MODELING AND SIMULATION OF TIME-DEPENDENT WILLIAMSON NANOFLUID FLOW WITH THERMAL RADIATION AND BIOCONVECTION

by

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Williamson squeezed nanofluid with mass transfer characteristics are assumed. Significance of microorganism and nanofluid are evaluated. From energy equation, thermal radiation, Joule heating, and Brownian motion gets assist. For alleviate affiliated expressions into ordinary differential system are used proper similarity transformations. Convergent series solutions are obtained, non-linear ODE are resolved numerically by using homotopy analysis method. The effect of bio convection Lewis, Prandtl, and Peclet numbers, magnetic parameters radiation, Brownian motion, and radiation are discussed. Main outcomes of current examination are prescribed in conclusion part.

Key words: *Williamson squeezed nanofluid, microorganism, thermal radiation, Brownian motion, homotopy analysis method*

Introduction

Non-Newtonian fluids are fluid under applied force viscosity changes with either more solid or more liquid, for instance ketchup has jiggle it becomes fluid. Molten polymers solutions, salt solutions, and materials like shampoo, paints, toothpaste, melted butter, corn

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starch, custard, starch suspensions are usually non-Newtonian fluids. On shear rate history or shear rate their viscosity is reliant. In few viscoelastic fluids viscosity is independent of the shear rate. Zou *et al.* [1] investigated a complex micro channel with a depth of 100 μm and spacing of 30 μm fabricated using the non-Newtonian fluid. Nino *et al.* [2] investigated bubble-size (distribution) for non-Newtonian flows (n-N F). Mehta *et al.* [3] investigated power-law model using heterogeneous surface and hydrodynamic characteristics for a pure electro osmotic flow. Sun *et al.* [4] scrutinized the non-locality of viscoelastic fluids by utilizing truncated fractional derivatives. To simulate seepage of non-Newtonian fluid Boltzmann method is used. More studies about Newtonian and non-Newtonian behavior are given [5-9].

Nanofluid represents a fluid that involves nanosize particles. For convective heat transfer applications rheological behavior of nanofluids is significant. Zhou *et al.* [10] studied effects of quantum carbon dots on the photo thermal stability of CQD nanofluids. Tabarhoseini and Sheikholeslami [11] investigated nanofluid for solar collector using CFD. For solar collector performance in variant volume fractions the performance of water-based Cu and MWCNT nanofluid was scrutinized. Mehta *et al.* [12] examined thermophysical and heat transfer properties for nanofluid applications with synthesis and stability. Hanafi *et al.* [13] examined Al_2O_3-Cu-water (hybrid-nanofluid) to get numerical solutions by utilizing ANSYS FLUENT. More works on nanofluid flow can be found in [14-17].

A microorganism is small size body, which might exist as a colony of cells. Along times ago the unseen existence of microbial life was suspects, such as in sixth century boundary conditions India in Jain scriptures with their observations Power and Moore [18] investigated molecular biology which provides a basic development for custom made microorganisms and industrial biotechnology. Wang *et al.* [19] examined removal of organic pollutants and phosphorus, nitrogen from water seeding type immobilized microorganisms. Li *et al.* [20] used analysis for bacterial community for various stages of dynamics processing of bacon smoked. Recently researchers used analysis to study, constrained decomposition method, isolated slug traveling in a voided line, study on interaction mechanism, and modeling segregation of polydisperse granular materials its further discussions on can be found in [21-24].

From aforementioned literature study it is noted that less attention is given towards radiative non-Newtonian fluid (Williamson model). The observation becomes narrow when nanoparticles and bioconvection are considered. The motivation of current research is to address swimming of microorganism in radiative Williamson nanofluid flow. Brownian motion and thermophoresis diffusions (Boungiarnos model) are utilized to incorporate nanofluid significance. Thermal radiation effects are considered. Non-linear systems of governing equations are reduced through appropriate similarity transformations and convergent solutions were achieved. Main results are highlighted in conclusion section.

