HOLDER TYPE INEQUALITY FOR NEW CONFORMABLE FRACTIONAL INTEGRAL AND SOME RELATED RESULTS

by

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The paper is concerned with the investigation of Holder's inequality. Firstly, we establish a new Holder's type inequality by using new conformable fractional integral which was introduced by Kajouni et al. [1]. Secondly, we give a reverse conformable fractional integral Holder's type inequality. Finally, we obtain some results related with conformable fractional integral Holder's type inequality.

Key words: *fractional integral Holder's inequality, reverse inequality fractional integral Minkowski's inequality*

Introduction

Assume that $\Pi_1(\ell)$, $\Pi_2(\ell)$ are continuous functions whose domain of definition is the interval $[\hbar_1, \hbar_2]$. Let $\xi, \beta \in \mathbb{R}$ satisfy the equation $1/\xi + 1/\beta = 1$, where $\xi > 1$. Then the integral form corresponding to the well-known inequality, established by Holder, can be expressed:

$$\int_{\hbar_1}^{\hbar_2} \left|\Pi_1(\ell)\Pi_2(\ell)\right| \mathrm{d}\ell \le \left(\int_{\hbar_1}^{\hbar_2} \left|\Pi_1(\ell)\right|^{\xi} \mathrm{d}\ell\right)^{1/\xi} \left(\int_{\hbar_1}^{\hbar_2} \left|\Pi_2(\ell)\right|^{\beta} \mathrm{d}\ell\right)^{1/\beta}$$

which can be found in [2, 3].

It needs to be pointed out that Hölder's inequality has important applications in dealing with some problems originated from modern pure mathematics. As a consequence, it has captivated many researchers to investigate extensively. For more detail, the reader is referred to [4-7]. Some reverse Holder's inequalities can also be found in [5-7].

As is well known, fractional calculus, different from traditional calculus, is among the most appropriate tools for investigating electromagnetism, speech signals, fluid mechanics, and so on. For more detail, we refer the readers to [8-12].

Recently, the authors in [13] applied the limit definition of the derivative to establish a new fractional derivative, which is called the conformable fractional derivative. The definition of the conformable fractional integral also was given in [13].

Very recently, Kajouni *et al.* [1] defined a new conformable fractional integral as follows.

Definition 1. Let $\alpha \in (0, 1)$ and $a \ge 0$, let $\hbar$ be a function defined on $(a, t]$. Then, the α-fractional integral of $\hbar$ is defined:

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$$I_\alpha^a(\hbar)(t)=\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar(\rho)\mathrm{d}\rho$$

which is different from the definition of [1].

The primary goal of this research is to establish a new conformable fractional integral Hölder's inequality, some related results also are obtained.

Main results

In this section, we shall state and prove the main results.

Theorem 1. (Holder's inequality) Let $\hbar_1(\rho)$ and $\hbar_2(\rho)$ be α-conformable fractional integrable and non-negative on (a, t), $0<\alpha<1$ and $1/\lambdabar_1+1/\lambdabar_2=1$ with $\lambdabar_1>1$. Then one has:

$$\begin{gathered}I_\alpha^a(\hbar_1\hbar_2)(t)=\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar_1(\rho)\hbar_2(\rho)\mathrm{d}\rho\le\\ \le\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar_1^{\lambdabar_1}(\rho)\mathrm{d}\rho\right)^{1/\lambdabar_1}\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar_2^{\lambdabar_2}(\rho)\mathrm{d}\rho\right)^{1/\lambdabar_2}=\\ =\left(I_\alpha^a(\hbar_1^{\lambdabar_1})(t)\right)^{1/\lambdabar_1}\left(I_\alpha^a(\hbar_2^{\lambdabar_2})(t)\right)^{1/\lambdabar_2}\end{gathered}\tag{1}$$

Proof. It is assumed, without any loss of generality, that:

$$\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar_1^{\lambdabar_1}(\rho)\mathrm{d}\rho\right)^{1/\lambdabar_1}\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar_2^{\lambdabar_2}(\rho)\mathrm{d}\rho\right)^{1/\lambdabar_2}\neq 0$$

and taking

$$\Pi_1=\frac{\hbar_1^{\lambdabar_1}(\rho)}{\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar_1^{\lambdabar_1}(\rho)\mathrm{d}\rho},\quad \Pi_2=\frac{\hbar_2^{\lambdabar_2}(\rho)}{\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar_2^{\lambdabar_2}(\rho)\mathrm{d}\rho}$$

