COST-OPTIMAL OPERATION OF HYBRID HEAT PUMP SYSTEMS WITH PROGRESSIVE ELECTRICITY TARIFFS

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Heat pumps are very significant for wide application of renewable energy sources and sustainable heating. Their market penetration and extensive application depend on economic performance. Optimization of the operation parameters of energy systems with heat pumps can result in lower costs and higher savings. Heat pumps operate under different electricity pricing structures, which affect the optimization process. This paper presents a methodology for cost-optimal operation optimization of hybrid energy systems with heat pumps that can be used with progressive electricity tariffs and their combination with time-of-use tariffs. It relies on mixed integer linear programming and includes the constraints that handle electricity tariff rules. The paper illustrates an example of the application of this methodology to a heating system with an air-source heat pump and an auxiliary heater. The results show the impact of the energy prices and electricity tariff structures on the operating regimes and the values of the objective function. This approach can enhance the quality of the optimization results and improve the comprehension of cost-optimal operation regimes.

Key words: *buildings, cost-optimal operation, heat pump, mixed integer linear programming, progressive electricity tariff*

1. Introduction

A heat pump (HP) uses electricity to provide heating, cooling, or hot water efficiently. HPs driven by electricity obtained from environmentally friendly sources can contribute substantially to sustainable heating [1]. They are a very important option for the wider utilization of renewable energy sources [2]. The environmental impact of HPs varies across countries, depending on the energy mix. Topić Božič *et al.* analyzed their impact for the cases of Slovenia and Serbia. The exergy analysis is given in [4] for an air-source HP (ASHP) and in [5] for a ground-source HP (GSHP).

Market penetration and broad adoption of HPs depend largely on their economic performance, which is related to many factors, *e.g.* investment costs, energy tariffs and prices, climate conditions, choice of system design and operation parameters, *etc*. Kossi and Rama [6] illustrated the impact of electricity prices on the economic feasibility of GSHP and underlined the importance of heat storage (HS). Jokić *et al.* [7] concluded that GSHP with closed loop is a feasible alternative to electric heating for households in Serbia.

Optimal operation of HPs can reduce operational costs and improve their attractiveness, especially when they are part of complex hybrid systems. Operation optimization (OO) can be applied while running energy systems, but also as a part of design optimization. Pesola [8] underlined the importance of cost optimization of hybrid systems that integrate ASHPs with existing centralized district heating systems. Mixed integer linear programming (MILP) is one of the best adopted optimization techniques for energy systems. It can guarantee the global optimality and precision of the solution. However, MILP models can be hard to formulate and computationally intensive to solve. Krützfeldt *et al.* [9] presented an integrated approach to the design and operation optimization of an HP system of a residential building with MILP. The study considered space heating and hot water preparation, and the system contained one HS for each type of demand. Aguilera *et al.* [10] used forecasts of the heat demand and exterior temperature to formulate an MILP model that reduces the operating cost of a large-scale HP with storage, considering the degradation of HP performance over time. Metaheuristic methods are also convenient to optimize HP operation. Compared to MILP, they are less precise but easier to formulate. For example, Li *et al.* [11] performed multi-objective optimization of a photovoltaic system (PVS), HP, and HS with the non-dominated sorting genetic algorithm II.

Heat pump energy systems can operate under various electricity pricing rules, depending on the country and sector. Tariff regulations can have a significant impact on the operating regime of an energy system and related costs. This paper focuses on progressive tariff (PT) systems, also known as increasing block tariff systems. The first block or tier of PT corresponds to low energy consumption and low price. When consumption exceeds the threshold of a tier, the price increases. There can be multiple blocks, corresponding to the increasing threshold and price values [12]. PT can be combined with a time-of-use tariff (TOUT) [13]. TOUTs have regular price variations during the day. Higher prices usually correspond to peak demand hours and lower prices to off-peak hours [14]. PT prices and thresholds can vary throughout the year [15].

Youn and Jin [16] examined the effects of the PT system in South Korea and concluded that it can be successful in reducing electricity consumption in households, especially during summer. In addition, they are more effective than feed-in tariffs, according to Prasanna [12]. However, Quan and Kim [17] have shown that South Korean PT led to increased consumption in some cases and appeared to be beneficial to the higher income population. Matar [18] emphasized the impact of building insulation and household income in the context of analyzing the dependence of electricity costs on the pricing scheme. Bae and Nam [19] analyzed the economic attractiveness and zero energy potential of a real system with GSHP and thermal-PVS with various pricing schemes, including PT.

The energy prices formulated within a PT system or a combination of PT and TOUT system, significantly increase the complexity of OO problems related to energy systems with HP, PVS, electric storage, *etc*. This happens because the price of electricity in each time step depends on the amount of electricity previously consumed. Since consumed electricity is usually either a decision variable or depends on decision variables, electricity price cannot simply be considered as an input constant.

Jung *et al.* [20] presented an MILP approach to multi-objective optimization of energy supply scheduling based on PVS, storage, and grid connection, under PT. The MILP problem is solved for one day horizon at a time. PT pricing is taken into account with an iterative procedure that involves updating the price of electricity and re-optimizing. This procedure is conveniently combined with the ε-constraint method. Kang *et al.* [21]

solved a similar problem with reinforcement learning (RL) and verified with the MILP-based approach from [20]. They concluded that the proximal policy optimization approach suggests the best decisions of all RL methods applied to residential buildings. Similarly, Dou *et al.* [22] concluded that the trust region policy optimization algorithm has the best performance among the four observed RL methods and is comparable to MILP, while having a simpler structure. In addition, RL is used to solve the multi-objective problem of sizing and scheduling a system with PVS and storage in [23]. PTs are also considered during the analysis of electricity price trading by Zhang *et al.* [24], and for the optimization of the price with the genetic algorithm by An *et al.* [15].

This paper presents and applies an MILP model for the OO of a hybrid heating system that contains HP and an auxiliary heater (AH). The model is suitable for the PT electricity prices defined on a monthly basis, as well as a combination of the PT and TOUT prices. It can be based solely on predicted data or on a combination of historical and predicted data. The objective function is the variable part of the annual energy cost, *i.e.* the part of the cost that depends on the operating regime of the system. The considered system is simple because the focus of the paper is on handling the constraints related to the tariff tiers. However, this model can be adjusted to handle complex systems with additional components and more detailed mathematical formulations.

In addition to the examples from the literature that optimize HP systems under PT with either approximate iterative MILP solutions or RL approaches, this paper offers a methodology that can solve the optimization problem entirely with MILP for the whole month, year, or several years. As such, it can find the global optimum and guarantee that the solution accuracy is within the predefined limits. The approach is flexible and extensible. It can be implemented in various cases, for many different energy system configurations, expanding the understanding of desirable operating regimes under complex pricing rules and their impact on the environment. This is particularly important for the case of Serbia that has PT and TOUT, and plans to significantly increase the application of HPs in buildings and district heating during the next 15 years [25].