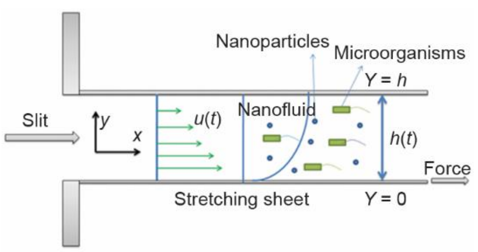

Figure 1. Physical flow model

Mathematical formulation

Here we have considered Williamson fluid between two horizontal permeable plates. The upper plate is at a distance h from the origin. The lower plate is placed at $y = 0$ and moving with some velocity U as shown in fig. 1. Movement of nanoparticles and microorganism are accounted.

The governing equations take the form [25]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+v\left(\frac{\partial u}{\partial y}\right)+u\left(\frac{\partial u}{\partial x}\right)=\frac{\mu_0}{\rho}\left[\left(\frac{\partial^2 u}{\partial y^2}\right)+\left(\frac{\partial^2 u}{\partial y^2}\frac{\partial u}{\partial y}\right)\sqrt{2}\Gamma\right]-u\left(\frac{\mu_0}{\rho k}+\frac{\sigma\beta_0^2}{\rho}\right) \tag{2}$$

$$u\left(\frac{\partial T}{\partial x}\right)+\frac{\partial T}{\partial t}+v\left(\frac{\partial T}{\partial y}\right)=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{1}{\rho C_p}\left(\frac{16T^3{}_\infty\sigma^*}{3K^*}\frac{\partial^2 T}{\partial y^2}\right)+\tau\left[\frac{D_T}{T_0}\left(\frac{\partial T}{\partial y}\right)^2 D_B\frac{\partial T}{\partial y}\frac{\partial c}{\partial y}\right] \tag{3}$$

$$u\left(\frac{\partial C}{\partial x}\right)+v\left(\frac{\partial C}{\partial y}\right)+\frac{\partial C}{\partial t}=\left[D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_0}\left(\frac{\partial^2 T}{\partial y^2}\right)\right] \tag{4}$$

$$\frac{\partial N}{\partial t}+\left(\frac{\partial N}{\partial x}\right)u+\left(\frac{\partial N}{\partial y}\right)v+\frac{bW_c}{(C_1-C_0)}\left[\frac{\partial}{\partial y}\left(\frac{\partial c}{\partial y}N\right)\right]=D_m\frac{\partial^2 N}{\partial y^2} \tag{5}$$

Boundary conditions are:

$$\begin{gathered}C=C_1,\quad N=N_1,\quad T=T_1,\quad U=u,\quad v=0\quad \text{at}\quad y=0,\\ \frac{\partial T}{\partial y}=\frac{\partial U}{\partial y}=\frac{\partial N}{\partial y}=\frac{\partial C}{\partial y}=0\quad \text{at}\quad h=y\end{gathered} \tag{6}$$

Considering the following similarity transformations [25]:

$$\begin{gathered}v=\frac{\partial\Psi}{\partial x}=-\left[\frac{\nu b}{1-at}\right]^{\frac{1}{2}}f'(\eta),\quad \eta=\sqrt{\frac{b}{\nu(1-at)}}y,\quad u=\frac{\partial\Psi}{\partial y}=\frac{bx}{(1-at)}f'(\eta),\\ \chi=\frac{N-N_0}{N_1-N_0},\quad \phi=\frac{C-C_0}{C_1-C_0},\quad \theta=\frac{T-T_0}{T_1-T_0}\end{gathered} \tag{7}$$

