# NUMERICAL STUDY OF FLOW AROUND TWO TEARDROP CYLINDERS IN TANDEM ARRANGEMENT AT LOW REYNOLDS NUMBER

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*In this paper, the numerical method is used to investigate the flow around two tandem teardrop cylinders for four different arrangements at low Reynolds numbers. The influence of Reynolds number and spacing ratio on the drag,lift coefficient and Strouhal number are calculated and analyzed. The Reynolds number ranges from 50 to 200, while the spacing ratio ranges from 0 to 15. The results indicate that the Reynolds number has a more significant effect than the spacing ratio on the two tandem teardrop cylinders. Moreover, the influence of Reynolds number and spacing ratio on drag and lift coefficients can be divided into three regions. The drag and lift coefficients exhibit regular variations in the front and rear regions, and irregularly in the intermediate region, which are 140~ 170,125 ~ 140,80 ~ 125 and 125 ~ 140 at Reynolds number and concentrated in* $2.4\text{-}2.8 \le S < 6$ *in spacing ratio for the four arrangements. A sudden increase present in both cylinders in all arrangements,except for the drag coefficient of the upstream cylinder which varies with Reynolds number.For a fixed spacing ratio or Reynolds numbers,constant vortex shedding is observed in cylinders with the various of Re or S. Arrangement A shows the latest onset of vortex shedding, regardless variations in the Reynolds number or the the spacing ratio.Additionally, intermittent vortex shedding is noted in Arrangements A and C before transitioning to a constant state.Overall, compared to the circular cylinders in tandem arrangement,the teardrop cylinders demonstrate superior flow characteristics.The results will provide a certain theoretical foundation for applications in engineering practice.*

Key words*: teardrop cylinders; tandem arrangement; drag coefficient; lift coefficient; strouhal number;spacing ratio*

## 1. Introduction

Flow around cylinders is widely encountered in engineering practice, such as the heat exchangers, chemical industry, ocean engineering, bridges and buildings etc. Regarding the fundamental physics of flow around cylinders, regular and periodic vortex shedding from the cylinders would induce vortex-induced vibration, which causes structural vibration fatigue and damage. Thus, an in-depth investigation of interactions between flow and cylinder-like structures and the hydrodynamic characteristics of wakes is particularly important.

Over the last century, the flow around a single or multiple cylinders has become a hot research topic due to its importance in practical engineering application, and a larger number of studies have investigated the drag, lift coefficient, Strouhal number, and wake characteristics of flow around cylinders through numerical and experimental methods. Compared to the flow around a single cylinder, the flow around multiple cylinders is more complex,the entire behaviour of the flow is not only depending on the the Reynolds number,but also on the distance between the cylinders and their orientation to the flow direction[1]. The tandem arrangement of cylinders results complicated,due to the mutual interaction of the upstream cylinder and the downstream cylinder shear layers shedding from the cylinder[2].And many researchers have investigated the flow around the circular,elliptic and square cylinders in tandem arrangement.

For the influence of Reynolds numbers, Prabhjot et al.[3] obtained that the flow around the cylinder is laminar and steady for Reynolds numbers below approximately 40~50, while Von Karman vortices can be observed for Reynolds numbers between approximately 40~200. Zhou et al.[4],Meneghini et al.[5],Mahir et al.[6] and Dehkordi et al.[7] numerical simulated the flow around two tandem arrangement circular cylinders in low Reynolds number.Skonecki et al.[2] numerical studied flow around two circular cylinders in tandem arrangements at Re=200 and 3900.Assi[8] experimental investigated the flow-induced vibration interference between two circular cylinders in tandem arrangement at Re from 3000 to 13000. Nazvanova et al.[1]performed numerical investigation at $\mathrm{Re}=3.6\times10^6$ and spacing ratio $S$ =1.56, 1.8, 2.5, 3, 3.7 and 4. Neumeister et al.[9] investigated the dominant frequency ranges of two cylinders in tandem arrangement at Re from $7.1\times10^3$ to $2.4\times10^4$ and spacing ratio $S$ =1.26, 1.4, 1.6 and 3.52.