2. Methodology

The methodology for finding the cost-optimal operation of a hybrid energy supply system with HP and AH is based on an MILP model. It can be applied to some variants of PTs, and combinations of PTs and TOUTs. The version of the model presented in this paper is suitable for three-tier PTs.

In addition to the usual decision variables and constraints that ensure component operation within predefined capacities, demand satisfaction, and energy balance, the model is extended to handle PT rules and limitations. The variable that represents the amount of electrical energy imported from the national electrical grid is divided into three decision variables, each of which corresponds to one PT tier or block.

2.1. Objective function

The objective is to minimize the annual variable cost of electricity and heat, C_{var}, expressed in Serbian dinars (RSD). The mathematical formulation of the objective function is given in Eq. (1):

$$\min C_{\text{var}} = \sum_{m} \left(\dot{c}_{\text{e,cap}}^{m} \dot{W}_{\text{e,cap}}^{m} + \sum_{\tau} \left(c_{\text{e,t1}}^{m,\tau} W_{\text{e,t1}}^{m,\tau} + c_{\text{e,t2}}^{m,\tau} W_{\text{e,t2}}^{m,\tau} + c_{\text{e,t3}}^{m,\tau} W_{\text{e,t3}}^{m,\tau} + \frac{c_{\text{h}}^{m,\tau} Q_{\text{h,ah}}^{m,\tau}}{\eta_{\text{h,ah}}^{m,\tau}} \right) \right) \quad (1)$$

where:

- m is the index of the month;
- τ is the index of the observed time step of a month;
- $\dot{c}_{\text{e,cap}}^{m}$ is the capacity price of electrical power for the month m, in RSD kW^{-1};
- $\dot{W}_{\text{e,cap}}^{m}$ is the charged electrical peak power for the month m, in kW;
- $c_{\text{e,t1}}^{m,\tau}$, $c_{\text{e,t2}}^{m,\tau}$, and $c_{\text{e,t3}}^{m,\tau}$ are the prices of electrical energy for all three tiers, for the month m and time step τ, in RSD kWh^{-1};
- $W_{\text{e,t1}}^{m,\tau}$, $W_{\text{e,t2}}^{m,\tau}$, and $W_{\text{e,t3}}^{m,\tau}$ are the amounts of electrical energy imported from the grid for all three tiers, for m and τ, in kWh;
- $c_{\text{h}}^{m,\tau}$ is the price of the energy source used to drive AH for m and τ, in RSD kWh^{-1};
- $Q_{\text{h,ah}}^{m,\tau}$ is the heating energy obtained from AH for m and τ, in kWh;
- $\eta_{\text{h,ah}}^{m,\tau}$ is the efficiency of AH for m and τ.

In some cases, depending on the PT formulation, $\dot{W}_{\text{e,cap}}^{m}$ can be predefined and considered as input to the model. Then the corresponding term can be removed from Eq. (1).

2.2. Decision variables

The continuous non-negative decision variables are the charged electrical power, $\dot{W}_{\text{e,cap}}^{m}$, in the cases when it is not predefined; electricity imported from the grid for all tiers, $W_{\text{e,t1}}^{m,\tau}$, $W_{\text{e,t2}}^{m,\tau}$, and $W_{\text{e,t3}}^{m,\tau}$; heat from AH, $Q_{\text{h,ah}}^{m,\tau}$; and heat obtained from HP, $Q_{\text{h,hp}}^{m,\tau}$, in kWh.

The model uses one auxiliary binary decision variable for each tier and each time step to properly constrain imported electricity for all tariff tiers. These variables are denoted with $\delta_{\text{t1}}^{m,\tau}$, $\delta_{\text{t2}}^{m,\tau}$, and $\delta_{\text{t3}}^{m,\tau}$, for the PT tiers t1, t2, and t3, respectively. They indicate whether electricity is charged within a particular tier during the month m and time step τ.

2.3. Constraints

The electricity imported from the grid for each time step of a month is bounded with the peak power for that month. As already mentioned, this upper bound can be either an input or a decision variable. Similarly, the heat obtained from HP and AH is constrained by their capacities. The capacity constraints for the month m and time step τ are given in Eqs. (2):

$$W_{\text{e,t1}}^{m,\tau} + W_{\text{e,t2}}^{m,\tau} + W_{\text{e,t3}}^{m,\tau} \leq \dot{W}_{\text{e,cap}}^{m} \Delta\tau \qquad \forall\,(m,\tau) \tag{2a}$$

$$Q_{\text{h,hp}}^{m,\tau} \leq \dot{Q}_{\text{h,hp,cap}}^{m,\tau} \Delta\tau \quad \forall\,(m,\tau) \tag{2b}$$

$$Q_{\text{h,ah}}^{m,\tau} \leq \dot{Q}_{\text{h,ah,cap}}^{m,\tau} \Delta\tau \quad \forall\,(m,\tau) \tag{2c}$$

where $\Delta\tau$ is the duration of a time step, in hours, while $\dot{Q}_{\text{h,hp,cap}}^{m,\tau}$ and $\dot{Q}_{\text{h,ah,cap}}^{m,\tau}$ are the HP and AH capacities, respectively, in kW. The capacities are inputs to the model, but can vary with time (*e.g.* depending on the ambient temperature in the case of ASHP).

The electricity from the grid satisfies the electricity demand and drives HP. Similarly, the heat generated by HP and AH fulfills the heat demand. The constraints that ensure energy balance and demand satisfaction are given in Eqs. (3):

$$W_{\text{e,t1}}^{m,\tau} + W_{\text{e,t2}}^{m,\tau} + W_{\text{e,t3}}^{m,\tau} - \frac{Q_{\text{h,hp}}^{m,\tau}}{\text{COP}_{\text{hp}}^{m,\tau}} = W_{\text{e,demand}}^{m,\tau} \quad \forall\,(m,\tau) \tag{3a}$$

$$Q_{\text{h,hp}}^{m,\tau} + Q_{\text{h,ah}}^{m,\tau} = Q_{\text{h,demand}}^{m,\tau} \quad \forall\,(m,\tau) \tag{3b}$$

where $\text{COP}_{\text{hp}}^{m,\tau}$ is the coefficient of performance (COP) of HP, which can vary with time, while $W_{\text{e,demand}}^{m,\tau}$ and $Q_{\text{h,demand}}^{m,\tau}$ are the demands for electricity and heat, respectively, in kWh.