Equation (1) satisfies and eqs. (2) to (5) take the form:

$$f'''-(f')^2-ff''+\mathrm{We}f''f'''-Mf'-S\left(f'\frac{\eta}{2}f''\right)-Kf'=0 \tag{8}$$

$$\theta''(Rd+1)-\mathrm{Pr}\left(\frac{S}{2}\eta\theta'-f\theta'+Nt\theta'^2+Nb\theta'\phi'\right)=0 \tag{9}$$

$$\frac{1}{\mathrm{Sc}}\phi''+\frac{Nt}{Nb}\frac{1}{\mathrm{Sc}}\theta''-\frac{s}{2}\phi'\eta+f\phi'=0 \tag{10}$$

$$\mathrm{Le}\chi''-\frac{s}{2}\eta\chi'+f\chi'-\mathrm{Pe}\left[(\chi+\Omega)\phi''+\phi'\chi'\right]=0 \tag{11}$$

where the dimensionless numbers are defined:

$$\mathrm{Pr}=\frac{\rho\nu C_p}{k}=\frac{\mu_0 C_p}{k},\quad \mathrm{We}=\Gamma x=\sqrt{\frac{2b^3}{\nu(1-at)^3}},\quad M=\frac{\rho\beta_0^2}{\rho\nu}(1-at),$$

$$K=\frac{\nu}{kb}(1-at),\quad Rd=\frac{16\sigma*T_0^3}{3kk^*},\quad S=\frac{a}{b},\quad Nb=\frac{\tau D_B}{\nu}(c_1-c_0),\quad \mathrm{Sc}=\frac{\nu}{D_B},$$

$$Nt=\frac{\tau D_T}{\nu T_0}(T_1-T_0),\quad \mathrm{Pe}=\frac{bW_c}{\nu},\quad \Omega=\frac{N_0}{N_1-N_0},\quad \mathrm{Le}=\frac{D_m}{\nu}$$

with:

$$\chi'(\beta)=0,\quad \theta(0)=1,\quad f(0)=0,\quad f'(0)=1,\quad f'''(\beta)=0,$$
$$\phi'(\beta)=0,\quad \phi(0)=1,\quad \theta'(\beta)=0,\quad \chi(0)=1 \tag{13}$$

Solution methodology

Here we utilize a familiar semi analytical technique homotopy analysis method (HAM) for solution purpose of the described defined problem. Lio [26] was first introduced HAM. This method gives convergent series solutions. Auxiliary parameter, $\hbar$, can control the convergence of solutions. We require proper linear operators and initial guesses, these are:

$$f_0(\eta)=\eta \tag{14}$$

$$\theta_0(\eta)=-\frac{1}{2}\eta^2+\beta\eta+1 \tag{15}$$

$$\phi_0(\eta)=-\frac{1}{2}\eta^2+\beta\eta+1 \tag{16}$$

$$\chi_0(\eta)=-\frac{1}{2}\eta^2+\beta\eta+1 \tag{17}$$

$$L_\theta(\theta)=\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2},\quad L_\chi(\chi)=\frac{\mathrm{d}^2\chi}{\mathrm{d}\eta^2},\quad L_f(f)=\frac{\mathrm{d}^4 f}{\mathrm{d}\eta^4},\quad L_\phi(\phi)=\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} \tag{18}$$

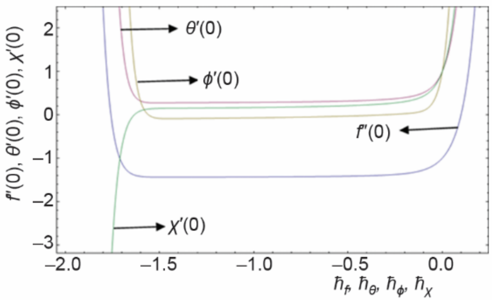

Figure 2. Convergence regions

Convergence

The convergence of solutions is mainly dependent on auxiliary variable $\hbar$. These regions are plotted in fig. 2.

Discussion

Graphical results are presented to show impact of fixing parameters for temperature, velocity and concentration for microorganisms and nanofluids.