In the light of the Young inequality in [14], it is easy to conclude:

$$\begin{gathered}\int_a^t \mathrm{e}^{(1-\alpha)\rho}\frac{\hbar_1(\rho)}{\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar_1^{\lambdabar_1}(\rho)\mathrm{d}\rho\right)^{1/\lambdabar_1}}\frac{\hbar_2(\rho)}{\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar_2^{\lambdabar_2}(\rho)\mathrm{d}\rho\right)^{1/\lambdabar_2}}\mathrm{d}\rho=\\ =\int_a^t \mathrm{e}^{(1-\alpha)\rho}\Pi_1^{1/\lambdabar_1}\Pi_2^{1/\lambdabar_2}\mathrm{d}\rho\le\\ \le\int_a^t \mathrm{e}^{(1-\alpha)\rho}\left(\frac{\Pi_1}{\lambdabar_1}+\frac{\Pi_2}{\lambdabar_2}\right)\mathrm{d}\rho=\\ =\frac{1}{\lambdabar_1}\int_a^t \mathrm{e}^{(1-\alpha)\rho}\frac{\hbar_1^{\lambdabar_1}(\rho)}{\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar_1^{\lambdabar_1}(\rho)\mathrm{d}\rho}\mathrm{d}\rho+\\ +\frac{1}{\lambdabar_2}\int_a^t \mathrm{e}^{(1-\alpha)\rho}\frac{\hbar_2^{\lambdabar_2}(\rho)}{\int_a^t \mathrm{e}^{(1-\alpha)\rho}\hbar_2^{\lambdabar_2}(\rho)\mathrm{d}\rho}\mathrm{d}\rho=1\end{gathered}$$

Theorem 2. (Reverse Holder's inequality) Let $\hbar_1(\rho)$ and $\hbar_2(\rho)$ be α-conformable fractional integrable and non-negative on (a, t), $0 < \alpha < 1$ and $1/\lambda_1 + 1/\lambda_2 = 1$ with $0 < \lambda_1 < 1$. Then one has:

$$I_\alpha^a(\hbar_1\hbar_2)(t) = \int_a^t e^{(1-\alpha)\rho}\hbar_1(\rho)\hbar_2(\rho)d\rho \geq$$
$$\geq \left(\int_a^t e^{(1-\alpha)\rho}\hbar_1^{\lambda_1}(\rho)d\rho\right)^{1/\lambda_1}\left(\int_a^t e^{(1-\alpha)\rho}\hbar_2^{\lambda_2}(\rho)d\rho\right)^{1/\lambda_2} = \tag{2}$$
$$= \left(I_\alpha^a(\hbar_1^{\lambda_1})(t)\right)^{1/\lambda_1}\left(I_\alpha^a(\hbar_2^{\lambda_2})(t)\right)^{1/\lambda_2}$$

Proof. It is supposed, without any loss of generality, that:

$$\left(\int_a^t e^{(1-\alpha)\rho}\hbar_1^{\lambda_1}(\rho)d\rho\right)^{1/\lambda_1}\left(\int_a^t e^{(1-\alpha)\rho}\hbar_2^{\lambda_2}(\rho)d\rho\right)^{1/\lambda_2} \neq 0$$

and set

$$\Theta_1 = \frac{\hbar_1^{\lambda_1}(\rho)}{\int_a^t e^{(1-\alpha)\rho}\hbar_1^{\lambda_1}(\rho)d\rho}, \quad \Theta_2 = \frac{\hbar_2^{\lambda_2}(\rho)}{\int_a^t e^{(1-\alpha)\rho}\hbar_2^{\lambda_2}(\rho)d\rho}$$

Based on the reverseYoung inequality in [14], it is not difficult to yield:

$$\int_a^t e^{(1-\alpha)\rho}\frac{\hbar_1(\rho)}{\left(\int_a^t e^{(1-\alpha)\rho}\hbar_1^{\lambda_1}(\rho)d\rho\right)^{1/\lambda_1}}\frac{\hbar_2(\rho)}{\left(\int_a^t e^{(1-\alpha)\rho}\hbar_2^{\lambda_2}(\rho)d\rho\right)^{1/\lambda_2}}d\rho =$$
$$= \int_a^t e^{(1-\alpha)\rho}\Theta_1^{1/\lambda_1}\Theta_2^{1/\lambda_2}d\rho \geq$$
$$\geq \int_a^t e^{(1-\alpha)\rho}(\Theta_1/\lambda_1 + \Theta_2/\lambda_2)d\rho =$$
$$= \frac{1}{\lambda_1}\int_a^t e^{(1-\alpha)\rho}\frac{\hbar_1^{\lambda_1}(\rho)}{\int_a^t e^{(1-\alpha)\rho}\hbar_1^{\lambda_1}(\rho)d\rho}d\rho +$$
$$+ \frac{1}{\lambda_2}\int_a^t e^{(1-\alpha)\rho}\frac{\hbar_2^{\lambda_2}(\rho)}{\int_a^t e^{(1-\alpha)\rho}\hbar_2^{\lambda_2}(\rho)d\rho}d\rho = 1$$

