

HEAT TRANSFER OVER A STRETCHING POROUS SHEET SUBJECTED TO POWER LAW HEAT FLUX IN PRESENCE OF HEAT SOURCE

by

Hitesh KUMAR

Department of Information Technology, IBRI College of Technology,
IBRI, Oman, Jodhpur, India

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In the present investigation the boundary layer steady flow and heat transfer of a viscous incompressible fluid due to a stretching porous sheet in presence of heat source are studied. The equations of motion and heat transfer are reduced to non-linear ordinary differential equations and the exact solutions are obtained in the form of confluent hypergeometric function (Kummer's function) for prescribed heat flux, when the wall is at prescribed second order power law heat flux or the prescribed heat flux at the stretching porous wall varies as the square of the distance from the origin. The effects of the various parameters entering into the problem on the temperature distribution and recovery temperature are discussed.

Key words: *heat transfer, porous medium, laminar boundary layer, heat source heat flux, Kummer's function*

Introduction

Flow and heat transfer study over moving smooth surfaces are of immense effect in many technological processes, such as the aerodynamic extrusion of plastic sheet, rolling, purification of molten metal from non-metallic inclusion by applying magnetic field and extrusion in manufacturing processes. In continuous casting, consists of pouring molten metal into a short vertical metal die or mould (at a controlled rate), which is open at both ends, cooling the melt rapidly and withdrawing the solidified product in a continuous length from the bottom of the mould at a rate consistent with that of pouring, the casting solidified before leaving the mould. The mould is cooled by circulating water around it. The process is used for producing blooms, billets, and slabs for rolling structural shaped, it is mainly employed for copper, brass, bronze, aluminum, and also increasingly with cast iron (CI) and steel.

In spite of this, in actual situation one has to fight face-to-face the boundary layer flow over a stretching sheet. For example, in a melt-spinning process, the banish is stretched into a filament or sheet while it is drawn from the die. Finally, this sheet or wire solidifies while it passes through controlled cooling system.

* Corresponding author; e-mail: hiteshrsharma@gmail.com

Sakiadis [1] first examined the boundary layer flow of a viscous fluid due to the motion of a plate in its own plane and Erickson *et al.* [2] and Gupta *et al.* [3] extended this problem to the case for which suction or blowing existed at the moving surface. Crane [4] and Mc Cormack *et al.* [5] studied the boundary layer flow of a Newtonian fluid caused by stretching of an elastic flat sheet which moves in its own plane with the velocity varying linearly with the distance from a fixed point due to the application of a uniform stress. The uniqueness of the exact analytical solutions followed by two different approaches, Crane [4], Mc Cormack *et al.* [5], was proved simultaneously by McLeod *et al.* [6] and Troy *et al.* [7]. Both the basic flow and the heat transfer problems for linear stretching of the sheet have since been extended in various ways. One may, for example, refer to Danberg *et al.* [8], Vleggaar [9], Soundalgekar *et al.* [10], Carragher *et al.* [11], Grubka *et al.* [12], Dutta [13], and Chen *et al.* [14]. Afzal *et al.* [15], Kuiken [16], and Banks [17] considered the power law stretching of the plate ($u\alpha x^m$). Banks *et al.* [18] considered the eigenvalue problem for boundary layer over the stretching plate. The hydromagnetic flow and heat transfer case for linearly stretching plate has been studied by Chakrabarti *et al.* [19], Chiam [20], and Abo-Eldahab *et al.* [21]. Series solution of unsteady boundary layer flow due to stretching sheet has been considered by Kechil *et al.* [22] and Liao [23].

