# TWO STAGES NON-LOCAL MEANS ALGORITHM BASED ON WAVELET TRANSFORM

by

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*This paper presents an innovative wavelet transform-based two-stage non-local means algorithm that offers a new approach to image de-noising. It first decomposes the noisy image into four sub-images with different frequencies, which provides a more detailed analysis. Then, it fuses three high frequency sub-images into one high frequency sub-image, which enhances the image quality. The low-frequency sub-image and the fused high frequency sub-image are de-noised using the non-local means algorithm, which is a highly effective and robust de-noising technique. Furthermore, the de-noised low-frequency sub-image and the high frequency sub-image are merged to form the initial de-noised image. Finally, the initial de-noised image is de-noised again using the non-local means algorithm to produce the final de-noised image. The proposed algorithm is able to effectively remove noise while preserving edges and details.*

Key words: *image de-noising, non-local means, two stages de-noising, wavelet transform*

## Introduction

Images are susceptible to noise contamination during acquisition and transmission due to external signal interference. This presents an exciting opportunity to enhance image quality through image research. Image de-noising pre-processing is an amazingly effective way to improve image quality, which is great news for scientists who can now do further research with much better results.

Spatial domain de-noising algorithms are an absolutely amazing branch of image de-noising algorithms. They analyze the pixel characteristics of the image itself and directly process the grayscale values of the image pixels one by one to achieve the goal of smoothing the image. However, spatial-domain de-noising algorithms [1-4] are based on simple local filtering ideas and do not fully utilize the structural information in the image, which presents an exciting opportunity for improvement. By fully utilizing the structural information in the image, we can avoid mistakenly identifying high frequency information such as image edges and details as noise, resulting in sharp, clear image edges, textures, and details. Fortunately, various algorithms have been proposed to protect the edge and detail features of the image. Buades *et al.* [5] came up with the brilliant non-local means algorithm, which has been a huge hit with many. This algorithm is a real game-changer. It fully exploits the redundant information and structural self-similarity in natural images, making it an absolute must-have for anyone

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looking to enhance their image processing skills. It is a truly remarkable process. It searches for similar image blocks around the pixel to be processed, assigns weights to them, and adjusts the grayscale values of the pixel based on these weights. This ensures the suppression of noise while maximizing the preservation of image edges and detail features.

In a thrilling development, researchers have discovered significant potential in the classic non-local means (NLM) algorithm and have proposed several improved NLM de-noising algorithms. Dabov *et al.* [6] introduced the block-matching and 3-D filtering method, which is an amazing new way of filtering image blocks with similar structures in the 3-D transform domain. It exploits the similarity between image blocks in a way that is truly impressive. Tasdizen *et al.* [7] came up with an amazing method based on principal component analysis that projects image neighborhood vectors into a low-dimensional subspace and computes similarity weights based on the distances in the subspace. Grewenig *et al.* [8] came up with a brilliant method to detect similar image blocks. Wu *et al.* [9] introduced an amazing new method called probability-based NLM (PNLM). This method uses probability distributions to determine the weights in the NLM algorithm, which makes it even more effective than before. Zhang *et al.* [10] created an amazing non-local image de-noising model based on singular value decomposition, which adheres to the principles of *two directions* and *seeking similarities while preserving differences*. Jacques [11] came up with an amazing acceleration method for the NLM algorithm using integral images, which avoids repetitive calculations by computing the square of the difference between two images. Cai *et al.* [12] introduced an amazing method based on candidate set selection. It first identifies image blocks with similar grayscale distributions to form a candidate set, and then selects image blocks with more similar structures from it. Nguyen *et al.* [13] made a significant improvement to the James Stein-type central pixel weight estimation method. Verma and Pandey [14] came up with an amazing non-local filtering method based on gray relation analysis that reselects the smoothing parameters for each pixel. Bo *et al.* [15] introduced an amazing method that uses fuzzy theory to redefine the similarity between image pixels. Xin *et al.* [16] utilized an asymptotic non-local means (ANLM) denoising algorithm which enhances the denoising process by applying multiple NLM filters with varying parameters to noisy images, thus achieving a notable improvement in PSNR compared to the classical NLM algorithm.