On the basis of *Theorems 1* and *2*, we can obtain the following generalizations.

Corollary 1. Let $\hbar_l(\rho)$ be α-conformable fractional integrable and non-negativeon (a, t), $0 < \alpha < 1$ and $\lambda_l \in R$, $l = 1, 2,..., m$, $1/\lambda_1 + 1/\lambda_2 + \cdots + 1/\lambda_m = 1$.

Then:

– For $\lambda_l > 1$, one has:

$$I_\alpha^a\left(\prod_{l=1}^m \hbar_l\right)(t) \leq \prod_{l=1}^m\left(I_\alpha^a(\hbar_l^{\lambda_l})(t)\right)^{1/\lambda_l} \tag{3}$$

– For $\lambda_l \in (0, l)$ and $\lambda_l < 0$, $l = 2, ..., m$, one has:

$$I_\alpha^a\left(\prod_{l=1}^m \hbar_l\right)(t) \geq \prod_{l=1}^m \left(I_\alpha^a(\hbar_l^{\lambda_l})(t)\right)^{1/\lambda_l} \tag{4}$$

Theorem 3. (Minkowski's inequality) If $\hbar_1(\rho)$ and $\hbar_2(\rho)$ are α-conformable fractional integrable and non-negative on (a, t), $0 < \alpha < 1$ and $\lambda_1 > 1$. Then one has:

$$\begin{gathered}\left(I_\alpha^a((\hbar_1+\hbar_2)^{\lambda})(t)\right)^{1/\lambda} = \left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_1(\rho)+\hbar_2(\rho))^{\lambda}\mathrm{d}\rho\right)^{\lambda} \leq \\ \leq \left(I_\alpha^a(\hbar_1^{\lambda})(t)\right)^{1/\lambda} + \left(I_\alpha^a(\hbar_2^{\lambda})(t)\right)^{1/\lambda}\end{gathered} \tag{5}$$

Proof. By direct computation, we have:

$$I_\alpha^a((\hbar_1+\hbar_2)^{\lambda})(t) = \int_a^t \mathrm{e}^{(1-\alpha)\rho}\left((\hbar_1(\rho)+\hbar_2(\rho))(\hbar_1(\rho)+\hbar_2(\rho))^{\lambda-1}\right)\mathrm{d}\rho =$$

$$= \int_a^t \mathrm{e}^{(1-\alpha)\rho}\left(\hbar_1(\rho)(\hbar_1(\rho)+\hbar_2(\rho))^{\lambda-1}\right)\mathrm{d}\rho + \int_a^t \mathrm{e}^{(1-\alpha)\rho}\left(\hbar_2(\rho)(\hbar_1(\rho)+\hbar_2(\rho))^{\lambda-1}\right)\mathrm{d}\rho$$

Thanks to the previous equation and *Theorem 1*, we have:

$$\begin{gathered}\int_a^t \mathrm{e}^{(1-\alpha)\rho}\left(\hbar_1(\rho)(\hbar_1(\rho)+\hbar_2(\rho))^{\lambda-1}\right)\mathrm{d}\rho + \int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_2(\rho)(\hbar_1(\rho)+\hbar_2(\rho))^{\lambda-1})\mathrm{d}\rho \leq \\ \leq \left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_1^{\lambda}(\rho))\mathrm{d}\rho\right)^{1/\lambda}\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_1(\rho)+\hbar_2(\rho))^{(\lambda-1)q}\mathrm{d}\rho\right)^{1/q} + \\ + \left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_2^{\lambda}(\rho))\mathrm{d}\rho\right)^{1/\lambda}\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_1(\rho)+\hbar_2(\rho))^{(\lambda-1)q}\mathrm{d}\rho\right)^{1/q} = \\ = \left[\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_1^{\lambda}(\rho))\mathrm{d}\rho\right)^{1/\lambda} + \left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_2^{\lambda}(\rho))\mathrm{d}\rho\right)^{1/\lambda}\right]\cdot \\ \cdot\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_1(\rho)+\hbar_2(\rho))^{(\lambda-1)q}\mathrm{d}\rho\right)^{1/q}\end{gathered}$$