Velocity

Figure 3 shows impact of velocity against magnetic parameter, *M*. The velocity decays for increasing values of *M*. Figure 4 depict the behavior of velocity against porosity parameter, *K*. Increasing *K* results in velocity decay. Figure 5 displays the behavior of velocity against unsteady parameter. The graph reveals a decreasing trend in velocity against large unsteady parameter.

Temperature

Figure 6 reveals the effect of Brownian motion against temperature field. For increasing Brownian motion parameter the temperature field enhances. Figure 7 represents behavior of thermophoresis against temperature. A boost in temperature occurs for increasing values of, *Nt*. Figure 8 elucidates temperature variations against Prandtl number. As the Prandtl number escalates the temperature rises. Figure 9 shows result for radiation parameter. Temperature increases for increasing the values of radiation. Figure 10 shows the influence of unsteadiness variable for temperature. Temperature enhancement occurs for large values of unsteadiness parameter, *S*.

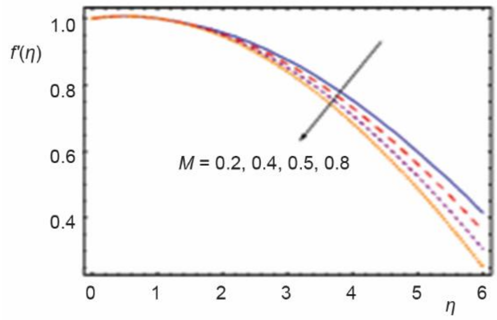

Figure 3. Magnetic parameter against velocity profile

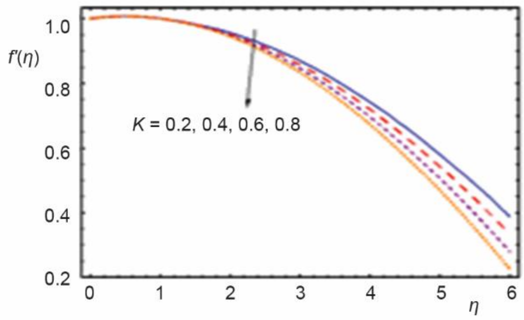

Figure 4. Porosity parameter against velocity profile

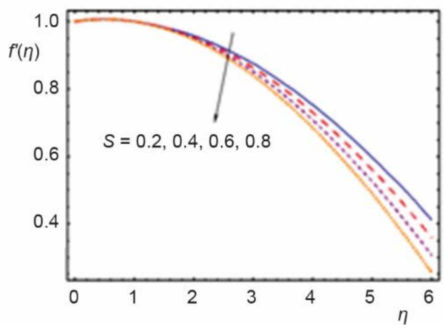

Figure 5. Unsteady parameter against velocity profile

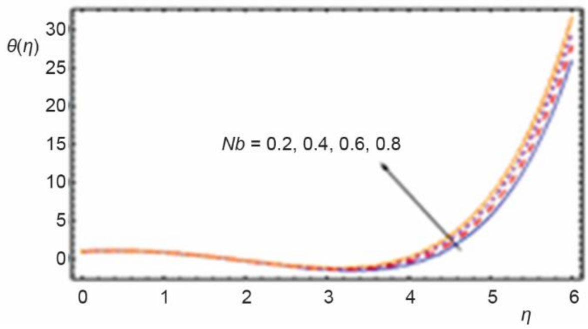

Figure 6. Brownian motion parameter against temperature profile

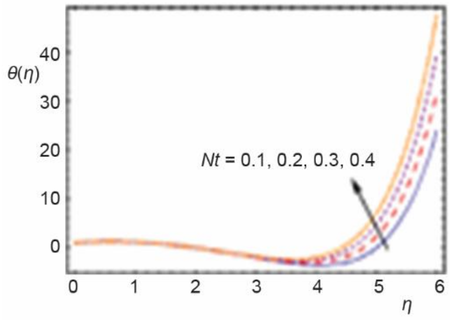

