{k} + g_{\mathrm{k}+1} = \gamma_\mathrm{k} \tag{20}$$

$$\frac{u_\mathrm{e}}{u'_\mathrm{e}}f_1\frac{\mathrm{d}\Lambda_\mathrm{k}}{\mathrm{d}s} = \left\{(\mathrm{k}-1)f_1 + \left[(2\mathrm{k}-1)/2\right]F_\mathrm{mp}\right\}\Lambda_\mathrm{k} + \Lambda_{\mathrm{k}+1} = \chi_\mathrm{k} \tag{21}$$

Applying the similarity transformations (11) and (13)-(16), *i. e.* (15'), a general mathematical model of the ionized gas flow adjacent the porous wall for the law (1) is obtained:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f_1}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]+\underline{\frac{g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)}+$$

$$+\frac{\Lambda_1}{B}\frac{\partial^2\Phi}{\partial\eta^2}=\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f_k}-\frac{\partial\Phi}{\partial f_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)+\sum_{k=1}^{\infty}\gamma_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial g_k}-\frac{\partial\Phi}{\partial g_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)+\right.$$
$$\left.+\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial\Lambda_k}-\frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)\right] \tag{22}$$

$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{\partial\bar h}{\partial\eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial\bar h}{\partial\eta}-\frac{2\kappa f_1}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2-$$
$$\underline{-\frac{2\kappa g_1}{B}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}}+\frac{\Lambda_1}{B}\frac{\partial\bar h}{\partial\eta}=\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar h}{\partial f_k}-\frac{\partial\Phi}{\partial f_k}\frac{\partial\bar h}{\partial\eta}\right)+\right. \tag{23}$$
$$\left.+\sum_{k=1}^{\infty}\gamma_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar h}{\partial g_k}-\frac{\partial\Phi}{\partial g_k}\frac{\partial\bar h}{\partial\eta}\right)+\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar h}{\partial\Lambda_k}-\frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial\bar h}{\partial\eta}\right)\right]$$

In the equations (22) and (23), the non-dimensional boundary layer characteristic B is determined with the expression:

$$B(s)=\int_0^{\infty}\frac{\partial\Phi}{\partial\eta}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\mathrm{d}\eta$$

For the non-dimensional function Q [11] and for the density ratio ρ_e/ρ [3], the following approximate formulae are used:

$$Q=Q(\bar h)=\sqrt[3]{\frac{\bar h_w}{\bar h}},\qquad \frac{\rho_e}{\rho}\approx\frac{\bar h}{1-\kappa} \tag{24}$$

Using the law (2), the underlined terms in (22) and (23) are:

$$\underline{-\frac{g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}} \tag{22'}$$

$$\underline{+\frac{2\kappa g_1}{B}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\left(\frac{\partial\Phi}{\partial\eta}\right)^2} \tag{23'}$$

while for the law (3), they are as follows:

$$\underline{-\frac{g_1}{B}\frac{\partial^2\Phi}{\partial\eta^2}\frac{\partial\Phi}{\partial\eta}} \qquad (22'')$$

$$\underline{+\frac{2\kappa g_1}{B}\frac{\partial^2\Phi}{\partial\eta^2}\left(\frac{\partial\Phi}{\partial\eta}\right)^2} \qquad (23'')$$

The transformed boundary conditions are:

$$\begin{aligned} &\Phi=\frac{\partial\Phi}{\partial\eta}=0, \qquad \bar{h}=\bar{h}_\mathrm{w}=\mathrm{const.} \quad \text{for} \quad \eta=0 \\ &\frac{\partial\Phi}{\partial\eta}\to 1, \qquad \bar{h}\to\bar{h}_\mathrm{e}=1-\kappa \quad \text{for} \quad \eta\to\infty \end{aligned} \qquad (25)$$