All these authors have studied the flow through non-porous medium and neglected the effects of internal heat generation. The analysis of the flow through porous medium has become the core of several scientific and engineering applications. This type of flow is important to a wide range of technical problems, such as flow through packed beds, sedimentation, environmental pollution, centrifugal separation of particles, and blood rheology. The study of flow and heat transfer in fluid past a porous surface has attracted many scientific investigators in view of its applications in engineering practice, particularly, in chemical industries; such as the cases of boundary layer control, transpiration cooling and gaseous diffusion. However, the study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution; consequently, the particle deposition rate in nuclear reactors, electronic chips, and semiconductor wafers. In fact, the literature is replete with examples dealing with the heat transfer in laminar flow of viscous fluids. Vajravelu *et al.* [24] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Abel *et al.* [25] studied convective heat and mass transfer in a viscoelastic fluid flow through a porous medium over a stretching sheet with variable viscosity. Bhargava *et al.* [26] have taken the problem of mixed convection micropolar fluid driven by a porous stretching sheet and found the solution by the finite element method. Rashad [27] has studied the radiative effects on heat transfer from a stretching surface in a porous medium. Shafie *et al.* [28] considered unsteady boundary layer due to a stretching sheet in a porous medium using Brinkman model. Veena *et al.* [29] had found the non-similar solution for heat and mass transfer flow in an electrically conducting visco-elastic fluid over a stretching sheet embedded in a porous medium. Recently Sharma *et al.* [30] studied effects of ohmic heating and viscous dissipation on steady MHD flow near a stagnation point on an isothermal stretching sheet, Abdullah [31] found an analytical solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect, and Hall effect using homotopy analysis method. Kumar [32] considered radiative heat transfer with hydromagnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux.

In the present study, flow over a stretching porous sheet in presence of heat source is discussed. The momentum and heat equation are solved analytically and the recovery temperature is obtained.

Governing equations and analysis

Consider a steady 2-D incompressible viscous laminar flow caused by a moving porous sheet, which is placed in quiescent fluid, in presence of heat source. The flow is assumed to be in the x -direction which is chosen along the sheet and the y -axis perpendicular to it. The sheet issues from a thin slit at the origin $(0, 0)$. It is assumed that the speed of a point on the plate is proportional to its distance from the slit fig. 1, the boundary layer approximations still applicable. In writing the following equations, it is assumed that the induced magnetic field, the external electric field, and the electric field due to the polarization of charges are negligible. Under these conditions, the governing boundary layer equations of momentum, energy for free convection flow and under Boussinesq's approximation are:

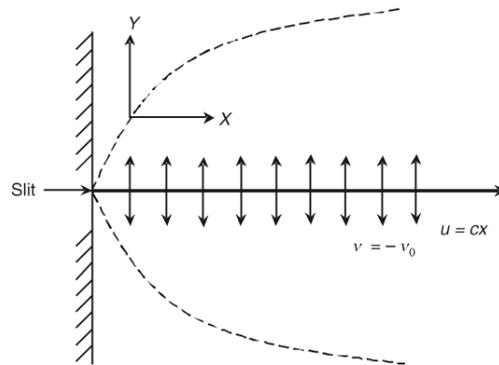


Figure 1. Boundary layer flow over a stretching porous surface

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g \frac{\partial^2 u}{\partial y^2} - \frac{g}{K} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p} (T_\infty - T) \quad (3)$$

The last term in the RHS of eq. (3) denotes the heat generation varying directly with the temperature.

The appropriate boundary conditions for the problem are:

$$u = cx, \quad v = -v_0 \quad \text{at } y = 0; \quad u = 0 \quad \text{as } y \rightarrow \infty \quad (4)$$

where c is the stretching rate and it is a positive constant.