Building on the aforementioned research, this paper presents a secondary NLM de-noising algorithm based on the cutting-edge wavelet transform. The traditional NLM algorithm has a few limitations when it comes to accurately distinguishing between flat regions and edges in de-noising. This can result in the blurring of image edges and details. However, the wavelet transform, which is known to accurately extract image edges, is used in this algorithm to overcome these limitations. This proposed method will be compared with the de-noising algorithms presented in [5, 9, 16], in section *Simulation experiment.*

## Non-local means algorithm

The NLM algorithm is a weighted average de-noising algorithm. The algorithm initiates by creating a search window surrounding the pixel to be processed. Within this window, the algorithm evaluates the degree of similarity between the image block containing the pixel to be processed and all other image blocks within the search window. Subsequently, weights are assigned to the pixels based on this similarity. The greater the degree of similarity between the image block containing the pixel to be processed and a specific image block within the search window, the higher the weight assigned to that pixel. Finally, a weighted sum of all pixels in the search window, with the exception of the pixel to be processed, is performed in order to obtain the grayscale value of the processed pixel, thereby achieving the desired filtering effect.

A natural image is denoted as $A$, and the image corrupted by additive Gaussian noise is denoted as $F$. The expression for $F$ is:

$$F = A + n = \{ f(i) \mid i \in \Omega \} \tag{1}$$

where $n$ represents Gaussian noise (with a mean of 0 and variance of $\sigma^2$), $\Omega$ – the image region, and $f(i)$ – the grayscale value of the pixel $i$.

Let the image resulting from de-noising the noisy image $F$ using the NLM algorithm be denoted as $F$. The expression for $F$ is:

$$\hat{F} = \{ \hat{f}(i) \mid i \in \Omega \} \tag{2}$$

where the weights $\hat{f}(i)$ are calculated using:

$$\hat{f}(i) = \frac{\sum_{j\in\Omega_i} w(i,j) f(j)}{\sum_{j\in\Omega_i} w(i,j)} \tag{3}$$

where $\Omega_i$ is the square region centered at $i$ with a size of $p \times p$, the weight function $w(i, j)$ measures the similarity between the image block $I(i)$ and $I(j)$, and $\Sigma_{j\in\Omega_i} w(i,j)$ is the normalization factor ensuring the sum of weights is 1. The weight function $w(i, j)$ is:

$$w(i,j) = \exp\left( -\frac{\mathrm{d}(i,j)}{h^2} \right) \tag{4}$$

where $h$ is the smoothing parameter, typically set as the standard deviation of the noise and $d(i, j)$ – the weighted Euclidean distance between the image block $I(i)$ and $I(j)$, expressed:

$$d(i,j) = \| I(i) - I(j) \|_{2,a}^2 = \| G_a \otimes [I(i) - I(j)] \|_2^2 \tag{5}$$

As shown in fig. 1, within the search window $\Omega_i$, $I(j)$ is more similar to $I(i)$ compared to $I(k)$, reflected in the formula as $d(i, j) < d(i,k)$, leading to $w(i, j) > w(i,k)$. This demonstrates the NLM algorithm capability to locate similar image blocks around the pixel to be processed and assign weights based on similarity, where higher similarity yields higher weights.

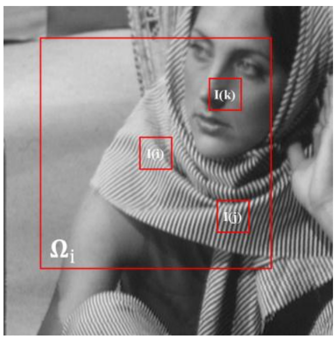

**Figure 1. Schematic diagram of NLM algorithm**

## Wavelet transform-based two stages non-local mean algorithm

The selection of the smoothing parameter is of paramount importance in the NLM algorithm. A low value of can result in sub-optimal de-noising effects, leaving significant noise in the image. Conversely, a large value of can result in excessive de-noising, which can lead to the loss of detailed features in the image. Consequently, the quality of the image can be optimized by selecting an appropriate smoothing parameter.

To effectively remove noise from images, the NLM algorithm typically selects a larger smoothing parameter, which can result in the blurring of edges and details of the image after de-noising. To address this issue, the improved algorithm employs wavelet decomposition and reconstruction techniques to propose a wavelet transform-based two-stage non-local mean algorithm, which is described as follows.

