DIMENSIONLESS ANALYSIS FOR THE 3-D PRINTING PROCESS

by

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The dimensionless analysis is a highly effective mathematical tool for the physical comprehension of intricate phenomena. This paper employs the aforementioned mathematical tool to study the 3-D printing process. A number of criteria for a precise printing process have been identified, which can be used to optimize the printing process and offer a new strategy to improve its printing accuracy.

Key words: *exact printing, dimensionless analysis, mathematical model*

Introduction

The precise methodology of printing has been the subject of considerable interest in the fields of material science, nanotechnology, and physics [1]. In particular, the printing process is of great importance, particularly in the context of soft robotic systems [2-5]. In order to control the printing process with extreme precision, the machine vision technology was employed to regulate the printing process [6, 7]. This technology enables the monitoring of the printing jet, allowing for the identification and correction of any minor disturbances, which are then fed back to the controller to adjust the printing parameters, such as the printing velocity. The process is complex, yet it can precisely monitor the printing jet, as illustrated in fig. 1.

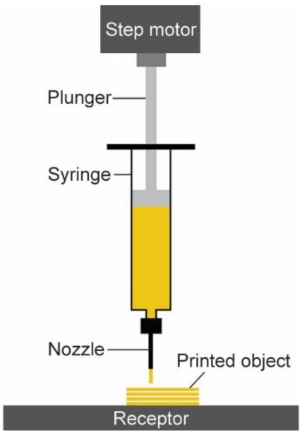

Figure 1. The 3-D printing process

Due to the solvent evaporation, it is nearly impossible to achieve precise control of the printing process, given the multitude of factors that influence the process [8-12]. It is of paramount importance to identify the primary factors that contribute to the instability of the printing jet. This paper employs the technique of dimensionless analysis to ascertain the extent to which each factor affects the diameter of the printed object.

In a recent study, Estrada-Diaz *et al.* [13] employed dimensionless analysis to investigate the electrospinning process, resulting in the development of a valuable mathematical model for electrohydrodynamics. This model has proven to be highly beneficial for the electrospinning process and the bubble electrospinning process [14-18], which are utilized for the

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fabrication of nanofibers. The aforementioned analysis can be extended to the 3-D printing process. Figure 1 depicts the experimental setup employed for the 3-D printing process.

Dimensionless analysis

The diameter of the printed object is affected by pressure, P, velocity, V, density, ρ, viscosity, η, nozzle diameter, d, receptor distance, h, by a similar analysis as that in [13], we obtain the following formulation:

$$D = k\frac{P^a d^{(1+a-b-c)} h^c}{\rho^b \eta^{a-b} V^{a+b}} \tag{1}$$

where k, a, b, and c are constants, which are relative to the fractal dimensions [19, 20]. According to eq. (1) we have some important criteria for the printing process.

Stability criterion

The receptor distance will greatly affect the instability of the printing process [12]. If a stable printing process can be guaranteed as discussed in [12], then eq. (1) reduces to the following formulation:

$$D = k\frac{P^a d^{(1+a-b)}}{\rho^b \eta^{a-b} V^{a+b}} \tag{2}$$

Viscosity criterion

The viscosity, η, is an important factor for a printable process. If it is too small, a discontinuous printing process is predicted. When it is too large, the process becomes impossible. If we can control the printing process, and let $a = b$, then eq. (2) can be further simplified:

$$D = kd\frac{P^a}{\rho^a V^{2a}} \tag{3}$$

Energy criterion

In eq. (3), we can introduce two new variables, the kinetic energy, K, and the pressure potential, E, defined:

$$K = \frac{1}{2}V^2 \tag{4}$$

$$E = \frac{P}{\rho} \tag{5}$$

Equation (3) becomes:

$$D = k_0 d\left(\frac{E}{K}\right)^a \tag{6}$$

where k_0 is a constant.

Equation (6) implies that the printing process is controlled by the kinetic energy and pressure potential.

Criterion for the exact printing

According to the Bernoulli equation for an inviscid and incompressible fluid, we have:

$$K + E = B \tag{7}$$

where B is a Bernoulli constant. For the printing process, eq. (7) is not valid, but it can be approximately used for controlling the values of K and E. When $K = E$, we have:

$$D = k_0 d \tag{8}$$

This is the exact printing process.

Conclusions

This article, for the first time ever, presents a physical approach to the exact printing process, akin to the spider's printing manner [21], illuminating a novel trajectory for 3-D printing technology. Furthermore, the 3-D printing technology can print metamaterials for Buffer's vibration attenuation [22], micro/nano devices [23-27] and even concretes [28, 29]. It can be also used to control bearing furface [30] by printing Babbitt melts onto the surface.

Now 3-D printing method presents the most advanced technology in various fields [31, 32]. Although a self-contained formulation is proposed in this article, which is of great significance for the precise printing process and the optimal design of the printing process, further experimental verification and high-precision numerical verification [33] are still required.

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