The cumulative import of electricity from the grid for the month m and time step T is the sum of the amounts of electricity imported from the beginning of the month to T, inclusive, as defined in Eq. (4):

$$W_{\text{e,cum}}^{m,T} = \sum_{\tau=1}^{T} \left(W_{\text{e,t1}}^{m,\tau} + W_{\text{e,t2}}^{m,\tau} + W_{\text{e,t3}}^{m,\tau} \right) \quad \text{and} \quad W_{\text{e,cum}}^{m,0} = 0 \tag{4}$$

Given the predefined electricity consumption thresholds for each month m, $W_{\text{e,lb}}^{m}$ and $W_{\text{e,ub}}^{m}$, in kWh, which represent the lower and upper bounds of the middle tier, respectively, *i.e.* the limits between the tiers, the constraints related to the PT rules are defined on a monthly basis in Eqs. (5)–(12).

Equations (5) impose the widest bounds on the grid electricity variables $W_{\text{e,t1}}^{m,\tau}$, $W_{\text{e,t2}}^{m,\tau}$, and $W_{\text{e,t3}}^{m,\tau}$, and connect them to the corresponding indicator variables $\delta_{\text{t1}}^{m,\tau}$, $\delta_{\text{t2}}^{m,\tau}$, and $\delta_{\text{t3}}^{m,\tau}$:

$$W_{\text{e,t1}}^{m,\tau} \leq W_{\text{e,lb}}^{m} \delta_{\text{t1}}^{m,\tau} \quad \forall\,(m,\tau) \tag{5a}$$

$$W_{\text{e,t2}}^{m,\tau} \leq (W_{\text{e,ub}}^{m} - W_{\text{e,lb}}^{m}) \delta_{\text{t2}}^{m,\tau} \quad \forall\,(m,\tau) \tag{5b}$$

$$W_{\text{e,t3}}^{m,\tau} \leq M \delta_{\text{t3}}^{m,\tau} \quad \forall\,(m,\tau) \tag{5c}$$

where M is a sufficiently large constant.

Equation (6a) requires that at least one indicator variable is positive for each time step, while Eq. (6b) prevents the possibility that electricity is charged in tiers 1 and 3 and not in tier 2, during the same time step:

$$\delta_{\text{t1}}^{m,\tau} + \delta_{\text{t2}}^{m,\tau} + \delta_{\text{t3}}^{m,\tau} \geq 1 \quad \forall\,(m,\tau) \tag{6a}$$

$$\delta_{\text{t1}}^{m,\tau} - \delta_{\text{t2}}^{m,\tau} + \delta_{\text{t3}}^{m,\tau} \leq 1 \quad \forall\,(m,\tau) \tag{6b}$$

Equations (7) ensure that electricity can be charged in tier 1 only if the cumulative electricity for the previous time step is lower than or equal to the threshold $W_{\text{e,lb}}^{m}$:

$$W_{\text{e,lb}}^{m} - W_{\text{e,cum}}^{m,\tau-1} \leq M \delta_{\text{t1}}^{m,\tau} \quad \forall\,(m,\tau) \tag{7a}$$

$$W_{\text{e,lb}}^{m} - W_{\text{e,cum}}^{m,\tau-1} \geq M (\delta_{\text{t1}}^{m,\tau} - 1) \quad \forall\,(m,\tau) \tag{7b}$$

According to Eqs. (8), electricity can be charged in tiers 2 or 3 only if the cumulative electricity for the current time step is not lower than the threshold $W_{\text{e,lb}}^{m}$:

$$W_{\text{e,lb}}^{m} - W_{\text{e,cum}}^{m,\tau} \leq M (1 - \delta_{\text{t2}}^{m,\tau}) \quad \forall\,(m,\tau) \tag{8a}$$

$$W_{\text{e,lb}}^{m} - W_{\text{e,cum}}^{m,\tau} \leq M (1 - \delta_{\text{t3}}^{m,\tau}) \quad \forall\,(m,\tau) \tag{8b}$$

$$W_{\text{e,lb}}^{m} - W_{\text{e,cum}}^{m,\tau} \geq -M (\delta_{\text{t2}}^{m,\tau} + \delta_{\text{t3}}^{m,\tau}) \quad \forall\,(m,\tau) \tag{8c}$$

Equations (9) ensure that electricity can be charged in tiers 1 or 2 only if the cumulative electricity for the previous time step is not greater than the threshold $W^{m}_{\text{e,ub}}$:

$$W^{m,\tau-1}_{\text{e,cum}} - W^{m}_{\text{e,ub}} \leq M(1 - \delta^{m,\tau}_{\text{t1}}) \qquad \forall\,(m,\tau) \tag{9a}$$
$$W^{m,\tau-1}_{\text{e,cum}} - W^{m}_{\text{e,ub}} \leq M(1 - \delta^{m,\tau}_{\text{t2}}) \qquad \forall\,(m,\tau) \tag{9b}$$
$$W^{m,\tau-1}_{\text{e,cum}} - W^{m}_{\text{e,ub}} \geq -M(\delta^{m,\tau}_{\text{t1}} + \delta^{m,\tau}_{\text{t2}}) \quad \forall\,(m,\tau) \tag{9c}$$

According to Eqs. (10), electricity can be charged in tier 3 only if the cumulative electricity for the current time step is greater than or equal to the threshold $W^{m}_{\text{e,ub}}$:

$$W^{m,\tau}_{\text{e,cum}} - W^{m}_{\text{e,ub}} \leq M\delta^{m,\tau}_{\text{t3}} \qquad \forall\,(m,\tau) \tag{10a}$$
$$W^{m,\tau}_{\text{e,cum}} - W^{m}_{\text{e,ub}} \geq M(\delta^{m,\tau}_{\text{t3}} - 1) \quad \forall\,(m,\tau) \tag{10b}$$

Equations (11) impose the constraint that electricity can be charged in tier 2 only if the sum of electricity from tier 1 and the cumulative electricity for the previous time step is equal to the threshold $W^{m}_{\text{e,lb}}$:

$$W^{m,\tau}_{\text{e,t1}} + W^{m,\tau-1}_{\text{e,cum}} - W^{m}_{\text{e,lb}} \leq M(1 - \delta^{m,\tau}_{\text{t1}}) \qquad \forall\,(m,\tau) \tag{11a}$$
$$W^{m,\tau}_{\text{e,t1}} + W^{m,\tau-1}_{\text{e,cum}} - W^{m}_{\text{e,lb}} \geq M(\delta^{m,\tau}_{\text{t1}} + \delta^{m,\tau}_{\text{t2}} - 2) \quad \forall\,(m,\tau) \tag{11b}$$

