SIMULATION MODEL FOR FRACTAL CHARACTERISTICS OF GRINDING WHEEL SURFACE

by

Chuhan SHANG[a,b*] and Huaian YI[a,b]

[a]Key Laboratory of Advanced Manufacturing and Automation Technology (Guilin University of Technology), Education Department of Guangxi Zhuang Autonomous Region, Guilin, China
[b]College of Mechanical and Control Engineering, Guilin University of Technology, Guilin, China

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This article proposes a simulation model for the fractal characteristics of grinding wheel surfaces generated based on specified roughness parameters. The new model is based on the relationship between roughness parameters and fractal dimension, and uses a random Weierstrass-Mandelbrot function and Johnson transformation system to obtain a surface point cloud matrix with non-Gaussian random distribution. Then, the spacing of this matrix was adjusted using random number algorithm and fractal interpolation algorithm to obtain a matrix of abrasive distribution with randomness and self-affinity. The ablation study proved that the model is superior to the fractal function model in calculating roughness parameters. This achievement is of great significance for optimizing the design and manufacture of grinding wheels and improving the quality of grinding operations.

Key words: *grinding wheel surface profile simulation, self-affinity, Weierstrass-Mandelbrot function, Johnson transformation system, fractal interpolation algorithm*

Introduction

Any surface morphology can be described as fractal. For example, water waves move along a fractal boundary [1], the permeability of a fabric depends upon its fractal surface [2], and it is easy to understand that the section of a porous medium is a fractal geometry [3]. Even the smooth lubrication surface, when measured on the nanoscale, exhibits fractal properties [4].

The current approach to simulating the contour of a grinding wheel surface primarily entails the simulation of the height distribution function of the grinding wheel surface and the generation of a simulated surface based on the aforementioned height distribution function [5, 6]. The fractal function method is a widely utilized approach for simulating rough surfaces. Its objective is to employ fractal functions to describe rough surfaces and achieve the simulation of various rough surfaces by adjusting the fractal dimension. Yan and Komvopoulos [7] proposed the use of the Weierstrass-Mandelbrot (W-M) fractal function to construct rough surfaces. Cai *et al.* [8] proposed a direct search method to circumvent the necessity of calculating the derivative of the minimum objective fractal function, thereby enhancing the effi-

* Corresponding author, e-mails: 1113019533@qq.com, sch1113019533@163.com

ciency of the simulation process. In their study, Lv *et al.* [9] employed multifractal methods to investigate the distribution characteristics of surface height on W-M surfaces and constructed rough surface simulation models under varying fractal dimensions.

The use of simulation models to expand the training data sets is a widely used method in current machine learning [10]. However, from the perspective of expanding machine learning training data sets, it can be found that the above methods have shortcomings. The simulation model based on fractal functions [7-9] can describe the self-affinity of the grinding wheel surface, but it does not have the ability to generate different simulation surfaces based on different roughness parameters, and is not suitable for expanding the training set of machine learning. To solve this problem, this paper obtains the surface fractal dimension for the specified roughness parameters based on the relationship between the fractal dimension and surface roughness proposed by Zhang *et al.* [11]. Using the Johnson transformation system, non-Gaussian random distribution sequences are obtained for the specified skewness, Rsk, and kurtosis, Rku. Finally, the obtained and are substituted into the fractal function to obtain the fractal function for the specified roughness parameters. In addition, another disadvantage of the fractal function-based grinding wheel simulation model is that the distance between the grinding wheel particles exported by the model is constant in both axial and circumferential directions. In the actual grinding process, the randomness and self-affinity of the distribution of grinding wheel particle spacing will directly affect the morphological characteristics of the machining surface wear marks. Therefore, the simulation model of the grinding surface generated by equidistant abrasive particles does not accurately calculate the motion trajectory of the abrasive particles. In this regard, in this article, based on the particle size of the grinding wheel, a random number algorithm and fractal interpolation algorithm are used to generate a distribution matrix of abrasive spacing with randomness and self-affinity.

Generate fractal grinding wheel surface based on specified roughness parameters

The simulated profile is usually represented by a height function. In this paper, we first generate the initial matrix $O(x, y)$ with the size of $m \times n$ and equal spacing, and then use the 3-D random W-M function as the height function, and combine the initial matrix with the height function to draw the simulation surface.