For the influence of spacing ratios $S$ (the ratio of suface-to-surface distance between the cylinders to the diameter of cylinder.),the results show that in diverse regimes of spacing ratios,the interaction of the upstream cylinder and the downstream cylinder is different.Zhou et al.[4] investigated the flow around two finte-length circular cylinders in tandem arrangement at Re=200,their results show that spacing ratio has a significant effect on the Strouhal number and on the lift and drag coefficients of cylinders.With an increase in $S$ , the average drag coefficient of the downstream cylinder increases, whereas that of the upstream cylinder first decreases and then increases. Additionally, as $4.5\le S\le 5$ , the average drag coefficient of the two cylinders changes suddenly. Zdravkovich et al.[10] study the continuous and discontinuous changes in vortex shedding of flow interference between two circular cylinders in various arrangements.Results show that in the shore separation regime ( $S\le 2.4$ ), both cylinders behave like a single bluff body. Vortex shedding is only observed from the downstream cylinder. In the intermediate separation regime ( $2.4\le S\le 4-6$ ), the flow is bi-stable and switches between the short and large separation regimes depending on the spacing ratio. Intermittent or constant vortex shedding is observed depending on the spacing ratio and the experimental conditions.Dehkordi et al.[7] study the laminar flow around tandem circular cylinders at Re=100,200 and S=1.5,2,3,4. Results observed that the downstream cylinder shows more unsteady behavior at Re=200 compared to Re=100,and for $S$ =1.5,2,3,the drag coefficients of the downstream cylinder are negative,the lift coefficient follows an organized periodicity and their amplitude were nearly close, especially for $S$ =2,3.At $S$ =4,the amplitude of the lift force becomes much larger and its fluctuations become more intense than in the three previous cases.

The flow around other shapes of cylinders in tandem arrangement has been studied. Ota et al.[11] experimentally investigated the flow around two elliptic cylinders in tandem arrangement through measurements of the surface static pressure distribution and estimations of flow parameters such as drag, lift and moment coefficients. Dung et al.[12] numerical simulated the flow around three elliptic cylinders with equal spacing and aspect ratio in tandem arrangements at Re=65~160. Mahrukh et al.[13],Ahmad et al.[14]and Qiu et al.[15] studied the flow around two square cylinders in tandem arrangements,while Zhai et al.[16] numerically investigated flow past two tandem triangular cylinders in a uniform flow at Re=100.

The teardrop cylinder has a better streamlined shape,a smaller drag coefficient, and is more widely used in practice. However, research on the hydrodynamics of single or multiple cylinders has mainly focused on circular,elliptical and square cylinders, etc. Studies on flow around single or multiple teardrop cylinders are rare. In this paper, a two-dimensional model of two tandem teardrop cylinders is built. A numerical method is used to study the flow characteristics of the two tandem teardrop cylinder as the Reynolds number varies from 50 to 200 and the spacing ratio varies from 0 to 15. The drag, lift coefficient and Strouhal coefficient of the cylinders are calculated and analyzed, along with the wake characteristics.This study establishes a theoretical foundation for the use of teardrop-shaped structures across various Reynolds numbers and spacing ratios. Furthermore, it provides theoretical guidance for selecting four tandem arrangements under different conditions.

## 2. Numerical methods and problem definition

### 2.1. Governing equations,drag,lift coefficient and Strouhal number

The Navier-Stokes equations govern two-dimensional unsteady incompressible fluids and are given as follows:

$$\rho\frac{\partial \mathrm{u}}{\partial t}+\rho(\mathrm{u}\cdot\nabla)\mathrm{u}=\nabla\cdot[-pI+\mu(\nabla\mathrm{u}+(\nabla\mathrm{u})^T)]+F \tag{1}$$

$$\rho\nabla\cdot\mathrm{u}=0 \tag{2}$$

Where $\rho$ is the density, $\mathrm{u}$ is the velocity vector, $p$ is pressure, $I$ is the identity matrix, $f$ is the volume force vector.

Based on the Navier-Stokes equations and boundary condition,the drag coefficient,lift coefficient and Strouhal number were be derived in detail by Yu et al.[17],and Thompson et al.[18] use it to .The total fluid force ( $F(F_x,F_y)$ , where $F_x$ and $F_y$ are the streamwise and transverse components of the total fluid force acting on the solid boundary, respectively) is written as:

$$F(F_x,F_y)=\sum_{all(x_f)}\sum_{\alpha=1}^{N_Q}e_{\bar{\alpha}}[f_{\alpha}^{+}(x_f,t)+f_{\alpha}^{+}(x_f+e_{\bar{\alpha}}\Delta t,t)][1-w(x_f-e_{\bar{\alpha}})]\frac{\Delta x}{\Delta t} \tag{3}$$