So, it can be derived that:

$$\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_1(\rho)+\hbar_2(\rho))^{(\lambda-1)q}\mathrm{d}\rho\right)^{1-1/q} \leq \left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_1^{\lambda}(\rho))\mathrm{d}\rho\right)^{1/\lambda} + \left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_2^{\lambda}(\rho))\mathrm{d}\rho\right)^{1/\lambda}$$

Since $1/\lambda + 1/q = 1$, we have the conclusion presented in *Theorem 3*.

Theorem 4. (reverse Minkowski's inequality) If $\hbar_1(\rho)$ and $\hbar_2(\rho)$ are α-conformable fractional integrable and non-negative on (a, t), $0 < \alpha < 1$, and $0 < \lambda < 1$, then one has:

$$\begin{gathered}\left(I_\alpha^a((\hbar_1+\hbar_2)^{\lambda})(t)\right)^{1/\lambda} = \left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_1(\rho)+\hbar_2(\rho))^{\lambda}\mathrm{d}s\right)^{\lambda} \geq \\ \geq \left(I_\alpha^a(\hbar_1^{\lambda})(t)\right)^{1/\lambda} + \left(I_\alpha^a(\hbar_2^{\lambda})(t)\right)^{1/\lambda}\end{gathered} \tag{6}$$

Proof. Let $0 < ƛ < 1$

$$\frac{1}{\gamma} = ƛ,\ \frac{1}{\lambda} = 1 - ƛ,\ a_k = \mu_k^{ƛ},\ b_k = \nu_k^{1/ƛ-1}$$

with the help of the discrete Holder's inequality:

$$\sum_{k=1}^{n} a_k b_k \le \left(\sum_{k=1}^{n} a_k^{\gamma}\right)^{1/\gamma} \left(\sum_{k=1}^{n} b_k^{\lambda}\right)^{1/\lambda},\ \gamma > 1\ \frac{1}{\gamma} + \frac{1}{\lambda} = 1$$

we can derive that:

$$\sum_{k=1}^{n} \mu_k^{ƛ} \nu_k^{1/ƛ-1} \le \left(\sum_{k=1}^{n} \mu_k\right)^{ƛ} \left(\sum_{k=1}^{n} \nu_k^{1/ƛ}\right)^{1-ƛ} \tag{7}$$

The expressions of Φ_1, Φ_2, and M can be defined:

$$\Phi_1 = \int_a^t \mathrm{e}^{(1-\alpha)\rho} \hbar_1^{ƛ}(\rho)\mathrm{d}\rho,\ \Phi_2 = a\int_a^t \mathrm{e}^{(1-\alpha)\rho} \hbar_2^{ƛ}(\rho)\mathrm{d}\rho$$

$$M = \left(\int_a^t \mathrm{e}^{(1-\alpha)\rho} \hbar_1^{ƛ}(\rho)\mathrm{d}\rho\right)^{1/ƛ} + \left(\int_a^t \mathrm{e}^{(1-\alpha)\rho} \hbar_2^{ƛ}(\rho)\mathrm{d}\rho\right)^{1/ƛ} = \Phi_1^{1/ƛ} + \Phi_2^{1/ƛ}$$

According to eq. (7), one has the result:

$$\mathrm{M} = \Phi_1^{1/ƛ} + \Phi_2^{1/ƛ} =$$

$$= \Phi_1^{1/ƛ-1} \int_a^t \mathrm{e}^{(1-\alpha)\rho} \hbar_1^{ƛ}(\rho)\mathrm{d}\rho + \Phi_2^{1/ƛ-1} \int_a^t \mathrm{e}^{(1-\alpha)\rho} \hbar_2^{ƛ}(\rho)\mathrm{d}\rho =$$

$$= \int_a^t \mathrm{e}^{(1-\alpha)\rho} \left[\hbar_1^{ƛ}(\rho)\Phi_1^{1/ƛ-1} + \hbar_2^{ƛ}(\rho)\Phi_2^{1/ƛ-1}\right]\mathrm{d}\rho \le$$

$$\le \int_a^t \mathrm{e}^{(1-\alpha)\rho} (\hbar_1(\rho) + \hbar_2(\rho))^{ƛ} (\Phi_1^{1/ƛ} + \Phi_2^{1/ƛ})^{1-ƛ}\mathrm{d}\rho =$$

$$= \int_a^t \mathrm{e}^{(1-\alpha)\rho} (\hbar_1(\rho) + \hbar_2(\rho))^{ƛ}\mathrm{M}^{1-ƛ}\mathrm{d}\rho =$$

$$= M^{1-ƛ} \int_a^t \mathrm{e}^{(1-\alpha)\rho} (\hbar_1(\rho) + \hbar_2(\rho))^{ƛ}\mathrm{d}\rho$$

Next, we give some generalizations of *Theorems 3 and 4* as follows.