The 3-D stochastic W-M function is a fractal surface function commonly used in engineering to simulate the contour features of rough surfaces. This function can fully adapt to various characteristics of rough surfaces, while its parameters are stable and not affected by changes in measurement scale [12]. Its standard form is:

$$Z(x,y) = \sum_{n=1}^{\infty} C_n \lambda^{-(3-D_s)n} \sin[\lambda^n (x\cos B_n + y\sin B_n) + A_n] \tag{1}$$

where n is the natural sequence number, λ – the constant greater than 1, used to characterize the height scale of the workpiece surface, D_s – the surface fractal dimension, C_n – the automatically generated Gaussian random distribution sequence, A_n and B_n – the random numbers, which follow a uniform distribution on $[0, 2\pi]$.

In order to construct a simulation model of the grinding wheel fractal function under specified roughness parameters, this paper first calculates the fractal dimension corresponding to the roughness parameters based on the correlation formula between the roughness parameters and the fractal dimension [13].

$$D = 1.539R_a^{-0.0478} \tag{2}$$

$$D_s = 1 + \bar{D} \tag{3}$$

where $\bar{D}$ is the average fractal dimension of each contour.

Previous studies have proved that the grinding wheel surface is a non-Gaussian rough surface [14, 15]. Therefore, based on the Johnson transformation system, this article transforms the Gaussian random distribution sequence C_n in the W-M function into a non-Gaussian random distribution sequence η' with a specified skewness, Rsk, and kurtosis, Rku.

The inverse transformation of Johnson transformation:

– Unbounded system (SU): $$\eta' = \xi + \lambda \arcsin\left(\frac{C_n - \gamma}{\delta}\right) \tag{4}$$

– Lognormal System (SL): $$\eta' = \xi + \lambda e^{(C_n-\gamma)/\delta} \quad (\eta' > \xi) \tag{5}$$

– Bounded System (SB): $$\eta' = \frac{\xi + (\xi + \lambda)e^{(C_n-\gamma)/\delta}}{1 + e^{(C_n-\gamma)/\delta}} \quad (\xi < \eta' < \xi + \lambda) \tag{6}$$

Among them, C_n is a Gaussian random sequence, η' is a non-Gaussian random sequence generated with specified skewness and kurtosis, and γ, δ, ξ, and λ are four system constant parameters calculated based on the specified skewness and kurtosis [13].

The fractal formula for the particle height distribution is obtained by substituting the non-Gaussian random sequence and the fractal dimension into the W-M function:

$$h(x,y) = \sum_{n=1}^{\infty} \eta' \lambda^{-(3-D_s)n} \sin[\lambda^n (x\cos B_n + y \sin B_n) + A_n] \tag{7}$$

However, because the distance between the points of the initial matrix of the model is a fixed value, the distance between the points in the axial and circumferential directions derived from the model is a fixed value that does not correspond to reality.

Optimization of abrasive particle spacing distribution

Regarding the problem of unfixed distribution of abrasive particle spacing, this paper develops a new method, based on the self-affinity property of the grinding wheel surface and the principle of random distribution [16, 17], to adjust the initial matrix so as to generate a 2-D matrix with random spacing and self-affinity property.

The fractal interpolation algorithm is an effective method that uses fractal theory to explore the patterns between complex data. This method analyzes the relationships between known data points, performs self-similar extension by iterating the affine system to obtain predicted data with a small deviation from the actual data [18, 19], and the fitted curves can better describe the affine characteristics of the surface morphology of the grinding wheel, thus improving the simulation accuracy.

We create the following time series pointset:

$$\{(x_i, Y_i), \quad i = 1, 2, ..., N\} \tag{8}$$

where x_i is the time series value and Y_i – the measured value.

By applying eq. (12) to standardize the time series point set, we can obtain a standardized point set of $\{(x_i, y_i),\ i = 1, 2, ..., N\}$:

$$y_i = \frac{Y_i - Y_{\min}}{Y_{\max} - Y_{\min}} \quad (i = 1, 2, ..., N) \tag{9}$$

where $Y_{\max}$ is the maximum measured data and $Y_{\min}$ – the minimum value of measured data.