– Perform wavelet decomposition and reconstruction on the noisy image to obtain the low-frequency sub-image $A$ and the high frequency sub-images $H, V, D$.
– Fuse the high frequency sub-images $H, V, D$ to obtain the high frequency sub-image $HVD$.
– Apply the NLM algorithm separately to the sub-image $A$ and the sub-image $HVD$ for de-noising, resulting in pre-processed sub-image $\bar{A}$ and pre-processed sub-image $\overline{HVD}$.
– Fuse the pre-processed sub-image $\bar{A}$ and pre-processed sub-image $\overline{HVD}$ to obtain a first stage de-noised image.
– Apply the NLM algorithm to the first de-noised image for a second round of de-noising, resulting in a second-order de-noised image.

Figure 2 shows the amazing de-noising process of the Cameraman image. Let us take a closer look at fig. 2. Figure 2(b) is the noisy image. Figure 2(c) is the low-frequency sub-image A after wavelet decomposition and reconstruction. Figure 2(d) is the high frequency sub-image $HVD$ after wavelet decomposition. Figure 2(e): now for the really cool part. We have preprocessed sub-image $\bar{A}$ using the NLM algorithm and sub-image $\overline{HVD}$ using the same algorithm. Then we fused the two images to create the first-order de-noised image. Figure 2(f) shows the result of applying the NLM algorithm to the initially de-noised image.

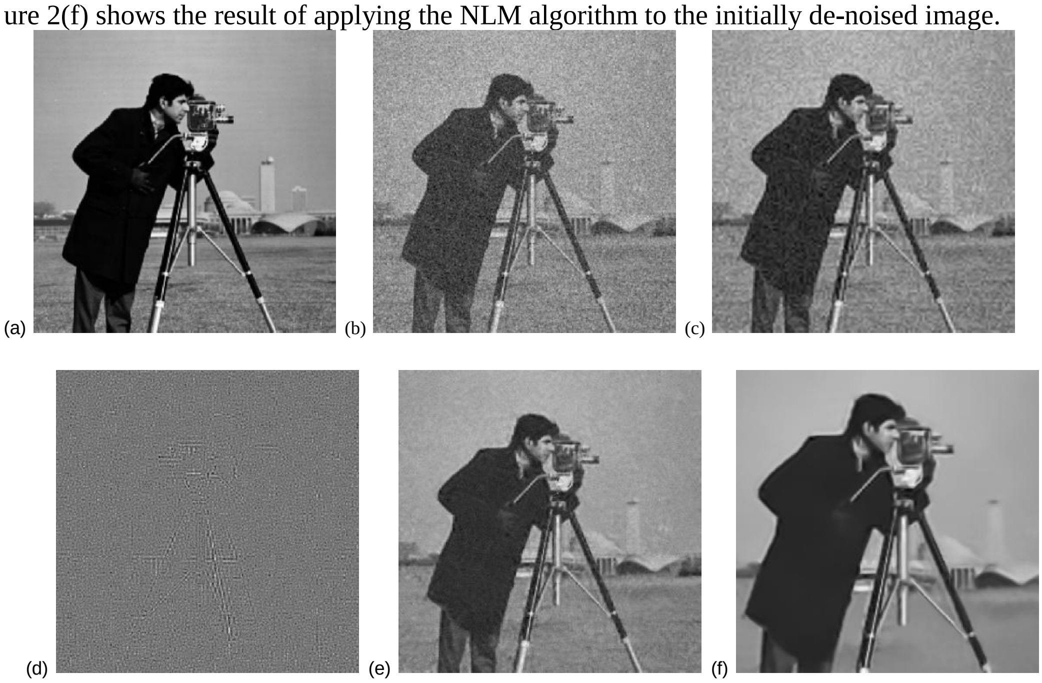


**Figure 2. De-noising process of the proposed algorithm; (a) original image, (b) noisy image, (c) low-frequency sub-image, (d) high frequency sub-image, (e) fused sub-image, and (f) final de-noised image**

## Simulation experiment

The simulation experiment uses six commonly used images for de-noising investigation. The six test images are shown in fig. 3, and the classical NLM [5], PNLM [9], and ANLM [16] algorithms were chosen for comparative analysis. In NLM, PNLM, and ANLM, the search window size was set to, while the similarity window size was set to. The appropriate NLM smoothing parameter h was chosen.