Figure 7. Thermophoresis parameter against temperature profile

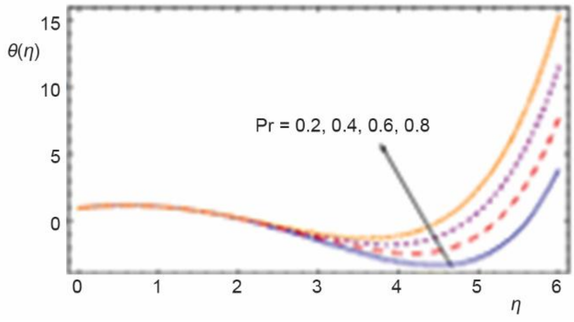

Figure 8. Prandtl number against temperature profile

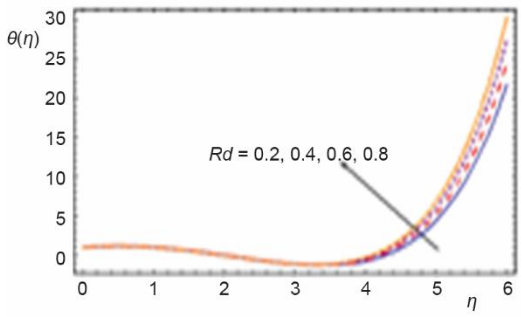

Figure 9. Radiation parameter against temperature profile

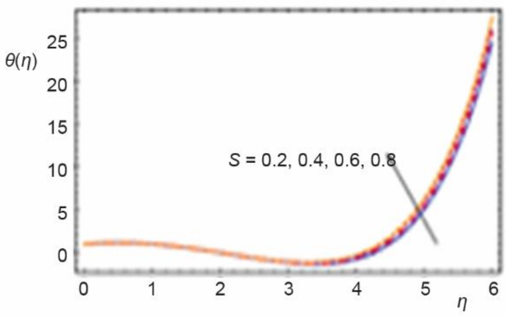

Figure 10. Unsteadiness parameter against temperature profile

Concentration

Figure 11 displays the influence of Schmidt number over concentration. Enhancing Schmidt number decays concentration profile. Figure 12 shows the impact of Brownian motion for concentration. Decay in nanofluid concentration is observed for an increment in *Nb*. Figure 13 reveals the impact of *Nt* for concentration. Nanofluid concentration enlarges with higher *Nt*.

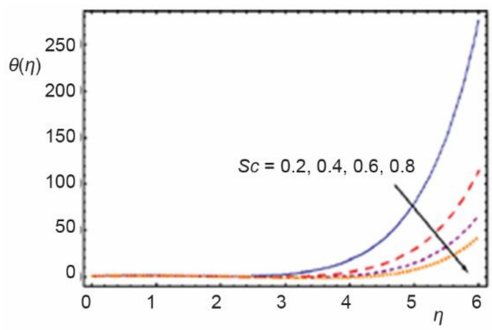

Figure 11. Schmidt number against concentration profile

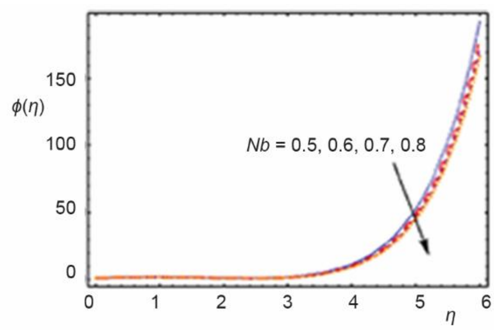

Figure 12. Brownian motion parameter against concentration profile

Microorganism

Figure14 displays the influence of Peclet number against microorganisms. Microorganisms field increases with large Peclet number. Figure 15 shows the characteristic of a microorganism concentration difference parameter, *Ω*, against microorganism field. Microorganism concentration enlargement occurs for increasing *Ω*. Figure 16 shows the outcomes of Lewis number against microorganism field. Microorganism concentration decreases for increasing Lewis number. Figure 17 reveals the impact of microorganisms against *S*. Microorganism concentration decreases for higher unsteadiness parameter.

