IMPROVED FRACTIONAL EXPONENTIAL CURVE MODEL FOR NUCLEAR ENERGY GENERATION IN ASIA-PACIFIC REGION

by

***Kai ZUO*[a] *and Wen-Qing WU*[b*]**

[a] School of Mathematics, Chengdu Normal University, Chengdu, China
[b] Faculty of Science, Civil Aviation Flight University of China, Guanghan, China

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To further improve the modelling and forecasting accuracy of the exponential curve model, a new type of exponential curve model is proposed by combining the fractional-order data processing method and a first-order polynomial term. The parameters involved are solved by the piecewise summation technique and the intelligent optimization algorithm. Furthermore, the new model is applied to nuclear energy generation in Asia-Pacific, and the results are compared with the exponential curve model, the modified exponential curve model, and the fractional-order modified exponential curve model. These results indicate that the newly proposed model can obtain more competitive results.

Key words: *exponential curve model, fractional-order operator, piecewise summation, particle swarm optimization*

Introduction

Energy generation refers to the process of converting various natural energy resources into electricity and mechanical energy that is usable by humans [1]. Some primary methods of energy generation include fossil fuel power generation (such as coal-fired, petroleum, and natural gas power generation), nuclear power generation, and renewable energy generation [2]. In this paper, we focus on nuclear energy generation in the Asia-Pacific region using an improved fractional exponential curve model. According to the *BP Statistical Review of World Energy 2024 (https://www.energyinst.org/statis tical-review)*, global electricity generation increased by 2.5% in 2023, reaching 29925 terawatt-hours (TWh), indicating a growing trend of electrification in the world's energy system. The electricity demand in the Asia-Pacific region increased by approximately 5%, while the share of nuclear energy remained stable at around 9%. Advanced nuclear fission and, ultimately, nuclear fusion are being considered as technologies for clean hydrogen production. Soto *et al.* [3] examined the environmental impact of nuclear energy generation using data from 1990 to 2022 in EU countries and demonstrated that nuclear energy generation entails significant land use requirements and can lead to ecosystem degradation. Nighoskar *et al.* [4] utilized the sophisticated regression modelling method, XGBoost, to forecast nuclear energy generation based on raw data from 2000 to 2020. Rahman *et al.* [5] integrated low temperature electrolysis and high temperature steam electrolysis for hydrogen production into a nuclear-renewable energy system.

The growth of energy data often does not follow a linear pattern, despite exhibiting a trend, it possesses non-linear characteristics. In such circumstances, trend curve models, in-

* Corresponding author, e-mail: wwqing0704@cafuc.edu.cn

cluding the parabolic curve model, the exponential curve model, the Logistic curve model, and the Gompertz curve model, are viable options [6, 7]. However, to the best of our knowledge, these mentioned curve models have primarily been applied to various fields without significant theoretical innovation. In fact, raw data rarely aligns perfectly with existing statistical prediction models, and most raw data fails to match these models well, leading to substantial modelling and fitting errors. If raw data can be preprocessed before model selection, it is likely to yield more accurate calculation results.

Inspired by this situation, in this paper, we first introduce fractional-order accumulation and fractional-order difference for preprocessing raw data and subsequently propose an improved exponential curve model incorporating a first-order polynomial term. The closed-form expression for system parameters is calculated using a neater method, and the fractional-order parameters are optimized using the particle swarm optimization algorithm. Furthermore, the newly developed model is applied to nuclear energy generation in the Asia-Pacific region.

The improved fractional exponential curve model

We first review the definition of the fractional-order operator [8], the exponential curve model and the modified exponential curve model, and then discuss the improved fractional-order exponential curve model.

The fractional-order operator

For an original sequence $Y^{(0)} = [y^{(0)}(1), y^{(0)}(2),..., y^{(0)}(n)]$, the r^{th} ($r \in R^+$) order accumulation sequence is defined as $Y^{(r)} = [y^{(r)}(1), y^{(-r)}(2),..., y^{(r)}(n)]$, and the r^{th} ($r \in R^+$) order difference is $Y^{(-r)} = [y^{(-r)}(1), y^{(-r)}(2),..., y^{(-r)}(n)]$, where the expressions are:

$$y^{(r)}(k) = \sum_{i=1}^{k} \begin{bmatrix} r \\ k-i \end{bmatrix} y^{(0)}(i), k = 1,2,\ldots,n \tag{1}$$