Where $N_Q$=8 is the number of non-zero lattice velocity vectors and $w(x_f-e_{\bar{\alpha}})$ is an indicator, which is set as 0 at $x_b$ and 1 at $x_f$ . The inner summation accounts for the momentum exchange between the nearest fluid cell ( $x_f$ ) and all possible neighbouring solid cells ( $x_b$ ). The outer summation computes the force contributed by all $x_f$ . The global hydrodynamic coefficients, such as the lift and drag coefficients, Strouhal number, mean drag coefficient, and root-mean-square value of lift coefficient are accordingly computed as:

$$C_D = \frac{F_x}{\frac{1}{2}\rho U_\infty^2 D_h} \quad (4)$$

$$C_L = \frac{F_y}{\frac{1}{2}\rho U_\infty^2 D_h} \quad (5)$$

$$S_t = \frac{f_t \cdot D_h}{U_\infty} \quad (6)$$

$$\overline{C}_D = \frac{1}{N}\sum_1^N C_D \quad (7)$$

$$C_L' = \sqrt{\frac{1}{N}\sum_1^N (C_L - \overline{C}_L)^2} \quad (8)$$

Where $U_\infty$ is the fluid velocity, $D_h$ is hydraulic diameter, $f_t$ is the vortex shedding frequency,which is closely related to the periodicity of lift coefficient.

### 2.2. Problem definition

The computational domain and related boundary conditions are illustrated in Fig.1. $O$ is the coordinate origin, $P_1$ and $P_2$ are the center of the cylinders,and $L$ is the surface-to-surface distance between the two teardrop cylinders.The size of the computational domain used in this study was $25D\times45D$,and diameter $D$ $D$ is the distance between the starting and ending points. A uniform flow $(u/U_\infty = 1, v = 0)$ in the streamwise direction was applied at the inlet of the domain,and $\partial p/\partial x = 0$, all wall boundary conditions are no-slip expect the inlet and outlet.

The two-dimensional model of teardrop cylinder is a compound curve defined by two Bézier curves,these curves share the same starting point and endpoint,and is symmetrical.The length between the starting point and endpoint is $D$. $h$ is the height of teardrop cylinder. The two tandem teardrop cylinders have four arrangement depending on their direction,which are shown in Fig.2.

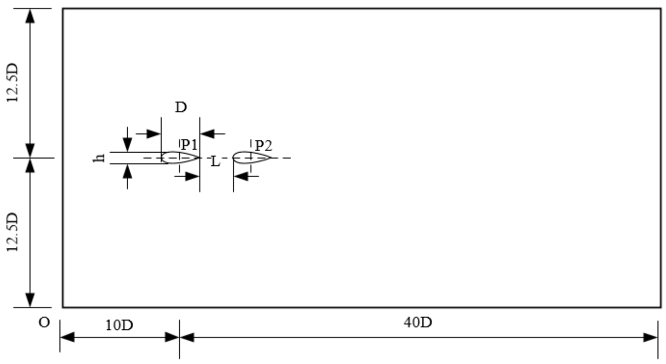


Fig 1. Computational domain for the flow past two teardrop cylinders

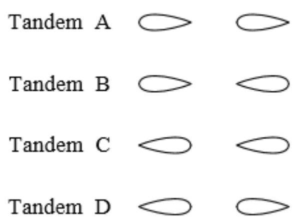


Fig.2 Four arrangement of two tandem teardrop cylinders

### 2.3 Computational method

The Bézier curves were derived from the Rational Bézier curve in the CFD model, which is defined as follows:

$$R(t)=\frac{\sum_{i=0}^{n}\omega_i B_i^n(t)P_i}{\sum_{i=0}^{n}\omega_i B_i^n(t)} \tag{9}$$

$B_i^n(t)=C(n,i)t^i(1-t)^{n-i}$ represents the Bernstein basis function, $C(n,i)$ denotes the number of combinations,and $t$ ($0\le t\le 1$)is a parameter. $P_i$ refers to the Control points,while $\omega_i$ indicates the weight coefficient. In the CFD model,only the control points and the weight coefficient are required. The control points for the two Bézier curves are ( $(9.5D, 12.5D)$ ,$(9.5D, 13D)$,$(10.5D, 12.5D)$) and( $(9.5D, 12.5D)$ , $(9.5D, 12D)$ ,$(10.5D, 12.5D)$) ,with the weight coefficients set to their default values of $(1, 1/\sqrt{2}, 1)$.

To minimize the time spent, the computational domain is divided into two meshed regions. The blue region represents mesh1, which has a length and height that are 1/2 and 1/4 of the total computational domain, respectively. The remaining area is referred to as mesh2, as illustrated in Fig. 3. To establish appropriate grid resolutions for the simulations, convergence studies were conducted on four computational grids within mesh1, as detailed in Tab. 1. In this study, the Reynolds number is 200, $D=L=0.1m$. To facilitate a better comparison of the convergence results, the results are presented with four significant digits after the decimal point.

