A SIQRS EPIDEMIC MODEL INCORPORATING PSYCHOLOGICAL FACTORS FOR COVID-19

by

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Since the initiation of the highest-level public health emergency response in Jilin Province, China, on January 25, 2020, awareness of COVID-19 has remained high. However, in March 2022, an outbreak of COVID-19, attributed to imported cases, occurred in Changchun, China. Considering the psychological impact of COVID-19 on humans, we developed a SIQRS epidemic model. Dynamic analyses were conducted on both deterministic and corresponding stochastic models. By employing non-linear least squares to fit COVID-19 data, we analyzed the influence of psychological factors on the disease's spread. Our aim was to explore effective prevention and control measures for COVID-19, taking into account the psychological dimension of the pandemic.

Key words: *deterministic model, stochastic model, psychological effect, stability analysis, extinction*

Introduction

Since December 2019, COVID-19 has spread widely in China [1]. By March 2020, China controlled the outbreak with Level 1 Emergency Response and lockdowns [2]. However, the virus spread globally, reaching 691456005 cases and 6899439 deaths worldwide by July 20, 2023 [3].

Mathematical modelling is commonly utilized to explore the transmission dynamics of diseases [4, 5]. Gong *et al.* [6] evaluated the psychological effects on individuals quarantined in Hubei due to the COVID-19 epidemic. Although the transmission of COVID-19 in China has been extensively studied, there is currently a lack of relevant research on mathematical models specifically designed to investigate the transmission of COVID-19 in Changchun, China, with a focus on psychological effects and quarantined compartment.

To study COVID-19 spread in Changchun, calculating both deterministic and stochastic basic reproduction numbers is key. The basic reproduction number estimates early transmission potential. We formulated a SIQRS model considering psychological effects and white noise. We computed both numbers and drew conclusions for both systems. Using daily case data, we validated the model and estimated transmission rates. Finally, we assessed the impact of psychological effects on disease progression.

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Model

Dynamics of deterministic model

In this section, we proposed a SIQRS disease model. In this model, The population N is divided into susceptible people $S(t)$, infected people $I(t)$, quarantined people $Q(t)$ and removed people $R(t)$, respectively. Here, $N = S(t) + I(t) + Q(t) + R(t)$. In the model, several assumptions are given in the following. Natural birth and death rates are not considered because of the short duration of the outbreak. When the number of infected human individuals increases, susceptible populations reduce the number of contacts with infected populations due to psychological effects, so we use a non-monotone incidence function describe the transmission of the COVID-19 from infected people to susceptible people. Based on the epidemiological patterns of COVID-19 in Changchun and previous work, the transmission dynamics of COVID-19 in Changchun is presented by differential equations:

$$\begin{aligned} \frac{\mathrm{d}S}{\mathrm{d}t} &= -\frac{\beta SI}{1+\alpha I^2} + \delta R \\ \frac{\mathrm{d}I}{\mathrm{d}t} &= \frac{\beta SI}{1+\alpha I^2} - \gamma_1 I - pI \\ \frac{\mathrm{d}Q}{\mathrm{d}t} &= pI - \gamma_2 Q \\ \frac{\mathrm{d}R}{\mathrm{d}t} &= \gamma_1 I + \gamma_2 Q - \delta R \end{aligned} \tag{1}$$

where the non-negative initial values, $S(0) > 0$, $I(0) > 0$, $Q(0) > 0$ and $R(0) > 0$. The flow diagram is shown in fig. 1. Table 1 provides a summary of the parameters and their respective meanings.

Firstly, we obtain the disease-free equilibrium $E_0(N,0, 0, 0)$. We consider if $I \neq 0$, then not satisfying the previous equation, so the Model (1) has not endemic equilibrium. Further, to obtain the expression for the basic reproduction number, R_0, we use the next generation matrix method, the basic reproduction number is calculated, where:

$$R_0 = \frac{\beta N}{\gamma_1 + p} \tag{2}$$

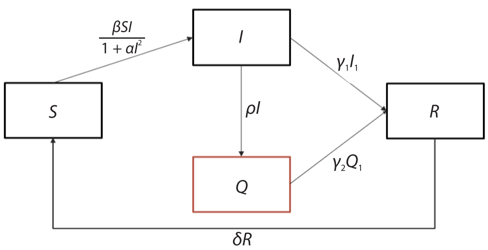

Figure 1. A schematic diagram of the system (1)

Next, we present the stability analysis theorem for the disease-free equilibrium.

Theorem 1. The disease-free equilibrium E_0 is locally asymptotically stable if $R_0 < 1$.

Proof. The proof process has been detailed with reference to [4], which is omitted here.