Solution of thus obtained equation system is possible only when there are a relatively small number of parameters. Assuming that:

$$\begin{aligned} &\kappa=f_0\neq 0, \quad f_1=f\neq 0, \quad g_1=g\neq 0, \quad \Lambda_1=\Lambda\neq 0 \\ &f_2=f_3=\ldots=0, \quad g_2=g_3=\ldots=0, \quad \Lambda_2=\Lambda_3=\ldots=0 \end{aligned} \qquad (26)$$

and that: $\partial/\partial\kappa=0,\ \partial/\partial g_1=0,\ \partial/\partial\Lambda_1=0$, the transformed equation system is significantly simplified. Furthermore, the order of the differential equations is decreased by the change:

$$\frac{u}{u_\mathrm{e}}=\frac{\partial\Phi}{\partial\eta}=\varphi=\varphi(\eta,\ \kappa,\ f,\ g,\ \Lambda) \qquad (27)$$

therefore, (22) and (23) in a four-parametric three times localized approximation take the following form:

$$\begin{aligned} &\frac{\partial}{\partial\eta}\left(Q\frac{\partial\varphi}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\varphi}{\partial\eta}+\frac{f}{B^2}\left(\frac{\rho_\mathrm{e}}{\rho}-\varphi^2\right)+\underline{\frac{g}{B^2}\frac{\rho_\mathrm{e}}{\rho}(1-\varphi)+} \\ &\qquad +\frac{\Lambda}{B}\frac{\partial\varphi}{\partial\eta}=\frac{F_\mathrm{mp}f}{B^2}\left(\varphi\frac{\partial\varphi}{\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial\varphi}{\partial\eta}\right) \end{aligned} \qquad (28)$$

$$\begin{aligned} &\frac{\partial}{\partial\eta}\left(\frac{Q}{\mathrm{Pr}}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f}{B^2}\frac{\rho_\mathrm{e}}{\rho}\varphi+2\kappa Q\left(\frac{\partial\varphi}{\partial\eta}\right)^2- \\ &\qquad \underline{-\frac{2\kappa g}{B^2}\frac{\rho_\mathrm{e}}{\rho}(1-\varphi)\varphi}+\frac{\Lambda}{B}\frac{\partial\bar{h}}{\partial\eta}=\frac{F_\mathrm{mp}f}{B^2}\left(\varphi\frac{\partial\bar{h}}{\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial\bar{h}}{\partial\eta}\right) \end{aligned} \qquad (29)$$

The following terms are different:

$$\underline{-\frac{g}{B^2}\frac{\rho_\mathrm{e}}{\rho}(1-\varphi)\varphi} \qquad (28')$$

$$+\frac{2\kappa g}{B^2}\frac{\rho_e}{\rho}(1-\varphi)\varphi^2 \tag{29'}$$

$$-\frac{g}{B}\frac{\partial\varphi}{\partial\eta}\varphi \tag{28''}$$

$$+\frac{2\kappa g}{B}\frac{\partial\varphi}{\partial\eta}\varphi^2 \tag{29''}$$

The boundary conditions (25) remain unchanged.
In this system, the subscript 1 is left out in the first parameters.

Numerical procedure

The obtained system is numerically solved using the ″passage method″. When this method is applied, some derivatives of the variables are replaced with the corresponding finite differences ratio, and then the solution of partial differential equations comes down to solution of a system of linear algebraic equations:

$$a^i_{M,K+1}\varphi^i_{M-1,K+1}-2b^i_{M,K+1}\varphi^i_{M,K+1}+c^i_{M,K+1}\varphi^i_{M+1,K+1}=g^i_{M,K+1} \tag{30}$$

$$\begin{gathered}a^j_{M,K+1}\bar{h}^j_{M-1,K+1}-2b^j_{M,K+1}\bar{h}^j_{M,K+1}+c^j_{M,K+1}\bar{h}^j_{M+1,K+1}=g^j_{M,K+1}\\ M=2,3,\dots,N-1;\quad K=0,1,2,\dots;\quad i,\, j=0,1,2,\dots\end{gathered} \tag{31}$$

$$\begin{aligned}&\Phi^i_{1,K+1}=\varphi^i_{1,K+1}=0,\quad \bar{h}^j_{1,K+1}=\bar{h}_w=\text{const.}\quad \text{for}\quad M=1\\ &\varphi^i_{N,K+1}=1,\qquad\qquad \bar{h}^j_{N,K+1}=1-\kappa\qquad \text{for}\quad M=N\end{aligned} \tag{32}$$