Similarly, Eqs. (12) allow electricity to be charged in tier 3 only if the total electricity from tiers 1 and 2 and the cumulative electricity for the previous time step is equal to the threshold $W^{m}_{\text{e,ub}}$:

$$W^{m,\tau}_{\text{e,t2}} - (W^{m}_{\text{e,ub}} - W^{m}_{\text{e,lb}}) \geq M(\delta^{m,\tau}_{\text{t1}} + \delta^{m,\tau}_{\text{t2}} + \delta^{m,\tau}_{\text{t3}} - 3) \quad \forall\,(m,\tau) \tag{12a}$$
$$W^{m,\tau}_{\text{e,t1}} + W^{m,\tau}_{\text{e,t2}} + W^{m,\tau-1}_{\text{e,cum}} - W^{m}_{\text{e,ub}} \leq M(1 - \delta^{m,\tau}_{\text{t2}}) \qquad \forall\,(m,\tau) \tag{12b}$$
$$W^{m,\tau}_{\text{e,t1}} + W^{m,\tau}_{\text{e,t2}} + W^{m,\tau-1}_{\text{e,cum}} - W^{m}_{\text{e,ub}} \geq M(\delta^{m,\tau}_{\text{t2}} + \delta^{m,\tau}_{\text{t3}} - 2) \qquad \forall\,(m,\tau) \tag{12c}$$

Depending on the actual PT rules, some of the existing constraints might be modified and additional constraints may be added to the model.

2.4. Additional considerations

The energy system modeled in this paper is simple because the focus is on handling electricity from the grid and particularly the rules of PTs. The model presented here can be extended to the level of details provided in [26, 27], including:

- Lower bounds of $Q^{m,\tau}_{\text{h,hp}}$ and $Q^{m,\tau}_{\text{h,ah}}$, as well as the dependence of $\text{COP}^{m,\tau}_{\text{hp}}$ and $\eta^{m,\tau}_{\text{h,ah}}$ on the part load ratio, by using semi-continuous and binary decision variables;
- Other components, such as HS, boilers, cogeneration, solar thermal, and PVS, as well as multiple components of the same kind;
- Cooling components and demand;
- Additional objectives, *i.e.* multi-objective optimization.

This approach is also not limited to three-tier applications.

The problem presented here can be decomposed over time into a sequence of smaller problems related to each month separately. They can be solved one at a time, reducing computational time and effort. If there are no constraints connecting the decision variables from distinct months, as in this simple case, the decomposition does not suffer from any loss of precision.

The model is implemented by the first author of the paper, in the Python programming language. MILP problems are solved with Gurobi Optimizer [28].

3. Results and discussion

The methodology for OO of hybrid energy systems is illustrated with an example of a residential building. The total floor area of the building is $180\ \mathrm{m}^2$. The energy demand is estimated with the EnergyPlus software [29], using the typical meteorological year for the City of Niš, Serbia. The electricity demand is satisfied with the energy from the national electricity grid. The heating demand is met with ASHP and AH. The capacity and COP of ASHP, and their dependence on the ambient conditions, are realistic and obtained from the manufacturer. AH is a hypothetical heating device.

The electricity prices are adopted according to the Serbian tariffs. This paper considers three electricity tariff cases: (1) PT, (2) combined PT and TOUT (CT), and (3) TOUT. Table 1 summarizes the prices for three-tier PT and CT. They are expressed without any taxes or incentives, including the value-added tax of 20%. However, taxes are taken into account in the calculations. The prices for TOUT and CT are given for peak hours (16 hours during the day) and off-peak hours (the remaining eight hours during the night and early morning). The TOUT-only case is hypothetical and takes the prices from the middle tier of CT. The capacity charge is fixed and therefore not taken into account for OO. The thresholds of the tiers are given for a period of 30 days. For longer and shorter months, they are corrected proportionally.

The price of heat generated by AH ranges parametrically from zero to $16\ \mathrm{RSD\ kWh}^{-1}$ and is constant throughout the year. It includes the price of the energy source used to drive AH and the efficiency of AH from Eq. (1). This assumption allows for the analysis of optimal operating regimes for various types of AH, *e.g.* gas or biomass boilers, district heating, solar thermal systems, *etc*.

The optimization procedure based on the presented methodology is carried out for a seven-month horizon, *i.e.* from October to April, with a time step of 1 h.

The values of the objective function defined in Eq. (1) vary significantly with the price of AH heat and the electricity tariff. Figure 1 illustrates these values. Each combination of heat price and tariff corresponds to a particular operating regime of the heating system.

The objective value increases with rising heat price. This is partly a direct consequence of the heat price impact on the variable cost but also caused by the decrease of the AH usage in the optimal operating regimes. For high heat prices, the objective becomes constant because AH is not used at all.

The variable cost has the highest values for the TOUT case and the lowest for CTs. The difference

Table 1. The prices of electricity, in RSD

		Progressive tariff	Combined tariff	
Tier name	Threshold [kWh]	All hours	Peek hours	Off-peak hours
Green zone	Up to 350	7.9706	9.1092	2.2773
Blue zone	From 350 to 1600	11.9558	13.6638	3.4160
Red zone	From 1600	23.9117	27.3276	6.8319