The simulation experiment uses six of the most commonly used images for de-noising investigation. Figure 3 shows the six test images, and we also present the classical NLM [5], PNLM [9], and ANLM [16] algorithms for comparative analysis. In NLM, PNLM, and ANLM, we set the search window size to 21 and the similarity window size to 5. We chose the appropriate NLM smoothing parameter *h*, which was the perfect choice.

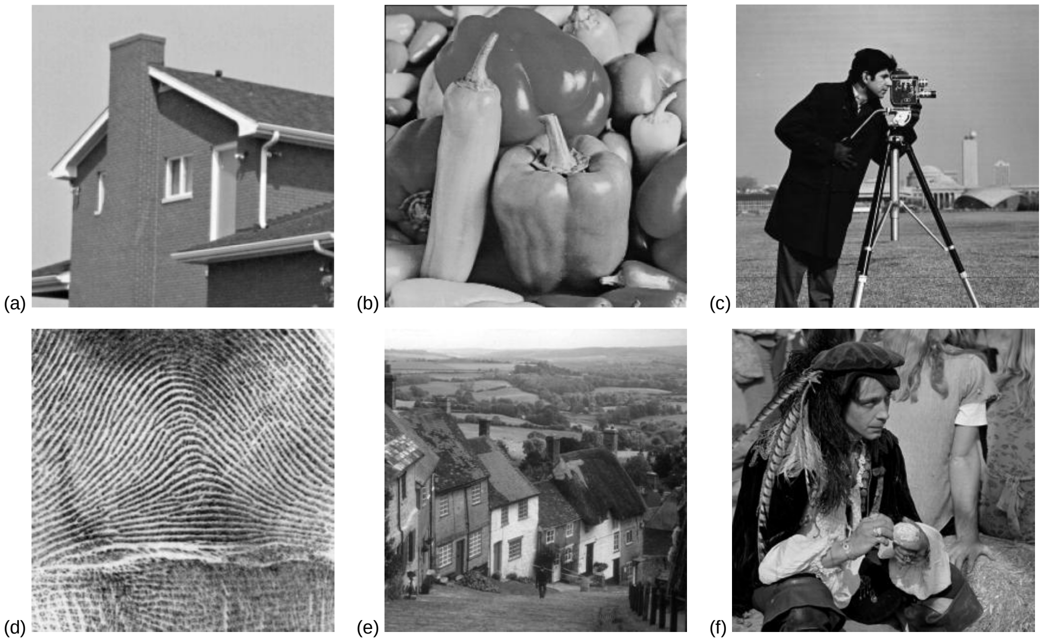


**Figure 3. Test images; (a) house, (b) peppers, (c) camera, (d) finger, (e) goldhill, and (f) man**

**Table 1. De-noising results (PSNR) of the different de-noising algorithms**

| Image | NLM | | PNLM | | ANLM | | Proposed | |
|---|---|---|---|---|---|---|---|---|
| | 25 | 30 | 25 | 30 | 25 | 30 | 25 | 30 |
| House 256 × 256 | 30.90 | 29.74 | 31.30 | 30.26 | 31.28 | 30.47 | 32.13 | 31.31 |
| Peppers 256 × 256 | 28.72 | 27.70 | 29.04 | 28.08 | 28.75 | 27.93 | 29.40 | 28.60 |
| Camera 256 × 256 | 28.23 | 27.27 | 28.39 | 27.58 | 28.07 | 27.43 | 28.34 | 27.66 |
| Finger 512 × 512 | 26.56 | 25.41 | 26.41 | 25.45 | 26.41 | 25.63 | 27.26 | 27.25 |
| Goldhill 512 × 512 | 28.30 | 27.40 | 28.37 | 27.51 | 28.69 | 27.94 | 29.29 | 28.56 |
| Man 512 × 512 | 28.33 | 27.46 | 28.45 | 27.62 | 28.63 | 27.89 | 29.17 | 28.45 |
| Average | 28.01 | 27.85 | 28.14 | 28.07 | 28.12 | 28.18 | 28.66 | 28.83 |