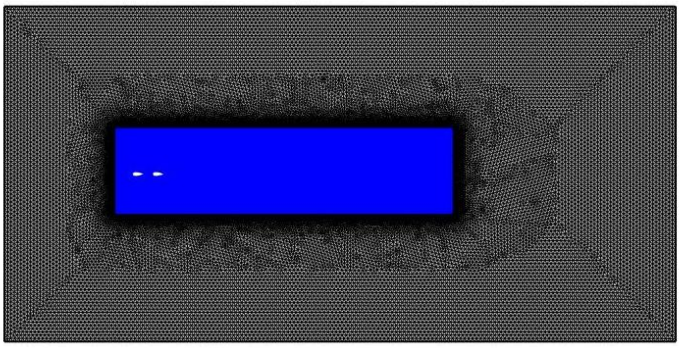

Fig.3 The mesh areas of computational domain(The blue region represents mesh1.)

Tab. 1 Results of grid convergence study

| | No. of cells(mesh1) | $\overline{C}_D$(UC) | $\overline{C}_D$(DC) | $C_L'$(UC) | $C_L'$(DC) |
|---|---|---|---|---|---|
| M1 | 19,154 | 0.7940 | 0.0078 | 0.0132 | 0.0131 |
| M2 | 40,600 | 0.8230 | 0.0400 | 0.0131 | 0.0130 |
| M3 | 81,374 | 0.8068 | 0.0266 | 0.0136 | 0.0136 |
| M4 | 150,296 | 0.8032 | 0.0243 | 0.0136 | 0.0136 |

As indicated in Tab.1, the results suggest that when the mesh1 areas contain a grid with a cell number exceeding approximately 81,374, additional grid refinement has a minimal effect on the hydrodynamic quantities obtained. Specifically, the lift coefficients( $C_L'$ ) of the upstream and downstream cylinders are nearly identical when the results are expressed with four significant figures after the decimal point. Furthermore, the drag coefficient( $\overline{C}_D$ ) of the upstream cylinders in case M3 is 0.448% greater than that of M4, while the difference for the downstream cylinders is 9.465%. Consequently, it can be concluded that the mesh configuration in case M3 could provide sufficient grid resolution for the simulations.

**2.4 Model validation**

In order to validate the numerical model and the results,the mean drag coefficient($\overline{C}_D$) and Strouhal number($S_t$) of two circular cylinders in tandem arrangement are computed at Re=200 and $S$ =0.5,1,2,3,the model parameters used in this paper are the same as those in the literature, results are compared in Tab.1.Their comparisons show good agreements,indicating that the present numerical method works well in predicting the flow parameters and vortex shedding frequency,and can be used to calculate the fluid dynamics of two teardrop cylinders in tandem arrangement.

Tab. 2 Comparison of flow parameters of two circular cylinders in tandem arrangement at Re=200 and $S$ =0.5,1,2,3.

| $L/D$ | | $\overline{C}_D$ | | | | | $S_t$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Present | Meneghini et.al.[5] | Mahir et al.[6] | Dehkodi et al.[7] | Skonecki et al.[2] | Present | Meneghini et.al.[5] | Mahir et al.[6] | Dehkodi et al.[7] | Skonecki et al.[2] |
| 0.5 | UC | 1.113 | 1.060 | - | 1.050 | 1.090 | 0.165 | 0.167 | - | 0.175 | 0.170 |
| | DC | -0.208 | -0.180 | - | -0.150 | -0.200 | 0.165 | 0.167 | - | 0.175 | 0.170 |
| 1 | UC | 1.086 | 1.030 | 1.060 | 1.030 | - | 0.135 | 0.130 | - | 0.138 | - |
| | DC | -0.213 | -0.17 | -0.210 | -0.160 | - | 0.135 | 0.130 | - | 0.138 | - |
| 2 | UC | 1.045 | 1.000 | 1.051 | 1.000 | 1.020 | 0.130 | 0.125 | 0.130 | 0.129 | 0.129 |
| | DC | -0.145 | -0.080 | -0.560 | -0.080 | -0.120 | 0.130 | 0.125 | 0.130 | 0.129 | 0.129 |
| 3 | UC | 1.270 | 1.180 | 1.340 | 1.160 | 1.310 | 0.175 | 0.174 | 0.181 | 0.179 | 0.182 |
| | DC | 0.705 | 0.380 | 0.558 | 0.520 | 0.590 | 0.175 | 0.174 | 0.181 | 0.179 | 0.182 |

## 3. Results and Discussion

### 3.1. Analysis of the influence of Reynolds number

For low Reynolds number laminar flow, the simulations are conducted at $50 \le \mathrm{Re} \le 200$ ,and $S=4$.The length $D$ is 0.1 m . In this case,the height $h$ of the teardrop cylinder is 0.04142 m.All four tandem arrangements of the two teardrop cylinders are simulated.