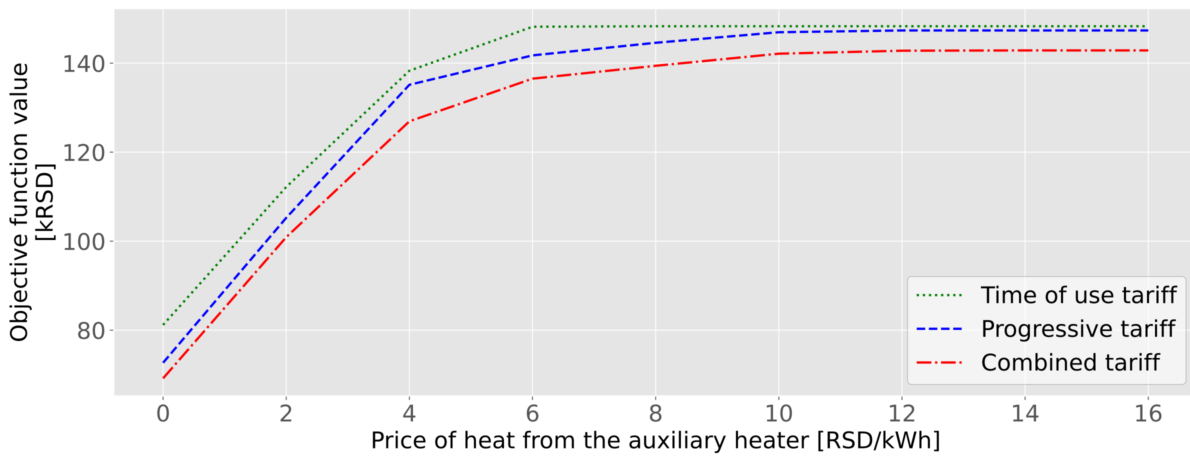

Figure 1. Objective function values for various tariffs and heat prices

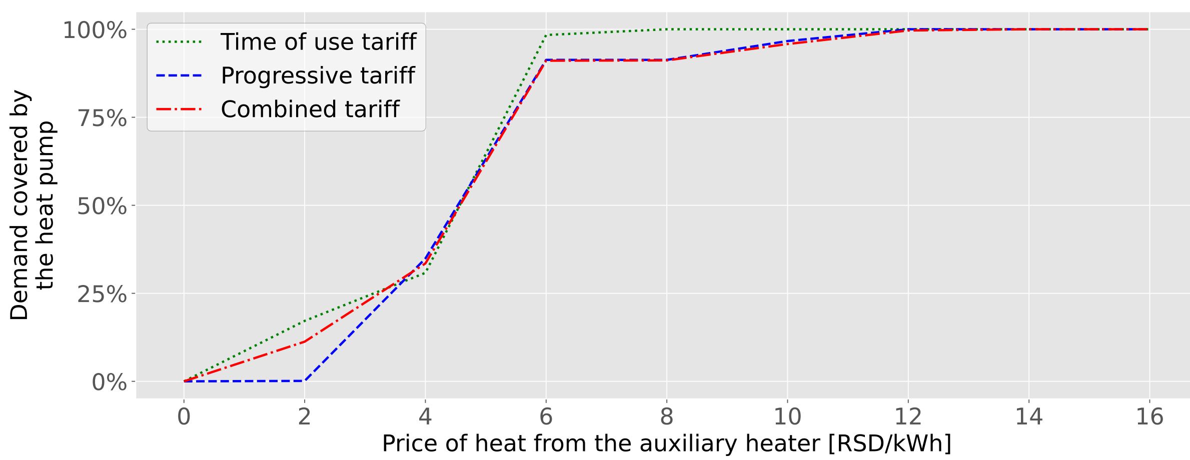

Figure 2. Annual heat demand covered by the heat pump for various tariffs and heat prices

between the two cases is approximately 17% for the zero heat price and decreases to less than 4% for high heat prices. This is mainly due to the fact that TOUT does not have low prices from the first tier (green zone). In contrast, CT has a very low electricity price in the first tier, during off-peak hours. Unlike TOUT, PT and CT also have high electricity prices from the third tier (red zone). However, they are active and impact the solution only when the AH heat price is large and HP covers most or all the demand.

Figure 2 shows the share of the annual heat demand covered by HP. The share is 0% for the zero heat price because it is not optimal to use HP at all due to the free heat from AH. It rises gradually with increasing the heat price of AH, until it reaches 100% for high heat prices. Higher AH heat price implies operating regimes that rely less on AH and more on HP.

The slowest increase in HP share is observed under PT. This is because PTs do not have low electricity prices off-peak. In addition, the threshold of the first tier is below the user electricity demand, therefore, the additional electricity consumed by HP increases the consumption in the second and third tiers. The fastest increase is under TOUT because it has off-peak prices and does not have the red-zone prices.

Figure 3 indicates that it is optimal to run HP and consume electricity in the third tier when the price of AH heat is very high, in this case at least 10 RSD kWh^{-1}. In addition, this is also acceptable during the

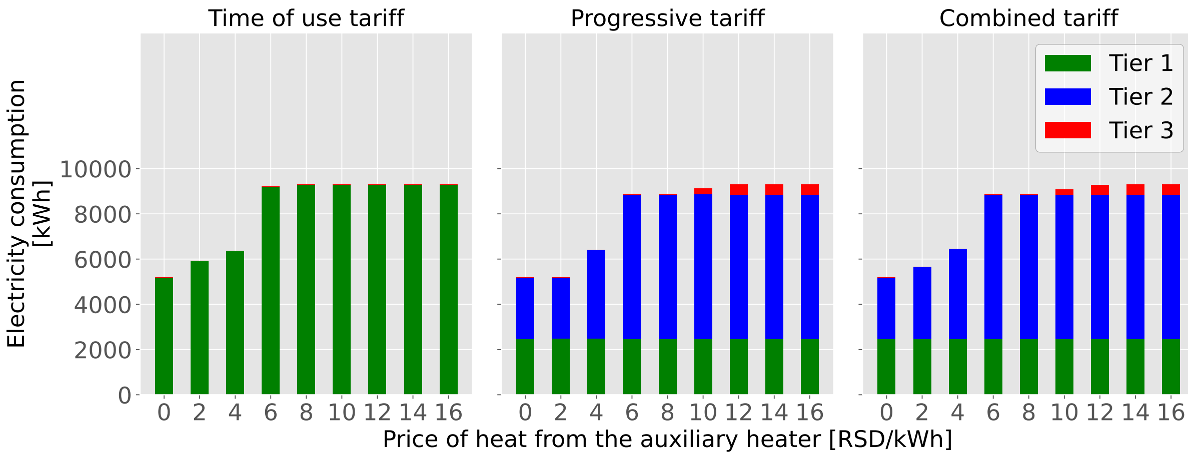

Figure 3. Electricity consumption for various tariffs and heat prices

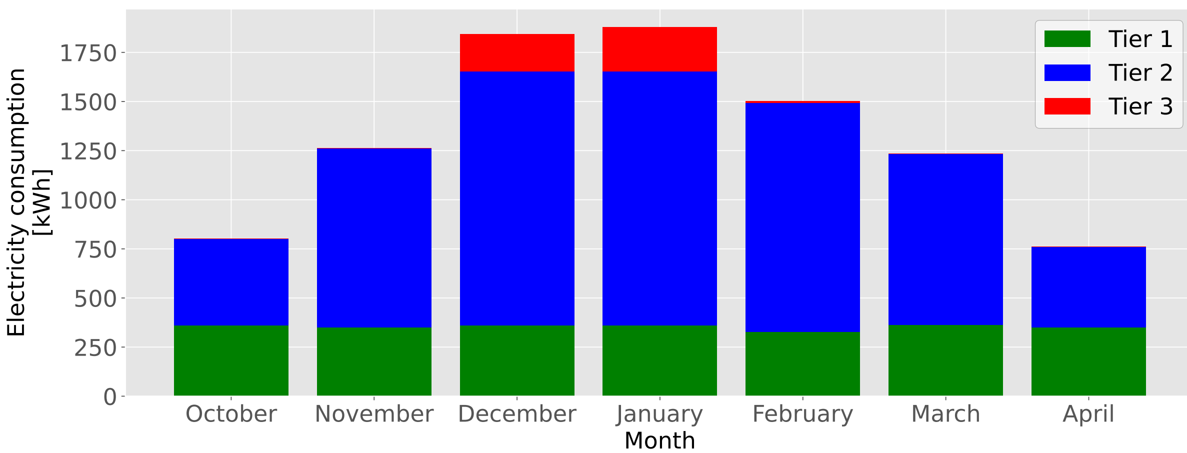

Figure 4. Example of electricity consumption by tier

off-peak hours for CT. Figure 4 provides an example of the monthly electricity consumption by tiers for the case when the AH heat price is 12 RSD kWh^{-1}. It can be seen that the thresholds of the tiers are such that electricity is consumed in the red zone only when the demand for heat is very high, *i.e.* during December, January, and February.