Table 1 compares the PSNR results of four algorithms. Table 1 indicates that the proposed algorithm has stronger de-noising capability than other methods. Specifically, when compared to the classic NLM algorithm, the proposed approach results in an average increase in PSNR of 2.32% and 3.52% for noise standard deviations of 25 and 30, respectively. In comparison to the PNLM algorithm, the proposed method achieves average PSNR improvements of 1.85% and 2.71% under the same noise conditions. Similarly, when compared to the ANLM algorithm, average PSNR enhancements of 1.92% and 2.31% are observed for the same levels of noise.

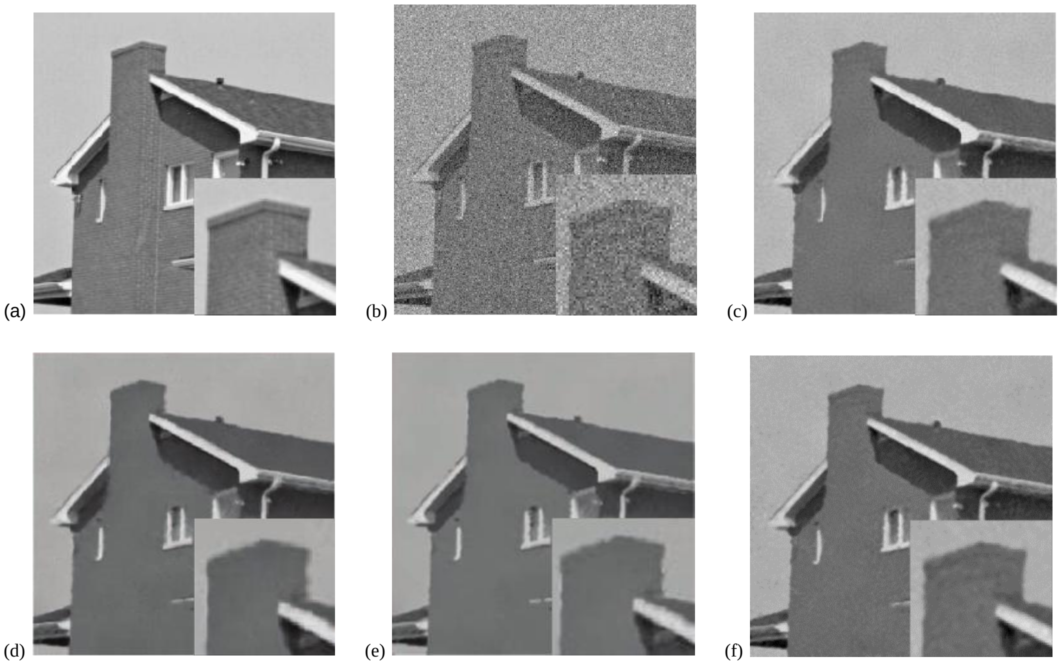


**Figure 4. De-noising results of the different de-noising algorithms; (a) original image, (b) noisy image ($\sigma$ = 30), (c) NLM (PSNR = 29.74), (d) PNLM (PSNR = 30.26), (e) ANLM (PSNR = 30.47), and (f) proposed (PSNR = 31.31)**

Figure 4 shows the amazing de-noising results of four algorithms on the house image. From figs. 4(d)-4(f), we can see that the PNLM and ANLM algorithms produce some pretty amazingly smooth results for the house image. In contrast, the proposed algorithm is able to protect the edges after de-noising, which is a fantastic result.

## Conclusion

The proposed wavelet transform-based two stages NLM algorithm cleverly integrates the concept of wavelet decomposition with the classic NLM algorithm. It has more advantages than the Chebyshev wavelet transform [17]. It applies NLM denoising twice on noisy images in different frequency domains, achieving significant improvements in PSNR in numerical experiments and effectively preserving the edge features of the images. While denoising approaches based on spatial and frequency domains have traditionally been treated as distinct methods in digital image denoising, this study highlights the great potential of innovatively combining these approaches to develop more advanced denoising algorithms.

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