Based on fractal theory, we can construct an iterative function system defined on the 2-D real domain R^2. We can ensure that its attractor precisely falls onto the continuous function $f : [x_0, x_n] \to R$ defined on point set $\{(x_i, y_i), i = 1, 2, ..., N\}$. The iterative function system is:

$$\omega_i \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_i & 0 \\ c_i & d_i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e_i \\ f_i \end{bmatrix}, \quad (i = 1, 2, ...N) \tag{10}$$

Equation (1) satisfies the endpoint condition:

$$\omega_i \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x_{i-1} \\ y_{i-1} \end{bmatrix}, \quad \omega_i \begin{bmatrix} x_N \\ y_N \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix}, \quad (i = 1, 2, ...N) \tag{11}$$

where ω_i is the affine transformation matrix to be solved. The a_i, c_i, d_i, e_i, and f_i, are the transformation parameters, which can be obtained from the following equation:

$$\begin{aligned} a_i &= \frac{(x_i - x_{i-1})}{(x_N - x_0)} \\ e_i &= \frac{(x_N x_i - x_0 x_{i-1})}{(x_N - x_0)} \\ c_i &= \frac{(y_i - y_{i-1})}{(x_N - x_0)} - d_i \frac{(y_N - y_0)}{(x_N - x_0)} \\ f_i &= \frac{(x_N y_{i-1} - x_0 y_i)}{(x_N - x_0)} - d_i \frac{(x_N y_0 - x_0 y_N)}{(x_N - x_0)} \end{aligned} \tag{12}$$

The vertical scale factor d_i is calculated:

$$d_i = \frac{y_i - y_{i-1}}{\varepsilon \sqrt{(y_{\max} - y_{\min})^2 + (y_i - y_{i-1})^2}} \tag{13}$$

where $y_{\max}$, $y_{\min}$ are the maximum and minimum values in interpolation points, ε = 1 + rand(1), where rand(1) represents a random number between 0 and 1.

According to the previous principle, the required affine transformation matrix system can be obtained, the measured data is substituted into eq. (16), and then the attractor is continuously iterated to obtain the fractal interpolation fitting curve.

In the actual grinding process, the random distribution of abrasive particles will directly affect the morphology of the grinding surface, so the spacing of abrasive particles cannot simply be regarded as a fixed value. In this paper, the definition is based on the grinding wheel grain size number, the average number of abrasive particles with different grain size numbers within the specified spacing is calculated, and the co-ordinate system *iOx* is constructed with the number of abrasive particles in the circumferential direction as the abscissa,

the circumferential distance from the end point of the abrasive particles as the ordinate, and the end point of the abrasive particles as the origin O, and the circumferential spacing sequence of abrasive particles based on the average spacing is obtained:

$$x = \frac{25.4}{T-1} i \tag{14}$$

where T is the particle size.

In the same way, the axial spacing sequence y of abrasive particles is generated, and the spacing distribution matrix $A(x, y)$ corresponding to different particle sizes is obtained.

Simultaneously, the definition of particle size limits the minimum value of particle spacing to be greater than 0 and the maximum value to be less than 25.4 mm (1 inch). To this end, this paper uses the rand function to generate a random array with a size of $(T - 1)\times 1$ and a Gaussian random distribution between 0 and 25.4 mm, and the sum of the array should not be greater than 25.4 mm. Then the array is used to adjust the matrix $A(x, y)$, so as to describe the randomness of the distribution of grinding wheel grain spacing, and the random matrix $B(x, y)$ corresponding to different grain sizes is obtained.

The random stacking of abrasive grains during the formation of the grinding wheel makes the abrasive grain spacing on the surface of the grinding wheel also self-affine. This article uses fractal interpolation algorithm to interpolate the matrix $B(x, y)$ through dual fractal interpolation, obtaining a random matrix $C(x, y)$ with self-affinity, and simulating the stacking of abrasive particles on the surface of the grinding wheel. Then, using the round function to take the x co-ordinates of each point in the matrix $C(x, y)$ as the nearest integer and take the y co-ordinates with the same x co-ordinates as the average value, and output a new matrix $D(x, y)$. It is worth noting that because $B(x, y)$ is a random matrix with Gaussian random distribution, the $C(x, y)$ generated each time is different, and $C(x, y)$ is self-affinity, so the $D(x, y)$ output each time is different, with randomness and self-affine.

Simulation results and experimental analysis

According to the data set obtained by the aforementioned algorithm, the spacing distribution function of abrasive particles with different particle sizes (taking #120 as an example) is drawn, as shown in fig. 1(a). Figure 1(b) is a partially enlarged image of fig 1(a).