Figure 5 shows the coverage of the heat demand with HP and AH when the price of heat is 6 RSD kWh^{-1}. It indicates that AH is used during the coldest months, either to avoid electricity consumption in the third tier, for PT and CT, or when the temperatures are very low and cause small values of HP COP. Figure 6 clarifies this further with examples of hourly schedules during a day in January. In the case of PT, HP is used when the outdoor temperature is relatively high, and so is COP. CT makes HP operation desirable during the off-peak hours due to a low electricity price, despite low COP values. However, in these two cases, HP usage is reduced to avoid the third tier. In the case of TOUT, this is not a concern, so HP can be used all the time.

Aggregating hourly values of heat generated by HP and AH can yield annual and monthly diagrams which show head demand coverage by components. They are given for the case where the price of AH heat is 6 RSD kWh^{-1}, in Figures 7 and 8. The former shows that HP is always used when the outdoor temperature exceeds $0°C$ for CT and even lower values for TOUT and PT. The latter shows that for the case of PT, the

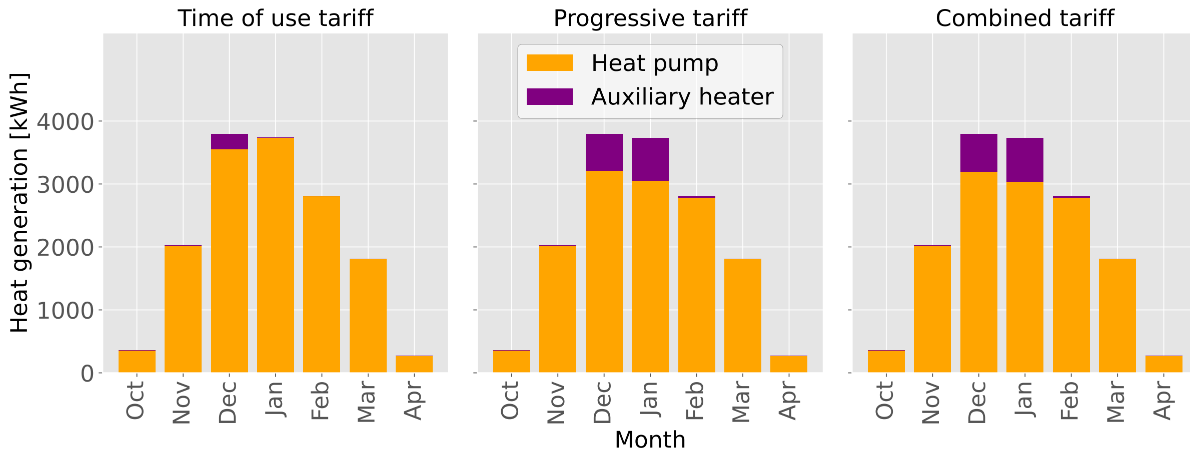

Figure 5. Monthly share of heat demand covered by the heat pump for various tariffs

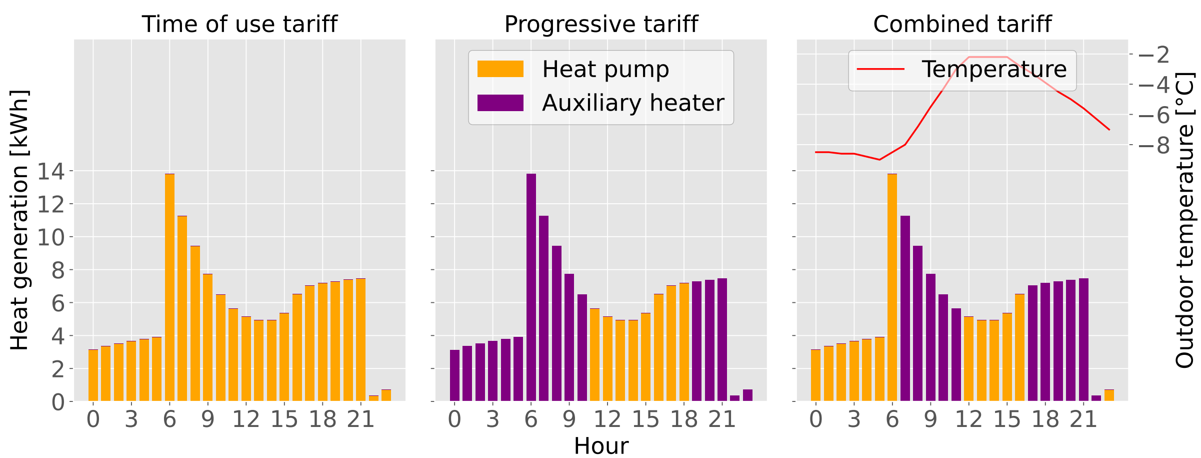

Figure 6. Example of daily heap pump and auxiliary heater scheduling

temperature of $-5°C$ represents the threshold value for using HP or AH. This value is different for each month and depends on prices, efficiencies, tier thresholds, total electricity and heat demand, *etc*. For TOUT and CT, the distribution also depends on peak and off-peak hours.

Optimization problems do not need to be solved for the entire year or month. It is possible to apply this methodology at a particular time during a month to optimize HP scheduling until the end of the current month. Historical values for that month can be used as input, along the future predicted values.

One option to avoid consuming electricity in high-price tariff tiers or during peak hours is the application of absorption [27] and adsorption [30] HPs and cooling devices. Krzywanski *et al.* [31] presented a method to evaluate their performance in heating and cooling systems, which might be combined with the approach shown in this paper. These devices can be driven with bio-fuels, geothermal energy, Solar energy, district heat, waste heat, *etc.*, but they are particularly interesting in the context of using waste heat [32]. This approach allows the integration of such components — either alone or in the combination with compressor HPs and chillers — and the examination of their feasibility and effects on the costs and environmental impact.