In addition of above, boundary conditions on the temperature are:

$$\frac{\partial T}{\partial y} = Bx^2 \quad \text{at } y = 0, \quad T = T_\infty \quad \text{as } y \rightarrow \infty \quad (5)$$

The solution of eqs. (1) and (2), satisfying the boundary conditions (4) is:

$$u = cx f'(\eta), \quad v = -\sqrt{g} c f(\eta), \quad \eta = \sqrt{\frac{c}{g}} y \quad (6)$$

where prime denotes differentiations with respect to η and:

$$f(\eta) = a + be^{-\alpha\eta}, \quad a = \frac{\alpha^2 - \lambda}{\alpha}, \quad b = -\frac{1}{\alpha}, \quad \alpha = \frac{m + \sqrt{m^2 + 4(1 + \lambda)}}{2}, \quad (7)$$

where $\lambda = g/cK$ and $m = v_0/(g c)^{1/2}$

Solution of heat transfer equation

In this case to solve the eq. (3) with corresponding boundary condition (5), we assume the dimensionless temperature $\theta(\eta)$ as:

$$T = T_\infty + B \sqrt{\frac{g}{c}} x^2 \theta(\eta) \quad (8)$$

By using eqs. (6), (7), and (8) eq. (3) in this case takes the following form:

$$\theta'' + \text{Pr} f \theta' - 2\text{Pr} f' \theta - \beta \theta = 0 \quad (9)$$

and the boundary conditions (5) reduces to:

$$\left. \begin{array}{l} \theta' = 1 \text{ at } \eta = 0 \\ \theta(\infty) = 0 \end{array} \right\} \quad (10)$$

where $\text{Pr} = \rho g c_p / k$ and $\beta = g Q / k c$.

To obtain the solution of eq. (9), we introduce a new variable ξ as:

$$\xi = -\frac{\text{Pr}}{\alpha^2} e^{-\alpha\eta} \quad (11)$$

hence the eq. (9) reduces to:

$$\xi \frac{d^2 \theta}{d\xi^2} + \left(1 - \text{Pr} \left(1 - \frac{\lambda}{\alpha^2}\right) - \xi\right) \frac{d\theta}{d\xi} + \left(2 - \frac{\beta}{\alpha^2 \xi}\right) \theta = 0 \quad (12)$$

The corresponding boundary conditions are:

$$\theta' \left(\xi = -\frac{\text{Pr}}{\alpha^2} \right) = \frac{\alpha}{\text{Pr}} \quad \text{and} \quad \theta(\xi = 0) = 0 \quad (13)$$

Solution of the hypergeometric equation, one may refer to Erdelyi [33],

$$x \frac{d^2 y}{dx^2} + (b' + a'x) \frac{dy}{dx} + \left(\beta' + \alpha'x + \frac{\gamma'}{x} \right) y = 0$$

is $y = x^{-b'/2} e^{-\alpha'x/2} W(\kappa, \mu, \zeta)$ where $k = (\beta' - a'b'/2)(a'^2 - 4\alpha')^{-1/2}$, $\mu = 0.5 [(b' - 1)^2 - 4\gamma']^{-1/2}$, and $\zeta = [(a'^2 - 4\alpha')^{1/2} x]$.

The solution of eq. (12) satisfying boundary conditions (13) in terms of η , using Kummer's confluent hypergeometric function is given by:

$$\theta(\eta) = Ce^{-\frac{\alpha(a_0+b_0)\eta}{2}} {}_1F_1\left(\frac{c_0}{2}, 1+b_0, -\frac{\text{Pr}}{\alpha^2}e^{-\alpha\eta}\right)$$

where $a_0 = \text{Pr}(1 - \lambda/\alpha^2)$, $b_0 = [\text{Pr}^2(1 - \lambda/\alpha^2)^2 + 4\beta/\alpha^2]^{1/2}$, $c_0 = a_0 + b_0 - 4$ and

$$C = \frac{1}{\frac{\text{Pr}}{\alpha} \frac{c_0}{2(1+b_0)} {}_1F_1\left(\frac{c_0}{2}, 2+b_0, -\frac{\text{Pr}}{\alpha^2}\right) - \frac{\alpha(a_0+b_0)}{2} {}_1F_1\left(\frac{c_0}{2}, 1+b_0, -\frac{\text{Pr}}{\alpha^2}\right)}$$

The recovery temperature at the stretching plate is given by:

$$\theta(0) = C {}_1F_1\left[\frac{c_0}{2}, 1+b_0, -\frac{\text{Pr}}{\alpha^2}\right]$$

Discussions and results

The steady 2-D incompressible viscous laminar flow of an electrically conducting fluid and heat transfer in presence of heat source through porous medium caused by stretching a porous wall is governed by parameter, namely, the permeability parameter λ , the injection parameter m , the Prandtl number Pr , heat source parameter β , and the wall temperature parameter s .