The variations of the mean drag coefficient and root-mean-square value of lift coefficient for both cylinders are presented in Fig.4 and Fig.5.The Strouhal number of both cylinders changes with the Reynolds number are the same, as shown in Fig.6, the power spectral density(PSD) of the vortex shedding frequency at Re = 200 is shown in Fig. 7.

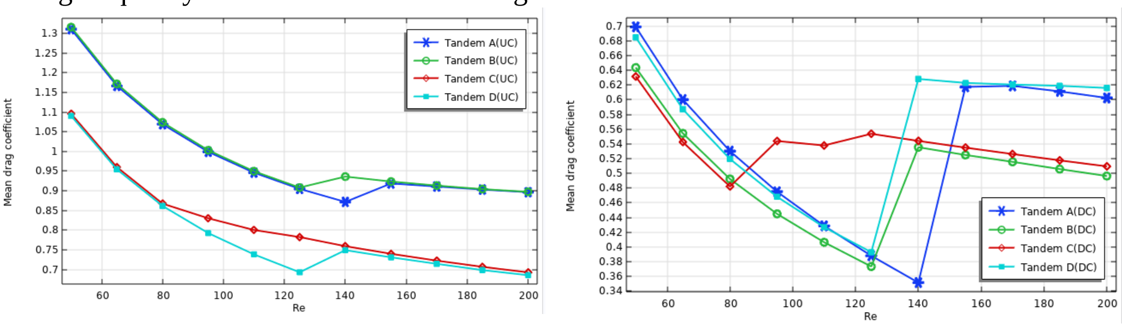


Fig.4 Mean drag coefficient $\overline{C}_D$ for teardrop cylinders of different tandem arrangement with variation of Reynolds number at $S=4$.Left column:upstream cylinder(UC),Right column:downstream cylinder(DC).

From Fig. 4, it can be seen that for the upstream cylinder, the mean drag coefficient ranges from 0.7 and 1.3 at $50 \le \mathrm{Re} \le 200$. With the increase in Reynolds number, the drag coefficient of all four arrangements gradually decreases, only fluctuating slightly at a few Reynolds numbers. Additionally, Arrangement A is similar to Arrangement B, and Arrangement C is similar to Arrangement D,and the drag coefficients of Arrangement C and D are smaller than those of Arrangement A and B, with Arrangement D having the smallest drag coefficient.

For downstream cylinders, when the Reynolds numbers varies from 50 to 200, due to the influence of the wake from the upstream cylinder, the drag coefficient of the downstream cylinder changes more complexly compared to the upstream cylinder. The trends of the four arrangements are vary similarly. As the Reynolds number increases, the drag coefficient initially decreases, then suddenly increases, and finally slowly decreases again. The Reynolds numbers of the four arrangements in which the drag coefficient suddenly increases are 140,125,80 and 125,with Arrangement A experiencing the most significant sudden increase in Reynolds number. The variation of the drag coefficient with Reynolds number is divided into three regions.The front and rear regions exhibit relatively stable variations, while the intermediate region experiences larger fluctuations.The intermediate region of four arrangements are 140~170, 125~140, 80~125 and 125~140.

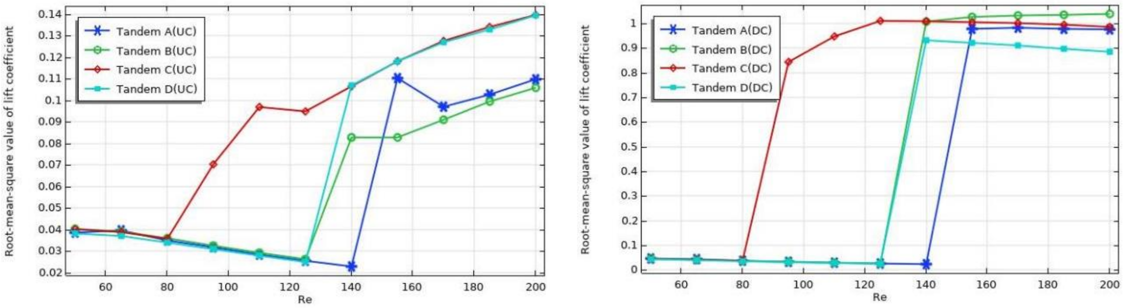


Fig.5 Root-mean-square value of lift coefficient $C_L'$ for teardrop cylinders of different tandem arrangement with variation of Reynolds number at $S = 4$.Left column: upstream cylinder(UC),Right column: downstream cylinder(DC).