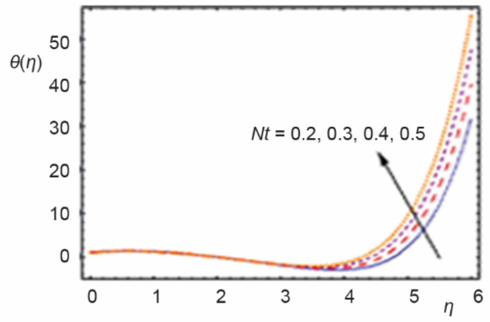

Figure 13. Thermophoresis parameter against concentration profile

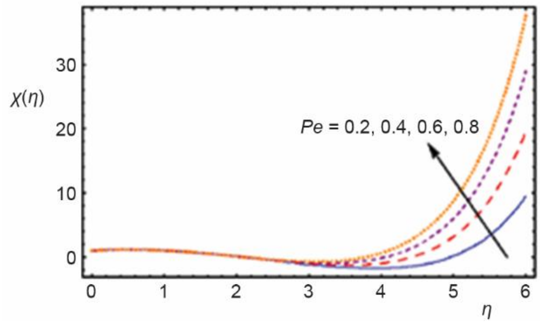

Figure 14. Peclet number against microorganisms profile

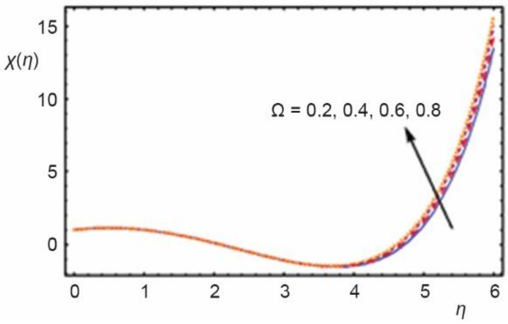

Figure 15. The *Ω* against microorganisms profile

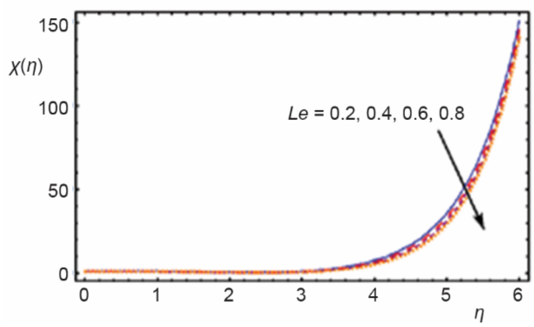

Figure 16. The Lewis number against microorganism concentration profile

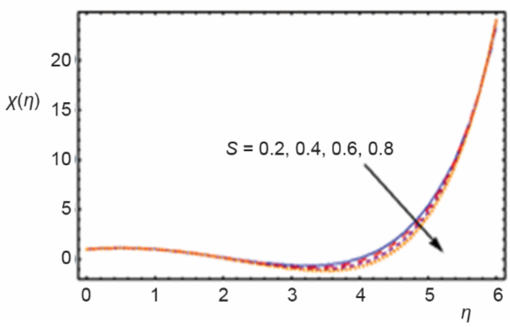

Figure 17. Unsteadyness parameter against microorganisms concentration profile

Conclusions

The major concluding points of the presented investigation are given below.

- For higher values of *K*, *M*, and *S* the velocity decreases.
- Temperature decays against *Nb*, *Nt*, Prandtl number, *Rd*, and *S*.

- Concentration alleviates for higher values of *Nb* and Schmidt number and an opposite behavior is noted against *S* and *Nt*.
- Higher values of Ω and Peclet number results in a rise in microorganism field on the other hand the microorganism field alleviate against *S* and Lewis number.

Conflict of interest statement

No, there is no conflict of interest.

Data availability statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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