$$y^{(-r)}(k) = \sum_{i=0}^{k-1} \begin{bmatrix} -r \\ i \end{bmatrix} y^{(0)}(k-i), k = 1,2,\ldots,n \tag{2}$$

where

$$\begin{bmatrix} r \\ k-i \end{bmatrix} = \frac{r(r+1)\cdots(r+k-i-1)}{(k-i)!} = \begin{pmatrix} r+k-i-1 \\ k-i \end{pmatrix}, \text{ and } \begin{bmatrix} 0 \\ i \end{bmatrix} = 0, \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 1$$

The exponential curve model and the modified exponential curve model

The exponential curve model:

$$y(k) = ab^k \tag{3}$$

where $a \neq 0$, $b \in (0, 1) \cup (1, \infty)$. Take logarithms on eq. (3) to derive the expression of parameters a and b, that is:

$$\ln y(k) = \ln a + k \ln b \tag{4}$$

By the least square method and from eq. (4), it obtains:

$$\ln a = \frac{1}{n}\sum_{i=1}^{n} \ln y(i) - \ln b \frac{1}{n}\sum_{i=1}^{n} i, \quad \ln b = \frac{n\sum_{i=1}^{n} i \ln y(i) - \sum_{i=1}^{n} i \sum_{i=1}^{n} \ln y(i)}{n\sum_{i=1}^{n} i^2 - \left(\sum_{i=1}^{n} i\right)^2} \tag{5}$$

Thus, the values of parameters a and b can be obtained by the antilogarithm.

The modified exponential curve model:

$$y(k) = ab^k + c \tag{6}$$

where $a \neq 0$, $b \in (0,1) \cup (1, \infty)$, and $c \in (0,\infty)$. By the three-sum method, model parameters are:

$$a = \left(\sum_{i=m+1}^{2m} y(i) - \sum_{i=1}^{m} y(i)\right)\frac{b-1}{(b^m-1)^2},\quad b = \left(\frac{\sum_{i=2m+1}^{3m} y(i) - \sum_{i=m+1}^{2m} y(i)}{\sum_{i=m+1}^{2m} y(i) - \sum_{i=1}^{m} y(i)}\right)^{1/m}$$
$$c = \frac{1}{m}\left(\sum_{i=1}^{m} y(i) - \frac{a(b^m-1)}{b-1}\right) \tag{7}$$

where generally m = floor(n/3).

The improved fractional exponential curve model

This subsection proposed the improved fractional exponential curve model:

$$y^{(r)}(k) = a_r b_r{}^k + c_r k + d_r \tag{8}$$

where $a_r \neq 0$, $b_r \in (0, 1) \cup (1, \infty)$, c_r, $r \in (-\infty, +\infty)$.

From eq. (8), we know the newly proposed model is a general model, and the classical exponential curve model, the modified exponential curve model, the fractional-order modified exponential curve model are all special case of it. Next, the piecewise summation technique is applied to derive the analytical expressions of parameters a_r, b_r, c_r, d_r.

Actually, the length of an any data sequence can be one of the four cases which is $4m$, $4m + 1$, $4m + 2$, and $4m + 3$, where m = floor($4/n$). Thus, we divide data sequence used for building different models into groups having m data. Set the four local summations of the r^{th} data sequence be $S_1^{(r)}$, $S_2^{(r)}$, $S_3^{(r)}$, and $S_4^{(r)}$:

$$S_1^{(r)} = \sum_{k=1}^{m} y^{(r)}(k),\ S_2^{(r)} = \sum_{k=m+1}^{2m} y^{(r)}(k),\ S_3^{(r)} = \sum_{k=2m+1}^{3m} y^{(r)}(k),\ S_4^{(r)} = \sum_{k=3m+1}^{4m} y^{(r)}(k) \tag{9}$$