Then, using the point data in the matrix $D(x, y)$, adjust the spacing of the initial matrix of the simulation model, and repeat the aforementioned algorithm for each row and column of the initial matrix, so as to obtain the initial matrix with random spacing and self-affine. With granularity number #120 and simulation model size 25.4 × 25.4 mm as an example, the results are shown in fig. 2.

The simulation program is written according to the aforementioned calculation method and the simulation experiment is carried out with the surface and roughness parameters of the grinding wheel with a grain size of #120 as an example. The simulation results are shown in fig. 3, and the roughness parameters Ra, Rq, Rsk, and Rku of the surface of the simulation model of the grinding wheel are calculated based on the point cloud.

Subsequently, ablation experiments were performed and each model was simulated five times. The average relative error between the simulation results and the specified roughness parameters was calculated, as shown in tab. 1. Comparative experiments A and B show that by introducing the Johnson transformation system, non-Gaussian distribution surfaces satisfying the specified roughness parameters were successfully generated. This improvement

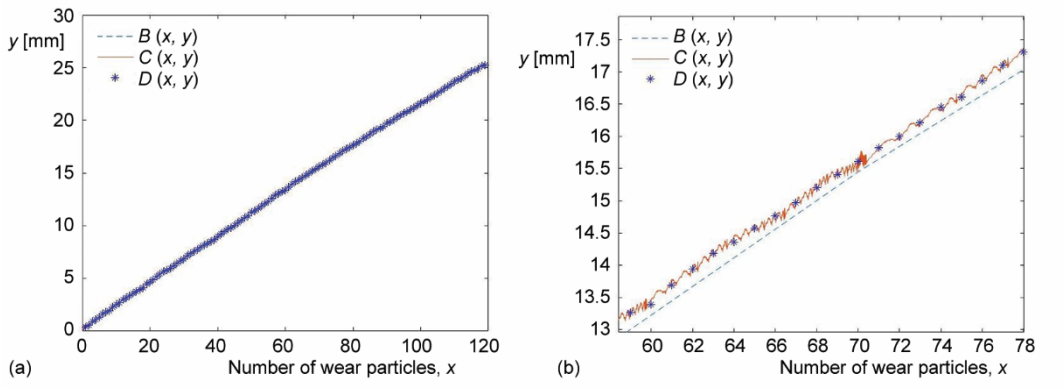

Figure 1. Plotting the distribution function of wear particle spacing based on the dataset obtained from the aforementioned algorithm; (a) abrasive particle spacing (T = 120) and (b) partial enlarged view

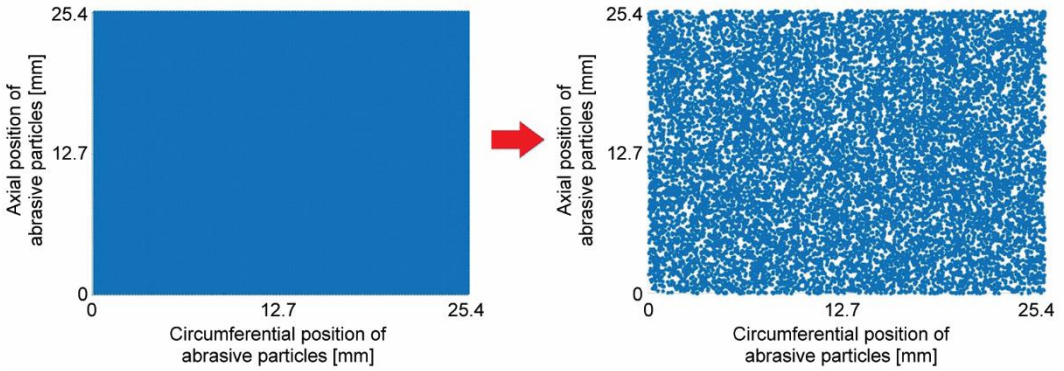

Figure 2. Schematic diagram of adjustment results

Table 1. Comparison of segmentation results between the proposed method and specified parameters

Simulation models	MRE of Ra	MRE of Rq	MRE of Rsk	MRE of Rku
A. FFM (baseline)	15.123%	15.242%	45.500%	7.565%
B. FFM+JT	9.092%	9.111%	7.143%	2.862%
C. FFM+FI	2.472%	2.670%	34.714%	7.439%
D. FFM+JT+FI	2.706%	1.758%	8.571%	3.051%

Note: MRE represents the average relative error between the simulation model and the specified roughness parameters, FFM stands for fractal function model, JT stands for Johnson transformation, FI stands for fractal interpolation.

significantly improves the accuracy of the simulation model in calculating the skewness and kurtosis of the roughness parameters. Meanwhile, the comparison experiments *A* and *C* show that the fractal interpolation algorithm has excellent performance in generating self-affine bottom point clouds on the grinding wheel surface, which improves the accuracy of the simulation model in calculating height parameters in roughness parameters, and the calculated roughness parameter values are too large.