Theorem 2. If $R_0 < 1$, then the disease-free equilibrium E_0 is globally asymptotically stable.

Proof. The proof process has been detailed with reference to [5], which is omitted here.

Table 1. The important events related of COVID-19 in Changchun

Variable initial values/parameter	Descriptions	Value	Source
β	The transmission rate of COVID-19	$7.0365 \cdot 10^{-5}$	Fitted
α	Psychological effect	0.2136	Fitted
γ_1	The recovery rate of infected people	$1.9267 \cdot 10^{-4}$	Fitted
p	The quarantine probability of infected	1/7	[7]
γ_2	The cure rate of quarantine infected people	1/14	[7]
δ	Immune loss rate	1/100	Estimated
$S(0)$	Initial number of susceptible people	11700	[3]
$I(0)$	Initial number of infected people	1	[3]
$Q(0)$	Initial number of quarantined people	3	[3]
$R(0)$	Initial number of removed people	0	[3]

Dynamics of stochastic model

In this subsection, we give the stochastic COVID-19 model defined by the stochastic differential equation system corresponding to the determination system (1):

$$
\begin{aligned}
\mathrm{d}S &= \left(-\frac{\beta SI}{1+\alpha I^2} + \delta R\right)\mathrm{d}t - \frac{\sigma SI}{1+\alpha I^2}\mathrm{d}B(t) \\
\mathrm{d}I &= \left(\frac{\beta SI}{1+\alpha I^2} - \gamma_1 I - pI\right)\mathrm{d}t + \frac{\sigma SI}{1+\alpha I^2}\mathrm{d}B(t) \\
\mathrm{d}Q &= (pI - \gamma_2 Q)\mathrm{d}t \\
\mathrm{d}R &= (\gamma_1 I + \gamma_2 Q - \delta R)\mathrm{d}t
\end{aligned}
\tag{3}
$$

Theorem 3. For any initial value ($S(0)$, $I(0$, $Q(0)$, $R(0)$) $\in \mathbb{R}_+^4$, there exists a unique solution ($S(0)$, $I(0$, $Q(0)$, $R(0)$) o system (3) on $t \geq 0$ and the solution will remain in $\mathbb{R}_+^4$ for all $t \geq 0$ almost surely.

Proof. The proof is omitted here for brevity, but it follows a similar reasoning as that presented in [8]. Readers are referred to this article for a detailed proof.

Next, we use Cai *et al*. [9] method to calculate a simple and rigorous formula for the computation of the basic reproduction number R_0^s for a general SDE epidemiological model:

$$
R_0^s = FV^{-1} = \frac{\beta N}{\gamma_1 + p} - \frac{\sigma^2 N^2}{2(\gamma_1 + p)} = R_0 - \frac{\sigma^2 N^2}{2(\gamma_1 + p)} \tag{4}
$$

Through the stochastic basic reproduction number R_0^s, we will focus on that how we can regulate the disease dynamics so that the COVID-19 will be eradicated.

Theorem 4. Suppose that one of the conditions holds:

(A) $R_0^s < 1$and $\sigma < (\beta/N)^{1/2}$

(B) $\sigma > \hat{\sigma} := \max\left\{\sqrt{\frac{\beta}{N}}, \frac{\beta}{\sqrt{2(\gamma_1 + p)}}\right\}$

Proof. The proof is omitted here for brevity, but it follows a similar reasoning as that presented in [8]. Readers are referred to this article for a detailed proof.

Numerical simulation

Data source

Data on newly identified COVID-19 cases in Changchun from March 3 to May 15 were obtained from the Health Commission of Jilin Province [3]. This dataset comprises the cumulative number of newly identified cases, quarantined cases, and recovered cases, all sourced from publicly accessible resources. Additionally, to provide a comprehensive context for these data, a summary of relevant policies and significant events during this period has been compiled and presented in fig. 2.