From the aspect of practical application, the main limitation of this method is the fact that it needs predicted inputs — electricity and heat demand, but also the variables that determine the HP performance —

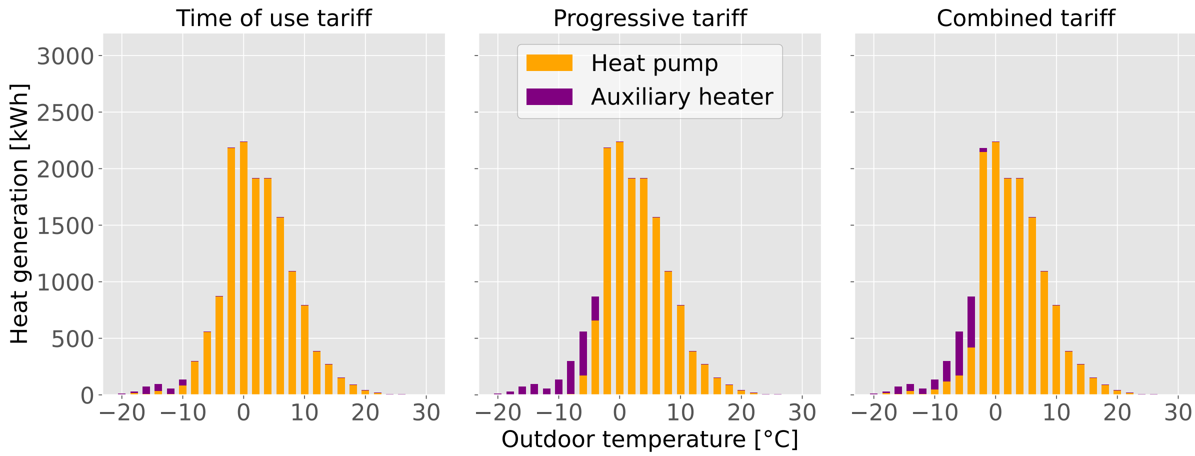

Figure 7. Dependence of the annual heat demand coverage on the outdoor temperature

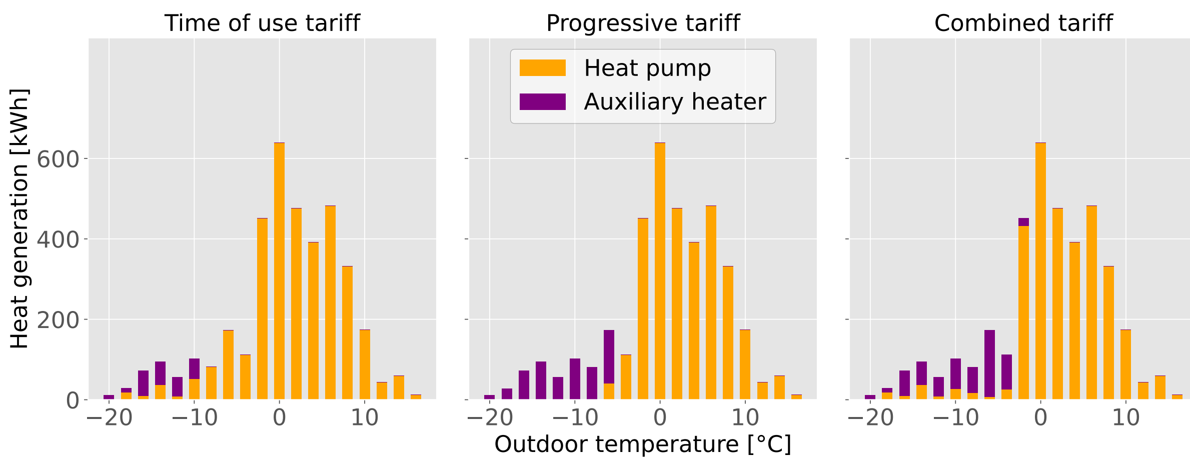

Figure 8. Dependence of the monthly heat demand coverage on the outdoor temperature

until the end of the observed month, when applied in real-time. It is possible to partly overcome this issue with machine learning methods that provide precise forecasts of input variables or learn rules of the optimal operation schedules.

The other potential limitation is the fact that the described MILP model is significantly more complex compared to the cases that consider only simple flat or TOUT tariffs. As a consequence of complexity, its implementation might and probably will be computationally intensive, requiring significant time and effort for problem solving.

4. Conclusion

This paper presents a methodology for determining cost-optimal operation regimes of hybrid energy systems with heat pumps. The methodology is suitable for the cases with progressive electricity tariffs, possibly combined with time-of-use tariffs. It uses mixed integer linear programming to solve the operation optimization problem that minimizes the variable operation cost. The most challenging issue related to the optimization with progressive tariffs is the fact that the price of electricity at each time step depends on the past energy consumption and therefore cannot be considered an ordinary input variable. For this reason, the mathematical model contains the constraints that handle tariff rules and limitations in addition to customary capacity and demand-satisfaction constraints.

The methodology is applied to a case of electricity supply and heating a residential building. The heat demand is covered with an air-source heat pump and an auxiliary heater, while electricity is imported from the national grid. Three cases with specific electricity pricing are observed: (1) time-of-use tariff, (2) progressive tariff, and (3) their combination.

The results indicate that the choice of the structure of the electricity tariff might have a significant impact on the cost-optimal operating regimes and the values of the objective function. The energy prices and their ratios, tier thresholds, and outdoor temperature that affect heat pump efficiency are also very important.

The value of this approach is that it can provide more precise results of operation optimization and, consequently, a better understanding of the potential to reduce costs and related consequences. It can also improve the comprehension of recommendable and preferable operation regimes in specific cases, but also optimal design solutions for heating systems.

There are two main limitations. The first is the requirement to predict input data until the end of the current month in real-time applications. It might be partially resolved using powerful machine learning techniques. The second is the increased computational intensity of the problem-solving process. Future work might focus on overcoming these limitations.

In the future work, this approach might be extended to analyze more complex energy supply systems that might include cooling applications, the integration of thermal storage, photovoltaic panels, as well as absorption and adsorption devices. Another direction could be adjusting the methodology to be able to define near-optimal operating strategies in the real time.

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References

[1] ****, The Future of Heat Pumps, International Energy Agency, Paris, France, 2022, https://www.iea.org/reports/the-future-of-heat-pumps

[2] Adebayo, P., *et al.*, Development, modeling, and optimization of ground source heat pump systems for cold climates: A comprehensive review, *Energy and Buildings 320* (2024) 114646, DOI: https://doi.org/10.1016/j.enbuild.2024.114646

[3] Topić Božič, J. *et al.*, Life cycle assessment of energy green transition goals in Slovenia and Serbia. Heat Pump Example, *Thermal Science 28* (2024), No. 6A, pp. 4709–4721, DOI: https://doi.org/10.2298/TSCI240618222T

[4] Vučković, G. D. *et al.*, Air-source heat pump performance comparison in different real operational conditions based on advanced exergy and exergoeconomic approach, *Thermal Science 25* (2021), No. 3A, pp. 1849–1866, DOI: https://doi.org/10.2298/TSCI200529237V

[5] Aryanfar, Y. *et al.*, A competitive study of a geothermal heat pump equipped with an intermediate economizer for R134a and R513a working fluids, *Thermal Science 27* (2023), No. 6B, pp. 5025–5038, DOI: https://doi.org/10.2298/TSCI230305133A