Fig.5 shows that for the upstream cylinders, the lift coefficient varies with the Reynolds number of four arrangements,which are more complicated. They all decrease slowly at first, then increase suddenly, and then increase slowly,the range of variation is between 0.02 and 0.14. The Reynolds number for the sudden increase in lift coefficient for the four arrangements are 140,125,80 and 125. Generally, Arrangement C has the largest lift coefficient, while Arrangement B has the smallest.

For the downstream cylinder, the variation in lift coefficient for the four arrangements is relatively simple compared to the upstream cylinder.All four arrangements exhibit a sudden increase in intermediate regions. The Reynolds numbers corresponding to the sudden increase in the lift coefficients of the four arrangements are 140, 125, 80 and 125,which are the same as those of the upstream cylinders.

From the above analysis, it can be observed that the variation in lift coefficient with Reynolds number is divided into three regions, similar to the variation in drag coefficient. The front and rear regions are relatively stable, while the intermediate region fluctuates greatly. The Reynolds numbers regions with large fluctuations in lift coefficient are 140 ~ 170,125 ~ 140,80 ~ 125 and 125 ~140. The variations in lift coefficient of the upstream cylinder and drag coefficient of the downstream cylinder with Reynolds number are intricate.

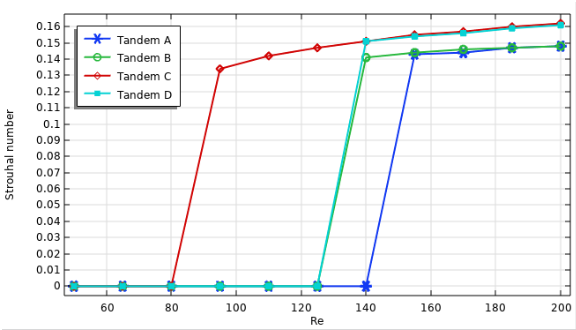


Fig.6 Strouhal number $S_t$ for teardrop cylinders of four arrangement with variation of Reynolds number.

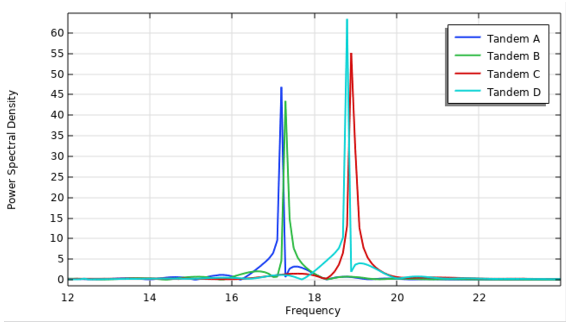


Fig.7 PSD diagram of the time-dependent variations of lift coefficient for teardrop cylinders of four arrangement with Re=200.

Fig. 6 shows the variation of Strouhal number of both the upstream and downstream cylinders in four arrangements with Reynolds number.The trend of the Strouhal number of the four arrangements is similar. When the Reynolds number is small, the Strouhal number is 0, indicating that there are no vortex shedding in the wake of both the upstream and downstream cylinders. As the Reynolds number increase, the Strouhal number obtains a value, and it increases gradually with the increase of the Reynolds number. At this time, constant vortex streets are generated in the wake of both the upstream and downstream cylinders,as shown in Fig.8. The Reynolds numbers of the vortex shedding generated by the wake of the upstream and downstream cylinders of the four arrangements are 155,140,95 and 140.Fig.7 shows the vortex shedding frequency of the four arrangements are 17.2Hz,17.3Hz,18.9Hz and 18.8Hz at Re=200.

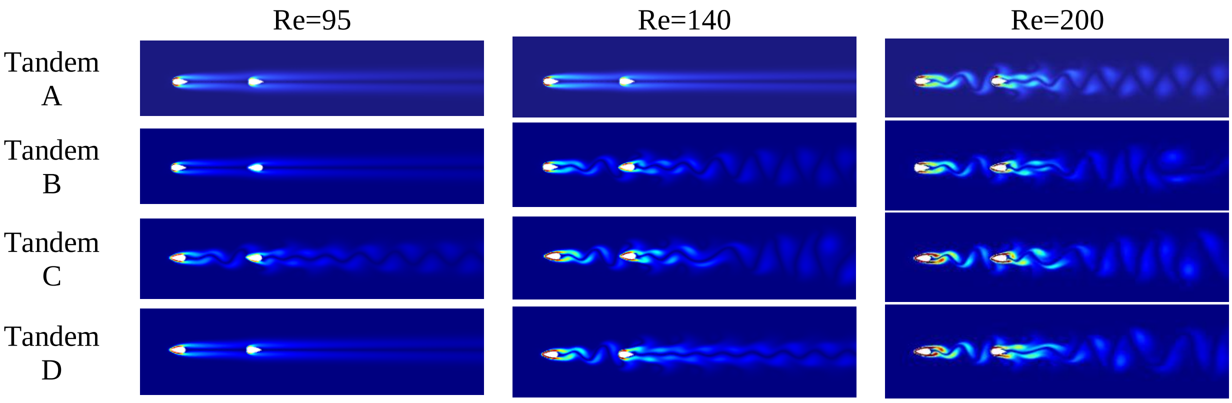


Fig.8 Vorticity contours for teardrop cylinders of four arrangement at Re=95,140,200.