The dimensionless temperature distribution $\theta(\eta)$ is plotted against η for different values of parameters λ , m , Pr , and β in figs. 2-5, respectively. It is being observed that $\theta(\eta)$ increases as λ or m or Pr or β increases. Physically $\beta > 0$ implies that $T_w > T_\infty$ and there will be a supply of heat to the flow region from the wall. Figure 3 shows that temperature comes down if more fluid is injected, due to the cost of heating the fluid has been inflated because of great loss of heat from hot injection. The profiles of the function $\theta(\eta)$ are almost all negative except in fig. 4, when $\text{Pr} = 50.0$. It has been concluded that if $\text{Pr} \geq 50.0$, the profile lies near the horizontal axis (where $\theta(\eta) = 0.0$).

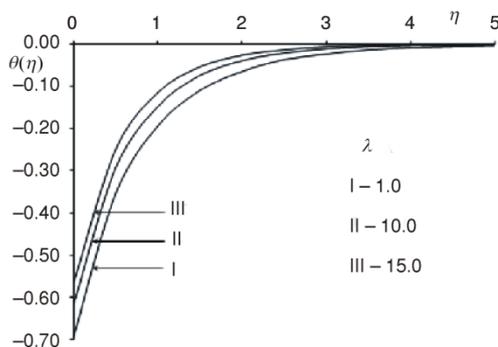


Figure 2. Dimensionless temperature against η for different values of λ with $m = 0.2$, $\text{Pr} = 1.0$, and $\beta = 0.1$

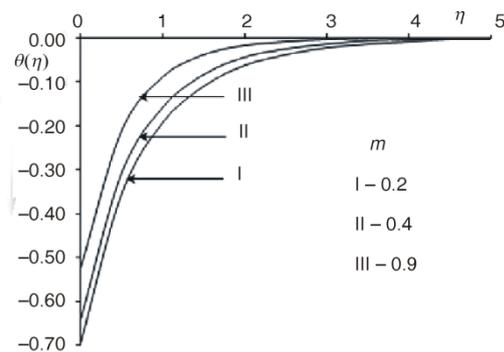


Figure 3. Dimensionless temperature against η for different values of m with $\lambda = 1.0$, $\text{Pr} = 1.0$, and $\beta = 0.1$

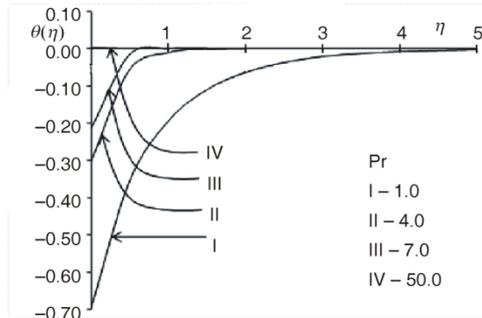


Figure 4. Dimensionless temperature against η for different values of Pr with $\lambda = 1.0$, $m = 0.2$, and $\beta = 0.1$

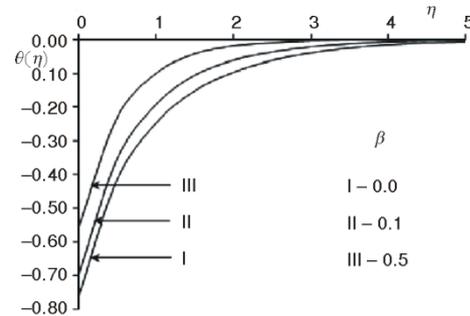


Figure 5. Dimensionless temperature against η for different values of β with $\lambda = 1.0$, $m = 0.2$, and $Pr = 1.0$