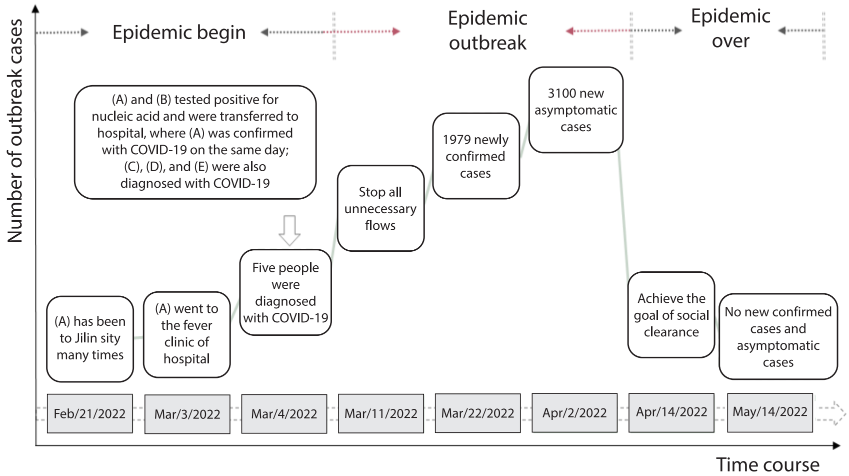

Figure 2. The important events related of COVID-19 in Changchun

Parameters estimation

In our experiment, we set the average quarantine duration was reported to be 14 days, leading us to set the transition rate from the quarantined compartment Q to the recovered compartment R, denoted as γ_2, to 1/14. Additionally, residents under home quarantine underwent nucleic acid testing every 3-7 days, prompting us to estimate the probability of detection, denoted as p, at 1/7, recognizing the midpoint of this range for our calculations.

Once individuals in quarantine recovered, they were required to complete an additional period of home isolation until the local lockdown measures were lifted. Given this scenario, we assigned a negligible value to δ, the transition rate from the recovered compartment R back to the susceptible compartment S, setting it to 1/100 as a practical approximation of zero movement in this direction.

According to official documents released by the Changchun Municipal Government, the daily mobile population within the urban area was reported to be 11700. Consequently, we initialized the susceptible population, $S(0)$, to this value. For the initial conditions of other compartments, we assumed a susceptible initial population of 1 (representing a single index case or the initial outbreak point), an initial quarantined population of 5 (reflecting early cases under quarantine), and an initial removed (recovered or deceased) population of 0.

Our focus in this study was to estimate three critical parameters: the transmission rate β. The psychological effect coefficient α, which captures the potential impact of behavioral changes in response to the pandemic; and the transition rate γ_1.

Fitting results

In this subsection, we present data fitting results using the non-linear least squares method on 74 data points. Figure 3(a) illustrates the good alignment between model predictions and observed data, with estimated parameter values provided in tab. 1. The consistency and adaptability of the model suggest its reliability and utility for exploring system behavior under various scenarios.

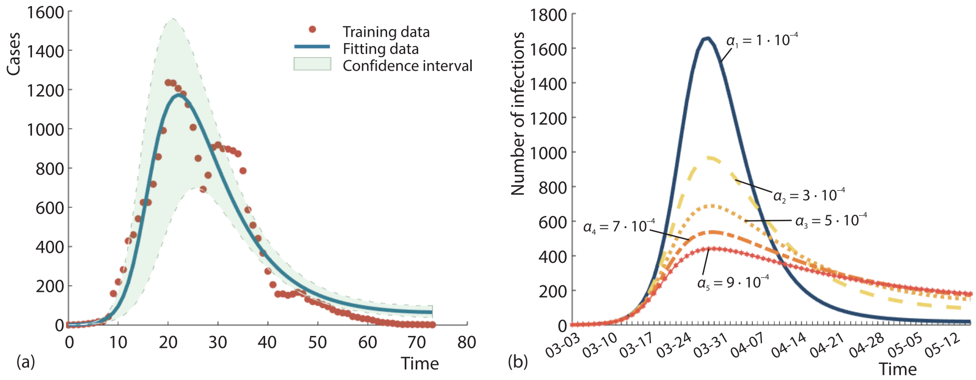

Figure 3. Combined analysis of disease transmission; (a) fitting daily new COVID-19 data in Changchun using non-linear least squares and (b) investigating the impact of varying psychological influence coefficient, *α*

Impact of psychological effect on COVID-19

This subsection explores the impact of psychological factors on disease transmission using the psychological effect coefficient, α. By analyzing parameter values from tab. 1 and fitting estimates, we found that adjusting α affects infected population dynamics. As α increases (indicating stronger psychological effects), the infection peak size decreases, though the peak timing remains unchanged, fig. 3(b). This suggests that heightened disease awareness and stricter measures reduce peak infections, influencing disease transmission patterns. Practically, enhancing disease propaganda and public awareness during outbreaks can increase α, lower peak infections, ease medical pressure, and provide time for effective responses.

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Nomenclature

I – population of infected humans, [–]
Q – population of quarantined humans, [–]
N – total population, [–]
p – infected people are under quarantine by the quarantined rate, [–]
R – population of removed humans, [–]
S – population of susceptible humans, [–]

Greek symbols

α – psychological effect, [–]
β – contact rate from infectious humans to susceptible humans, [–]
γ_1 – infected people are removed by the recovery rate, [–]
γ_2 – quarantined people are removed by the cure rate, [–]
δ – removed people become susceptible people by the immunity loss rate, [–]
σ – disturbance intensity, [–]

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