### 3.2. Analysis of the influence of spacing ratio

The spacing ratio is also an important factor that affects the hydrodynamic performance of multiple cylinders, and its influence has been studied in many literatures. In order to investigate the influence of spacing ratios on the hydrodynamic performance of the two teardrop cylinders in tandem arrangement, the drag, lift coefficient and Strouhal number of both the upstream and downstream cylinders were analyzed by varying the spacing ratios at Re = 200. The spacing ratios were varied within a range of 0 ~ 15, with an interval of 0.2 in the range 0 ~ 3,and the interval is 0.5 when the spacing ratio is greater than 3. The length $D$ and the height $h$ of teardrop cylinder are the same with the influence analysis of the Reynolds number.

Results of the drag coefficient for both the upstream and downstream cylinders with varying spacing ratios are presented in Fig. 9 and 10. The lift coefficients can be observed in Fig. 11 and 12, while the Strouhal numbers are depicted in Fig. 13 and 14.

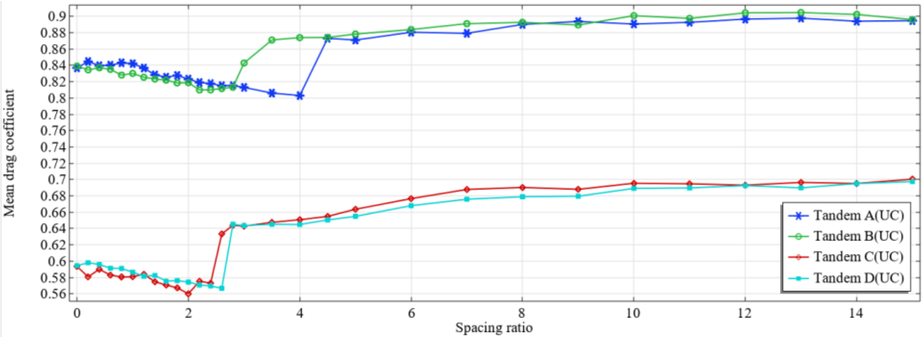


Fig.9 Mean drag coefficient $\overline{C}_D$ for upstream cylinder(UC) of different tandem arrangement with variation of $S$ at Re=200.

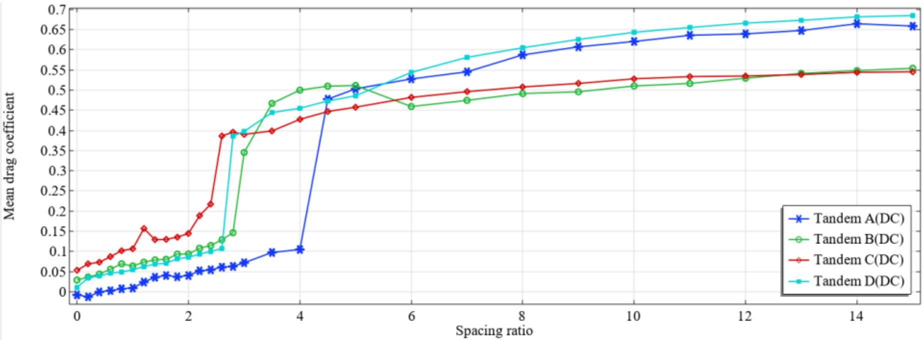


Fig.10 Mean drag coefficient $\overline{C}_D$ for downstream cylinder(DC) of different tandem arrangement with variation of $S$ at Re=200.

The drag coefficient with varying spacing ratios of the upstream cylinder is shown in Fig. 9. The drag coefficient of Arrangements A and B is significantly greater than that of Arrangements C and D. The trend in the variation of the drag coefficients for the four arrangements with spacing ratios first shows a slow decrease, followed by a sudden increase, and finally stabilizes. The spacing ratios $S$ at which the drag coefficient suddenly increases for the four arrangements are 4, 2.8, 2.4 and 2.6, respectively. The spacing ratios of Arrangements B, C and D are relatively close, ranging between 2.4 and 2.8. When the spacing ratio $S$ exceeds 6, the drag coefficient of the four arrangements tends to stabilize, indicating that the spacing ratio has less influence on the drag coefficient.