[6] Kossi, P., Rama, M., Improving the accuracy of heat pump feasibility assessment, *Thermal Science 28* (2024), No. 5B, pp. 4381–4394, DOI: https://doi.org/10.2298/TSCI2405381K

[7] Jokić, A. I. *et al.*, Feasibility of closed loop ground source heat pump for residential heating and cooling applications in Serbia, *Thermal Science 24* (2020), No. 2A, pp. 965–976, DOI: https://doi.org/10.2298/TSCI181219279J

[8] Pesola, A., Cost-optimization model to design and operate hybrid heating systems – Case study of district heating system with decentralized heat pumps in Finland, *Energy 281* (2023) 128241, DOI: https://doi.org/10.1016/j.energy.2023.128241

[9] Krützfeldt H., *et al.*, MILP design optimization of heat pump systems in German residential buildings, *Energy and Buildings 249* (2021) 111204, DOI: https://doi.org/10.1016/j.enbuild.2021.111204

[10] Aguilera, J. J., *et al.*, Operation optimization in large-scale heat pump systems: A scheduling framework integrating digital twin modelling, demand forecasting, and MILP, *Applied Energy 376* (2024) 124259, DOI: https://doi.org/10.1016/j.apenergy.2024.124259

[11] Li, B., *et al.*, Economy and energy flexibility optimization of the photovoltaic heat pump, system with thermal energy storage, *Journal of Energy Storage 100* (2024) 113526, DOI: https://doi.org/10.1016/j.est.2024.113526

[12] Prasanna, A. *et al.*, Recent experiences with tariffs for saving electricity in households, *Energy Policy, 115* (2018), pp. 514–522, DOI: https://doi.org/10.1016/j.enpol.2018.01.044

[13] Kim, K. *et al.*, Public preferences and increasing acceptance of time-varying electricity pricing for demand side management in South Korea, *Energy Economics, 119* (2023), 106558, DOI: https://doi.org/10.1016/j.eneco.2023.106558

[14] S. Ouédraogo *et al.*, Impact of electricity tariffs and energy management strategies on PV/Battery microgrid performances, *Renewable Energy, 199* (2022), pp. 816–825, DOI: https://doi.org/10.1016/j.renene.2022.09.042

[15] An, J. *et al.*, Determining the optimal trading price of electricity for energy consumers and prosumers, *Renewable and Sustainable Energy Reviews, 154* (2022), 111851, DOI: https://doi.org/10.1016/j.rser.2021.111851

[16] Youn, H., Jin, H. J., The effects of progressive pricing on household electricity use, *Journal of Policy Modeling, 38* (2016), 6, pp. 1078–1088, DOI: https://doi.org/10.1016/j.jpolmod.2016.06.001

[17] Quan, S. J., Kim, K., Did new electricity progressive tariff system change energy usage pattern in Seoul apartments? Evidence from integrated multisource dataset and combined analytical models, *Energy and Buildings, 287* (2023), 112979, DOI: https://doi.org/10.1016/j.enbuild.2023.112979

[18] Matar, W., Households' response to changes in electricity pricing schemes: Bridging microeconomic and engineering principles, *Energy Economics, 75* (2018), pp. 300–308, DOI: https://doi.org/10.1016/j.eneco.2018.08.028

[19] Bae, S., Nam, Y., Feasibility analysis for an integrated system using photovoltaic-thermal and ground source heat pump based on real-scale experiment, *Renewable Energy, 185* (2022), pp. 1152–1166, DOI: https://doi.org/10.1016/j.renene.2021.12.094

[20] Jung, S. *et al.*, An optimal scheduling model of an energy storage system with a photovoltaic system in residential buildings considering the economic and environmental aspects, *Energy and Buildings, 209* (2020), 109701, DOI: https://doi.org/10.1016/j.enbuild.2019.109701

[21] Kang, H. *et al.*, Reinforcement learning-based optimal scheduling model of battery energy storage system at the building level, *Renewable and Sustainable Energy Reviews, 190* (2024), Part A, 114054, DOI: https://doi.org/10.1016/j.rser.2023.114054

[22] Dou, Z. *et al.*, Innovative energy solutions: Evaluating reinforcement learning algorithms for battery storage optimization in residential settings, *Process Safety and Environmental Protection, 191* (2024), Part B, pp. 2203–2221, DOI: https://doi.org/10.1016/j.psep.2024.09.123

[23] Kang, H. *et al.*, Multi-objective sizing and real-time scheduling of battery energy storage in energy-sharing community based on reinforcement learning, *Renewable and Sustainable Energy Reviews, 185* (2023), 113655, DOI: https://doi.org/10.1016/j.rser.2023.113655

[24] Zhang, R. *et al.*, Peer-to-peer trading price and strategy optimization considering different electricity market types, tariff systems, and pricing models, *Energy and Buildings, 300* (2023), 113645, DOI: https://doi.org/10.1016/j.enbuild.2023.113645

[25] ****, Energy Sector Development Strategy of the Republic of Serbia up to 2040 with Projections up to 2050, *Official Gazette of the Republic of Serbia*, No. 94 (2024) (In Serbian)

[26] Stojiljković, M. M., *et al.*, Greenhouse gases emission assessment in residential sector through buildings simulations and operation optimization, *Energy 92* (2015), Part 3, pp. 420–434, DOI: https://doi.org/10.1016/j.energy.2015.05.021

[27] Stojiljković, M. M., Bi-level multi-objective fuzzy design optimization of energy supply systems aided by problem-specific heuristics, *Energy 137* (2017), pp. 1231–1251, DOI: https://doi.org/10.1016/j.energy.2017.06.037

[28] ****, Gurobi Optimizer Reference Manual, Gurobi Optimization, LLC, 2024, https://www.gurobi.com, [accessed 10-Dec-2024]

[29] ****, EnergyPlus, Energy Simulation Software, https://energyplus.net/, [accessed 10-Dec-2024]

[30] Shmroukh, A. N. *et al.*, Adsorption working pairs for adsorption cooling chillers: A review based on adsorption capacity and environmental impact, *Renewable and Sustainable Energy Reviews 50* (2015), pp. 445–456, DOI: https://doi.org/10.1016/j.rser.2015.05.035

[31] Krzywanski, J. *et al.*, Adsorption chiller in a combined heating and cooling system: simulation and optimization by neural networks, *Bulletin of the Polish Academy of Sciences Technical Sciences 69* (2021), No. 3, e137054, DOI: https://doi.org/10.24425/bpasts.2021.137054

[32] Sztekler, K. *et al.*, Performance Evaluation of a Single-Stage Two-Bed Adsorption Chiller With Desalination Function, *Journal of Energy Resources Technology 143* (2021), No. 8, 082101, DOI: https://doi.org/10.1115/1.4048771

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