Corollary 2. Let $\hbar_l(\rho)$ be α-conformable fractional integrable and non-negative on (a, t), $0 < \alpha < 1$, $l = 1, 2,..., k$. Then:

– For $ƛ > 1$, one has:

$$\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho} \left(\sum_{l=1}^{k} \hbar_l(\rho)\right)^{ƛ} \mathrm{d}\rho\right)^{1/ƛ} \le \sum_{l=1}^{k} \left(\int_a^t \mathrm{e}^{(1-\alpha)\rho} (\hbar_l(\rho))^{ƛ}\mathrm{d}\rho\right)^{1/ƛ} \tag{8}$$

– For $0 < \lambda < 1$, one has:

$$\left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}\left(\sum_{l=1}^k \hbar_l(\rho)\right)^{\lambda} \mathrm{d}\rho\right)^{1/\lambda} \geq \sum_{l=1}^k \left(\int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_l(\rho))^{\lambda}\mathrm{d}\rho\right)^{1/\lambda} \tag{9}$$

From *Corollary 2*, one has the following concl usions.

Corollary 3. Let $\hbar_l(s)$ be α-conformable fractional integrable and non-negative on (a, t), $0 < \alpha < 1$, $l = 1, 2,.., \mathrm{k}$. Then:

– For $\lambda > 1$, one has:

$$\int_a^t \mathrm{e}^{(1-\alpha)\rho}\left(\sum_{l=1}^k \hbar_l(\rho)\right)^{\lambda} \mathrm{d}\rho \geq \sum_{l=1}^k \int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_l(\rho))^{\lambda}\mathrm{d}\rho \tag{10}$$

– For $0 < \lambda < 1$, one has:

$$\int_a^t \mathrm{e}^{(1-\alpha)\rho}\left(\sum_{l=1}^k \hbar_l(\rho)\right)^{\lambda} \mathrm{d}\rho \leq \sum_{l=1}^k \int_a^t \mathrm{e}^{(1-\alpha)\rho}(\hbar_l(\rho))^{\lambda}\mathrm{d}\rho \tag{11}$$

Proof. (1) Assume that $\lambda > 1$, by taking $s = \lambda$, $r = 1$ in Jensen's inequality:

$$\hbar_1 + \hbar_2 + \cdots + \hbar_k \geq \left(\hbar_1^{\lambda} + \hbar_2^{\lambda} + \cdots + \hbar_k^{\lambda}\right)^{1/\lambda}$$

obviously, the inequality

$$\left(\hbar_1 + \hbar_2 + \cdots + \hbar_k\right)^{\lambda} \geq \hbar_1^{\lambda} + \hbar_2^{\lambda} + \cdots + \hbar_k^{\lambda}$$

holds, by integration, the desired result can be derived.

Proof. (2) Suppose that $0 < \lambda < 1$, by taking $s = \lambda$, $r = 1$ in Jensen's inequality, we can deduce:

$$\hbar_1 + \hbar_2 + \cdots + \hbar_k \leq \left(\hbar_1^{\lambda} + \hbar_2^{\lambda} + \cdots + \hbar_k^{\lambda}\right)^{1/\lambda}$$

with the help of the aforementioned inequality, it can be concluded that:

$$\left(\hbar_1 + \hbar_2 + \cdots + \hbar_k\right)^{\lambda} \leq \hbar_1^{\lambda} + \hbar_2^{\lambda} + \cdots + \hbar_k^{\lambda}$$

according to the integration of the previous inequality, it follows that the desired result is obtained.

Conclusion

In this paper the classical integral Holder's inequality has been generalized to the conformable fractional integral Holder's inequality. This generalization has been obtained using Young inequality. The reverse conformable fractional integral Holder's inequality also has been given. Further, we have established the conformable fractional integral Holder's inequality involving multiple functions and its reverse versions, some related results have been obtained.

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