Based on the algebraic equations (30)-(32), the following formulae are obtained:

$$\varphi^i_{N,K+1}=1 \tag{33}$$

$$\varphi^i_{M,K+1}=K^i_{M,K+1}+L^i_{M,K+1}\varphi^i_{M+1,K+1} \tag{34}$$

$$\varphi^i_{1,K+1}=0 \tag{35}$$

$$\bar{h}^j_{N,K+1}=1-\kappa \tag{36}$$

$$\bar{h}^j_{M,K+1}=K^j_{M,K+1}+L^j_{M,K+1}\bar{h}^j_{M+1,K+1} \tag{37}$$

$$\begin{gathered}\bar{h}^j_{1,K+1}=\bar{h}_w=\text{const.}\\ M=N-1,\, N-2,\,\dots,3,\,2\\ i,\, j=1,\,2,\,3,\dots\end{gathered} \tag{38}$$

Based on these formulae, the values of the functions φ and $\bar{h}$ can be calculated at discrete points, in the direction of decrease of the index *M*. Here, *K* and *L* are passage coefficients that depend on the coefficients *a*, *b*, *c* and *g*. These coefficients vary depending on the applied electroconductivity variation law (1)-(3).

For a concrete solution of the generalized equation system (28)-(29), *i. e.* the corresponding system expressed by finite differences (33)-(38), a program in FORTRAN has been written. Investigations performed using thermo-dynamic tables for air have undoubtedly shown that Pr number for ionized gas can be considered approximately constant. In this paper, equations are solved for $\mathrm{Pr} = 0.712$. The optimal values for the constants *a* and *b* are: $a = 0.4408$, $b = 5.7140$. The accepted values for characteristic functions B and F_{mp} at a zero iteration [3] are $B^0_{K+1} = 0.469$ and $F^0_{\mathrm{mp},K+1} = 0.4411$.

Results and discussion

In order to perceive the influence of the magnetic field on the characteristics of the boundary layer, the equation system (28)-(29) is solved for different, in advance given, values of the compressibility parameter $\kappa = f_0$, porosity parameter Λ and magnetic parameter *g*. Based on the obtained results for all three applied electroconductivity change laws, diagrams (figures 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11) are drawn and some interesting conclusions on the influence of certain quantities on the solution of the flow problem are made. Especially important are the following conclusions concerning the influence of the magnetic field:

- The magnetic field has no significant influence on distribution of non-dimensional flow velocity u/u_e (figures 2 and 3). At different cross-sections of the boundary layer, independently on the electroconductivity variation, this velocity converges towards unity very fast;
- A change in the magnetic field, *i. e.* in the magnetic parameter has an insignificant influence on distribution of the non-dimensional enthalpy (figures 4 and 5) in the boundary layer (for all three used electroconductivity variation laws);
- The magnetic parameter *g*, and hence the magnetic field, have a great influence on the boundary layer quantities B, ζ and F_{mp} (figures 6, 7, 8, 9, 10, and 11). An increase in the value of magnetic parameter causes increases in these quantities (laws (2) and (3)), while for the law (1) an opposite effect is noticed;
- The influences of the magnetic field on the non-dimensional friction function ζ, and consequently on the boundary layer separation point is particularly pointed out. Figures 8 and 9 show that when the value of the magnetic field is increased (laws (2) and (3)), the friction increases and the boundary layer separation point moves down the fluid flow. Therefore, the influence of the magnetic field is positive because it postpones the separation of the boundary layer. When the law (1) is applied, the effect of the magnetic field is negative, and