Meanwhile, $S_1^{(r)}$, $S_2^{(r)}$, $S_3^{(r)}$, and $S_4^{(r)}$ can also be obtained:

$$S_1^{(r)} = \sum_{k=1}^{m}(a_r b_r{}^k + c_r k + d_r) = \frac{a_r b_r(1-b_r^m)}{1-b_r} + c_r\frac{m+1}{2}m + md_r \tag{10}$$

$$S_2^{(r)} = \sum_{k=m+1}^{2m}(a_r b_r{}^k + c_r k + d_r) = \frac{a_r b_r^{m+1}(1-b_r^m)}{1-b_r} + c_r\frac{3m+1}{2}m + md_r \tag{11}$$

$$S_3^{(r)} = \sum_{k=2m+1}^{3m}(a_r b_r{}^k + c_r k + d_r) = \frac{a_r b_r^{2m+1}(1-b_r^m)}{1-b_r} + c_r\frac{5m+1}{2}m + md_r \tag{12}$$

$$S_4^{(r)} = \sum_{k=3m+1}^{4m}(a_r b_r{}^k + c_r k + d_r) = \frac{a_r b_r^{3m+1}(1-b_r^m)}{1-b_r} + c_r\frac{7m+1}{2}m + md_r \tag{13}$$

Then, the values of $S_{i+1}^{(r)} - S_i^{(r)} = 1, 2, 3$ are computed:

$$S_2^{(r)} - S_1^{(r)} = \frac{a_r b_r(1-b_r^m)}{1-b_r}(b_r^m - 1) + m^2 c_r \tag{14}$$

$$S_3^{(r)} - S_2^{(r)} = \frac{a_r b_r^{m+1}(1-b_r^m)}{1-b_r}(b_r^m - 1) + m^2 c_r \tag{15}$$

$$S_4^{(r)} - S_3^{(r)} = \frac{a_r b_r^{2m+1}(1-b_r^m)}{1-b_r}(b_r^m - 1) + m^2 c_r \tag{16}$$

Then, we have:

$$(S_3^{(r)} - S_2^{(r)}) - (S_2^{(r)} - S_1^{(r)}) = \frac{a_r b_r (1-b_r^m)}{1-b_r}(b_r^m - 1)^2 \tag{17}$$

$$(S_4^{(r)} - S_3^{(r)}) - (S_3^{(r)} - S_2^{(r)}) = \frac{a_r b_r^{m+1}(1-b_r^m)}{1-b_r}(b_r^m - 1)^2 \tag{18}$$

Thus, we obtain:

$$b_r = \left(\frac{\sum_{k=3m+1}^{4m} y^{(r)}(k) - 2\sum_{k=2m+1}^{3m} y^{(r)}(k) + \sum_{k=m+1}^{2m} y^{(r)}(k)}{\sum_{k=2m+1}^{3m} y^{(r)}(k) - 2\sum_{k=m+1}^{2m} y^{(r)}(k) + \sum_{k=1}^{m} y^{(r)}(k)} \right)^{1/m} \tag{19}$$

$$a_r = \frac{\left(\sum_{k=2m+1}^{3m} y^{(r)}(k) - 2\sum_{k=m+1}^{2m} y^{(r)}(k) + \sum_{k=1}^{m} y^{(r)}(k) \right)(1-b_r)}{b_r (1-b_r^m)(b_r^m - 1)^2} \tag{20}$$

$$c_r = \frac{\sum_{k=m+1}^{2m} y^{(r)}(k) - \sum_{k=1}^{m} y^{(r)}(k)}{m^2} - \frac{a_r b_r (1-b_r^m)(b_r^m - 1)}{m^2(1-b_r)} \tag{21}$$

$$d_r = \frac{\sum_{k=1}^{m} y^{(r)}(k)}{m} - \frac{a_r b_r (1-b_r^m)}{m(1-b_r)} - \frac{m+1}{2} c_r \tag{22}$$

Therefore, the analytical expressions of model parameters are obtained.

The optimization problem for the fractional order

We develop an optimization problem where the $\mathrm{MAPE_{simu}}$ is the objective function, and fractional-order r is a decision variable, namely:

$$\arg\min_r \mathrm{MAPE_{simu}} = \frac{1}{4m-1} \sum_{k=2}^{4m} \mathrm{APE}(k) \times 100\% \tag{23}$$

where

$$\mathrm{APE}(k) = \left| \frac{y^{(0)}(k) - \hat{y}^{(0)}(k)}{y^{(0)}(k)} \right| \times 100\%, \ k = 1, 2, \ldots, n$$

Due to the non-linearity of eq. (23), deriving an analytical expression for fractional-order r will be a challenging task. Therefore, the particle swarm optimization (PSO) algorithm is applied to numerically search for the optimal value due to its simple rules and small number of adjustable parameters. The detailed information of PSO algorithm can be seen in references [9, 10].