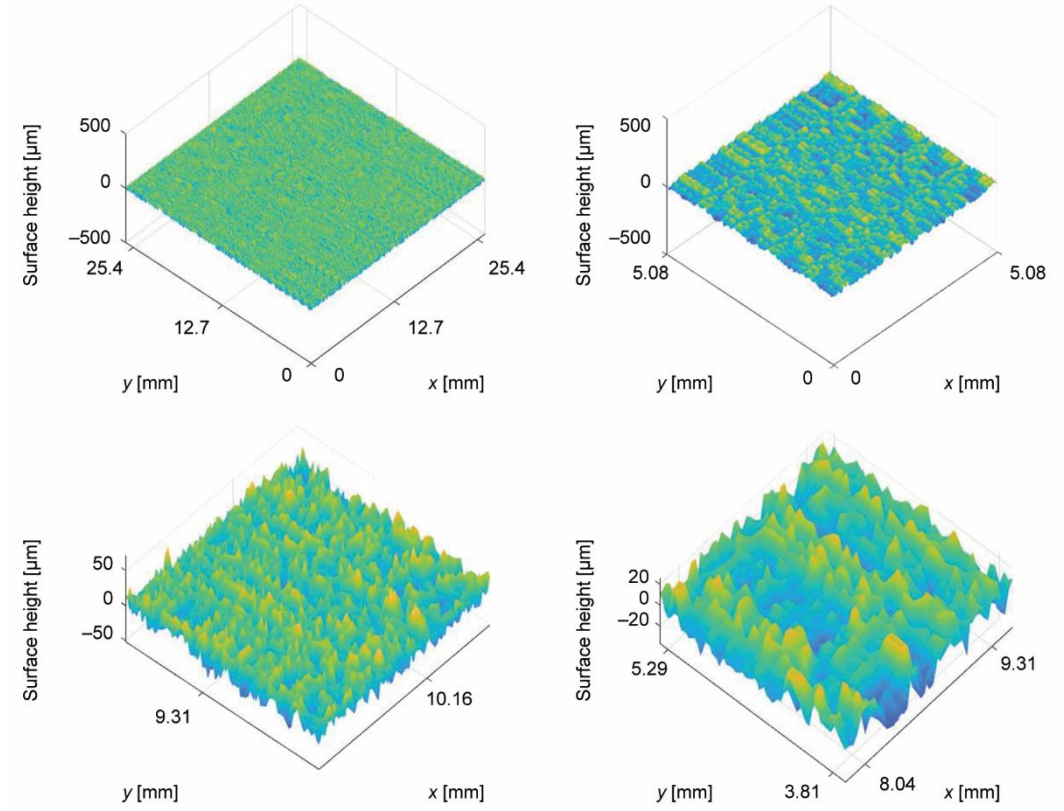

Figure 3. Simulated surfaces with fractal characteristics

Conclusion

In this paper, a simulation model of a grinding wheel with self-affine and random distribution of abrasive spacing is designed based on fractal theory. The model accepts the measured grinding wheel surface roughness parameters as input parameters and generates the roughness parameters from the simulation model surface as output parameters. A fractal surface simulation model of a grinding wheel with non-Gaussian random distribution and random variation of abrasive spacing is established based on the W-M function, the Johnson transformation system, a random function, and a fractal interpolation algorithm. The fractal function is employed in the construction of the surface roughness function, while the fractal interpolation algorithm and the random number algorithm are utilized to adjust the distribution array of the abrasive spacing. Consequently, the model is capable of describing the self-affinity of the surface roughness of the grinding wheel and the randomness and self-affinity of the distribution of the abrasive spacing. The recently developed two-scale fractal theory [20, 21] can be also employed in the present study.

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