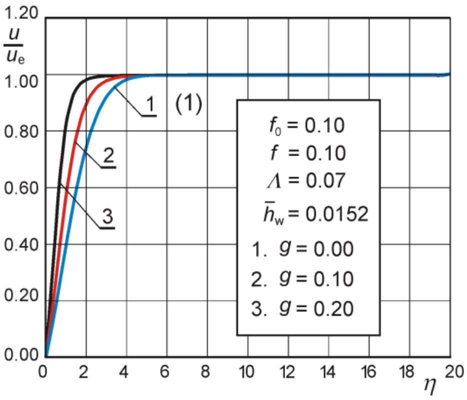

Figure 2. Distribution of the non-dimensional velocity for the law (1)

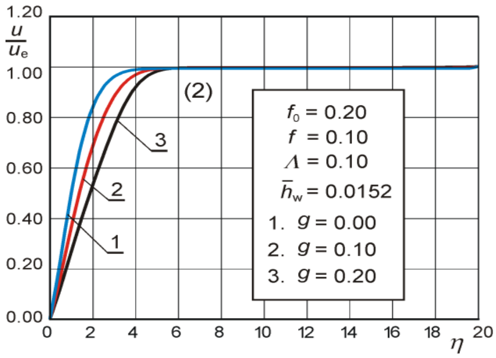

Figure 3. Distribution of the non-dimensional velocity for the law (2)

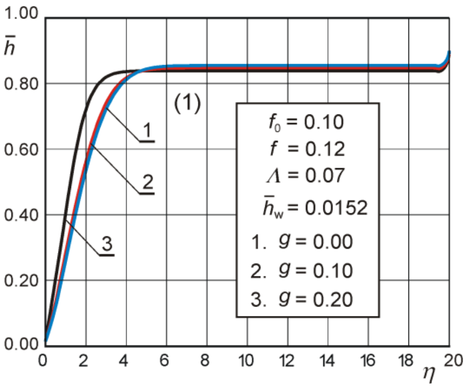

Figure 4. Diagram of the non-dimensional enthalpy for the law (1)

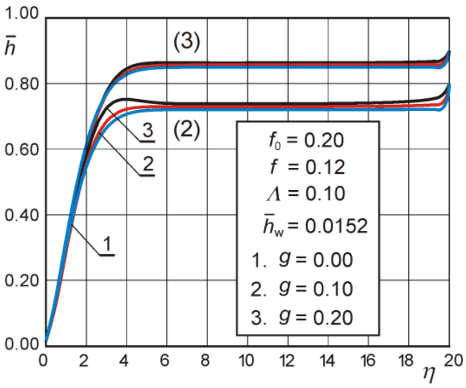

Figure 5. Diagram of the non-dimensional enthalpy for the laws (2) and (3)

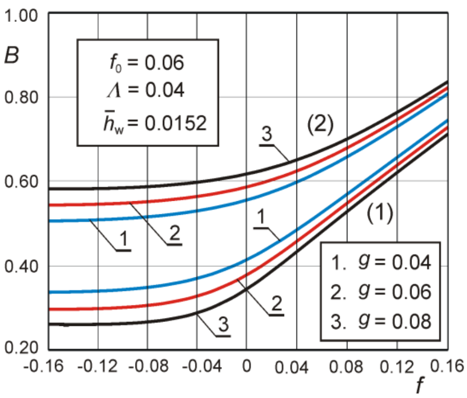

Figure 6. The characteristic of boundary layer *B* for the laws (1) and (2)

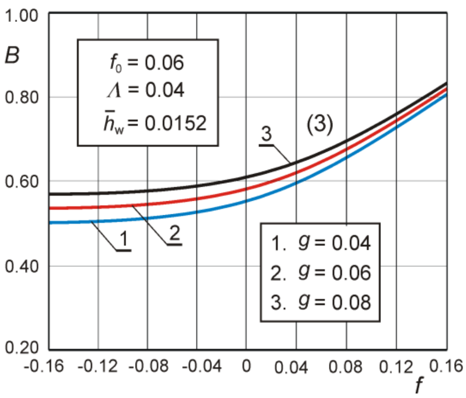

Figure 7. The characteristic of boundary layer *B* (3)