Application in the nuclear energy generation in Asia-Pacific

In this section, we consider the total nuclear energy generation in Asia-Pacific, where data are collected from *BP Statistical Review of World Energy 2024* (https://www.energyinst.org/statistical- review). All data are displayed in the following tab. 1.

Table 1. The total nuclear energy generation in Asia-Pacific [TWh]

Year	Raw data	Year	Raw data	Year	Raw data	Year	Raw data	Year	Raw data
2010	582.95	2013	344.10	2016	467.70	2019	646.91	2022	737.81
2011	483.13	2014	371.37	2017	493.58	2020	654.99	2023	781.05
2012	342.91	2015	419.75	2018	553.59	2021	714.14		

The first part data from 2010 to 2021 are applied for building the exponential curve model (ECM), the modified exponential curve model (MECM), the fractional-order modified exponential curve model (FMECM), and the newly developed model (IFECM), and the left data from 2022 to 2023 are used for testing. The optimal fractional-order r of the FMECM and IFECM by PSO algorithm are 1.64 and 0.003774, respectively, and the numerical results are listed in tab. 2.

Table 2. Results of the nuclear energy generation in Asia-Pacific by the ECM, MECM, FMECM, and IFECM models

Year	Data	ECM	APE [%]	MECM	APE [%]	FMECM	APE [%]	IFECM	APE [%]
2010	582.95	438.27	24.82	602.12	3.29	577.60	0.91	602.12	3.30
2011	483.13	438.27	9.28	437.74	9.40	483.10	0.00	435.42	9.87
2012	342.91	438.27	27.81	369.13	7.65	355.21	3.59	366.31	6.83
2013	344.10	438.27	27.37	355.20	3.23	351.08	2.03	352.21	2.36
2014	371.37	438.27	18.02	372.47	0.30	371.15	0.07	369.32	0.56
2015	419.75	438.27	4.41	407.55	2.91	403.99	3.74	404.16	3.70
2016	467.70	438.19	6.31	452.80	3.19	446.55	4.52	449.08	3.98
2017	493.58	437.67	11.33	503.85	2.08	498.07	0.91	499.72	1.24
2018	553.59	436.10	21.22	558.22	0.84	558.67	0.92	553.60	0.00
2019	646.91	441.13	31.81	614.47	5.01	628.97	2.77	609.32	5.81
2020	654.99	536.61	18.07	671.81	2.57	709.91	8.38	666.07	1.69
2021	714.14	1155.79	61.84	729.76	2.19	802.70	12.41	723.39	1.30
$MAPE_{simu}$ [%]			21.59		3.58		3.58		3.39
2022	737.81	3006.73	307.52	788.06	6.81	908.76	23.17	781.04	5.86
2023	781.05	-2946.04	477.19	846.57	8.39	1029.80	31.84	838.87	7.40
$MAPE_{fit}$ [%]			392.36		7.60		27.51		6.63
$MAPE_{total}$ [%]			78.63		4.20		7.26		3.89

From tab. 2, the ECM is not suitable for the nuclear energy generation, and cannot identify the trend of data. The other models have successfully caught the trend of nuclear energy generation, but values by the new model IFECM are much closer to raw data than those by the other two models. The $MAPE_{simu}$, $MAPE_{fit}$, and $MAPE_{total}$ of the IFECM are 3.39%, 6.63%, and 3.89%, respectively, which also means the new model is suitable for the nuclear energy generation in Asia-Pacific.

Conclusion

In this paper, the fractional-order operator is introduced to preprocess raw data, and then a new improved fractional-order exponential curve model is built. The analytical expression of system parameters are derived by the piecewise summation method and the fractional-order is determined by PSO algorithm. Further, the nuclear energy generation in Asia-Pacific is considered as an example. It shows that the improved fractional-order exponential curve model is suitable for this case.

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Nomenclature

$y^{(0)}(k)$ – nuclear energy generation, [TWh]
$\hat{y}^{(0)}(k)$ – calculated nuclear energy generation, [TWh]
$Y^{(r)}$ – r^{th} accumulation sequence, [TWh]
$Y^{(-r)}$ – r^{th} difference sequence, [TWh]

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