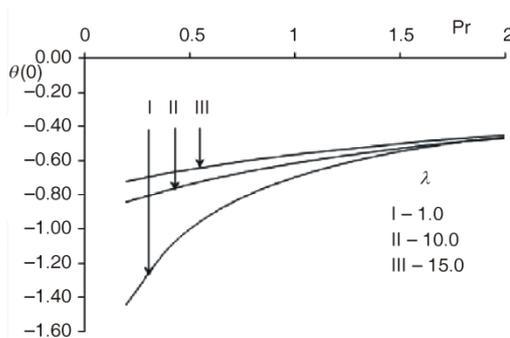


Figure 6. Recovery temperature against Pr for different values of λ with $m = 0.2$, and $\beta = 0.1$

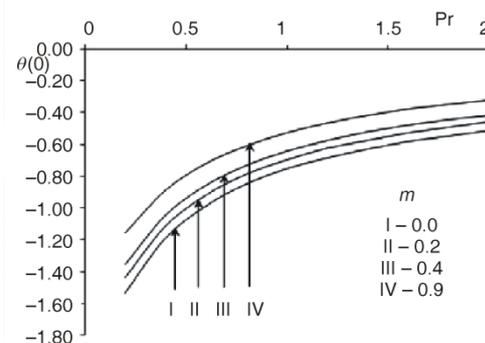


Figure 7. Recovery temperature against Pr for different values of m with $\lambda = 1.0$, and $\beta = 0.1$

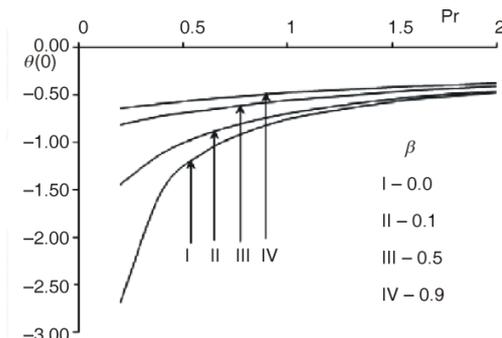


Figure 8. Recovery temperature against Pr for different values of β with $\lambda = 1.0$ and $m = 0.2$

The recovery temperature $\theta(0)$ is plotted against Pr for various values of parameters λ , m , and β in figs. 6-8, respectively. It has been noted that $\theta(0)$ increases as λ or m or β increases. Also, the function $\theta(0)$ is negative for all the cases.

The curve-I in fig. 5 and fig. 8 represents the case of absence of heat source.

Conclusions

In the present investigation, the effects of heat source and porosity on temperature

distribution are obtained. In the complete study it is concluded that:

- inclusion of porosity or heat source increases the temperature, and
- existence of porosity or heat source increases the recovery temperature.

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Nomenclature

B	– dimensional temperature coefficient	v_0	– suction velocity across the stretching sheet
c	– stretching rate	W	– Whittaker funktion
C_p	– specific heat at constant pressure	x, y	– cartesian co-ordinates
${}_1F_1$	– Kummer's confluent hypergeometric function	<i>Greek symbols</i>	
K	– thermal conductivity	β	– stretching rate
k	– permeability of the porous medium	γ	– constant
m	– injection parameter	λ	– permeability parameter
Pr	– Prandtl number	η	– dimensionless co-ordinate in y-direction
Q	– heat source coefficient	μ	– constant
T	– temperature	ρ	– density of the fluid
T_∞	– temperature on the sheet at infinity	ϑ	– kinematic viscosity
T_w	– wall temperature on the sheet at $y = 0$	θ	– dimensionless temperature
u, v	– fluid velocities in the x- and y-directions		

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