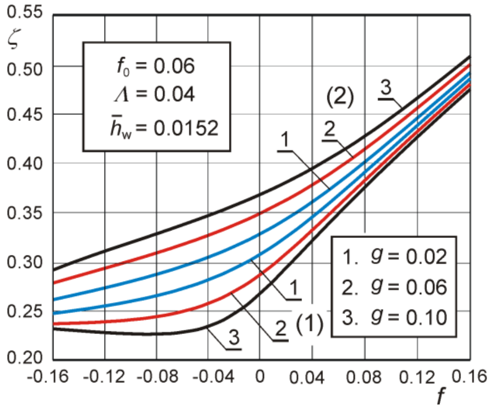

Figure 8. The non-dimensional friction function for the laws (1) and (2)

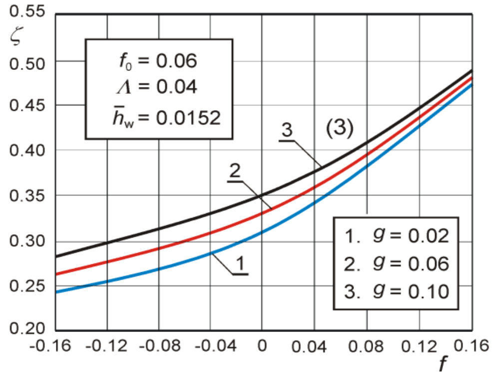

Figure 9. The non-dimensional friction function (3)

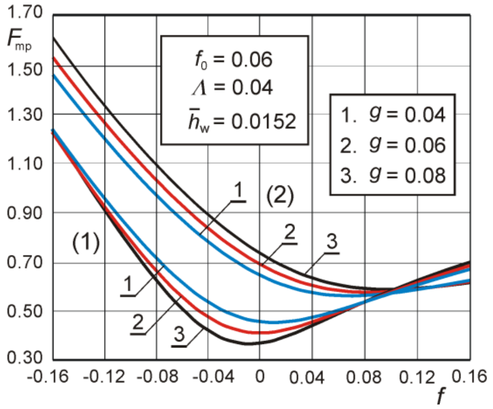

Figure 10. Distribution of the non-dimensional function F_{mp} for the laws (1) and (2)

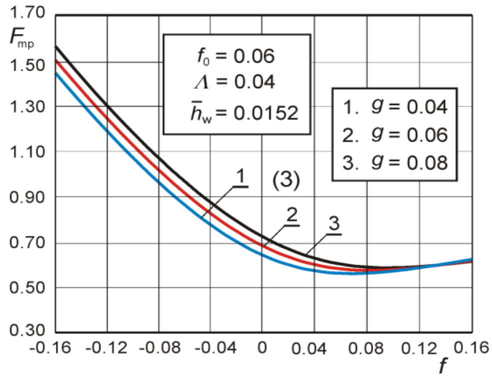

Figure 11. Distribution of the non-dimensional function F_{mp} (3)

– The magnetic parameter in the conffuser region of the boundary layer, for all three electroconductivity variation laws, has a relatively small influence on the characteristic function F_{mp}. In the diffuser region, however, this influence increases, laws (2) and (3), while for the law (1) the influence is the greatest at the beginning of the region and then it decreases (figures 10 and 11).

Conclusions

This paper investigates the influence of the transversal magnetic field on the ionized gas flow in the boundary layer adjacent the body in the conditions of the mass transfer through a porous wall.

Complex flows are of great importance and have a practical application in the area of high velocity aerodynamics which is usually associated with thermal protection of the aircraft surface. This is a topic of a great current interest.

Taking into consideration the importance of electroconductivity variation both for theoretical investigations and practical applications, a general mathematical model is obtained for three most frequently used electroconductivity variation laws. Saljnikov's version of the general similarity method, which should be taken as approximate, was used in the paper. Application of approximate methods has numerous advantages. Not only are solutions obtained fast, but they also facilitate establishing correlations with different areas and reaching general conclusions. Furthermore, when flow physics is involved, complex mathematical problems appear and then approximate methods, like general similarity method used in this paper, become the only possible. Therefore development of approximate methods should be by all means encouraged.

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Nomenclature

- a, b – constants, [–]
- B – boundary layer characteristic, [–]
- B_m – induction of outer magnetic field [$= B_m(x)$], [Vsm^{-2}]
- c_p – specific heat of ionized gas at constant pressure, [$Jkg^{-1}K^{-1}$]
- F_{mp} – characteristic boundary layer function, [–]
- f_1 – first form parameter ($= f$), [–]
- f_k – set of form parameters, [–]
- g_1 – first magnetic parameter ($= g$), [–]
- g_k – set of magnetic parameters, [–]
- H – boundary layer characteristic, [–]
- H_1 – boundary layer characteristic, [–]
- h – enthalpy, [Jkg^{-1}]
- $\bar{h}$ – non-dimensional enthalpy, [–]
- h_e – enthalpy at the outer edge of the boundary layer, [Jkg^{-1}]
- h_w – enthalpy at the wall of the body within the fluid, [Jkg^{-1}]
- h_1 – enthalpy at the front stagnation point of the body within the fluid, [Jkg^{-1}]
- i, j – iteration number, [–]
- M – discrete point, [–]
- Pr – Prandtl number ($=\mu c_p/\lambda$), [–]
- Q – non-dimensional function, [–]
- s – new longitudinal variable, [m]
- u – longitudinal projection of velocity in the boundary layer, [ms^{-1}]
- u_e – velocity at the boundary layer outer edge, [ms^{-1}]
- V_w – conditional transversal velocity, [ms^{-1}]
- v – transversal projection of velocity in the boundary layer, [ms^{-1}]
- v_w – velocity of injection (or ejection) of the fluid, [ms^{-1}]
- x, y – longitudinal and transversal coordinate, [m]
- Z^{**} – function, [s]
- z – new transversal variable, [m]

Greek letters

- Δ^* – conditional displacement thickness, [m]
- Δ_1^* – conditional thickness, [m]
- Δ^{**} – conditional momentum loss thickness, [m]
- Δ_1^{**} – conditional thickness, [m]
- ζ – non-dimensional friction function, [–]
- η – non-dimensional transversal coordinate, [–]

κ – local compressibility parameter $(= f_0)$, [–]

Λ_1 – first porosity parameter $(= \Lambda)$, [–]

Λ_k – set of porosity parameters, [–]

λ – thermal conductivity coefficient, $[\mathrm{Wm^{-1}K^{-1}}]$

μ – dynamic viscosity, [Pas]

μ_0 – known values of dynamic viscosity of the ionized gas, [Pas]

μ_w – given distributions of dynamic viscosity at the wall of the body within the fluid, [Pas]

ν_0 – kinematic viscosity at a concrete point of the boundary layer, $[\mathrm{m^2s^{-1}}]$

ρ – density of ionized gas, $[\mathrm{kgm^{-3}}]$

ρ_e – ionized gas density at the outer edge of the boundary layer, $[\mathrm{kgm^{-3}}]$

ρ_0 – known values of density of the ionized gas, $[\mathrm{kgm^{-3}}]$

ρ_w – given distributions of density at the wall of the body within the fluid, $[\mathrm{kgm^{-3}}]$

σ – electroconductivity, $[\mathrm{Nm^3V^{-2}s^{-1}}]$

τ_w – shear stress at the wall of the body within the fluid, $[\mathrm{Nm^{-2}}]$

Φ – non-dimensional stream function, [–]

ψ – stream function, $[\mathrm{m^2s^{-1}}]$

ψ^* – new stream function, $[\mathrm{m^2s^{-1}}]$

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