The variation of the drag coefficient of the downstream cylinder with spacing ratios is shown in Fig. 10. The trends of the four arrangements are quite similar. Compared to the trend of the upstream cylinder, there is also a sudden increase in the drag coefficient with the spacing ratio, where the values are the same as those of the upstream cylinder. When the spacing ratio is less than 2.4 ~ 2.8, the drag coefficient of Arrangement C is the largest, and the Arrangement A has the smallest drag coefficient. In the intermediate region,the spacing ratio is 2.4 ~ 2.8 to 6, variations in the drag coefficient of all four arrangement become complex. When the spacing ratio is greater than 6, the drag coefficients of the four arrangements increase slowly with the spacing ratio, and then gradually stabilize. The drag coefficients of Arrangements A and B are greater than those of Arrangements C and D.

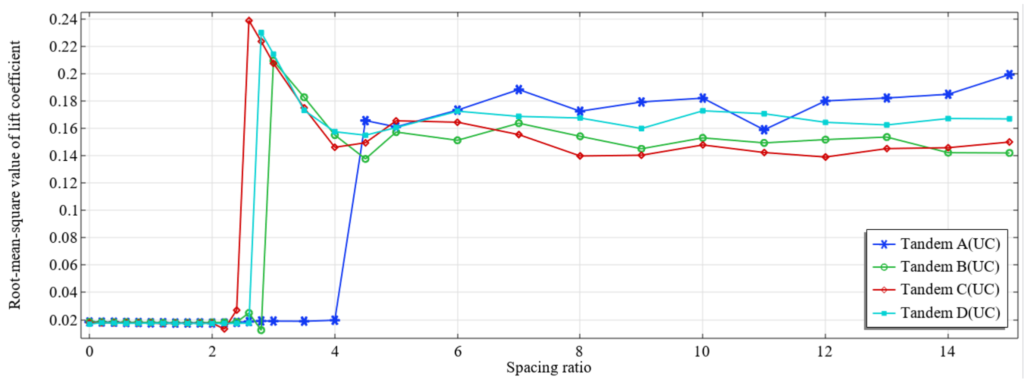


Fig.11 Root-mean-square value of lift coefficient $C_L'$ for upstream cylinder(UC) of different tandem arrangement with variation of $S$ at Re=200.

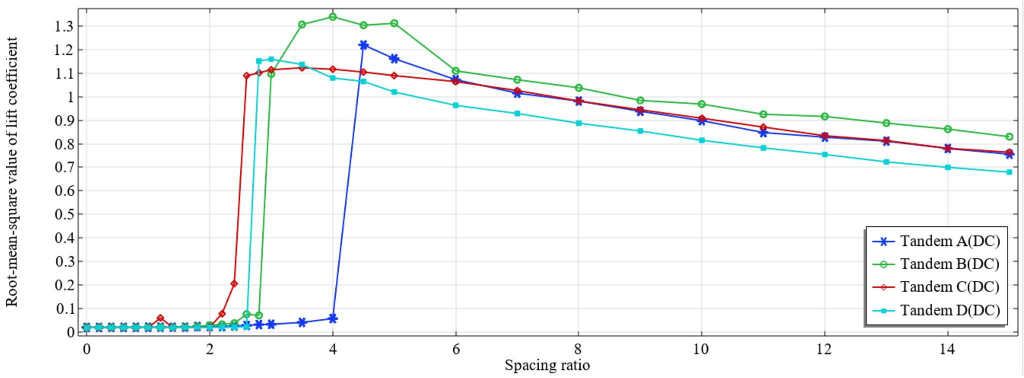


Fig.12 Root-mean-square value of lift coefficient $C_L'$ for downstream cylinder(DC) of different tandem arrangement with variation of $S$ at Re=200.

From Fig. 11, it can be observed that Arrangement A varies more gradually after the sudden increase at $S=4$. The variation trend of the lift coefficient of Arrangements B, C and D is similar,there is a sudden increase in the process when the spacing ratio is 2.4~2.8, followed by a faster rate of decrease, and then the variation tends to stabilize when the spacing ratio is larger than 4. Overall, the variation of the lift coefficients of the four arrangements with the spacing ratio can be divided into three regions, when $0 \le S < 2.4\text{-}2.8$, the lift coefficients of the upstream cylinder in all the four arrangements are essentially the same, and their values are very small; in the intermediate region $2.4\text{-}2.8 \le S < 6$, the lift coefficients of Arrangements B, C, and D are more similar and the lift coefficient of Arrangement A is the smallest; in the region $S \ge 6$, the variation of the lift coefficients in all four arrangements tends to be stable, with Arrangement A having the largest lift coefficient and Arrangement C having the smallest lift coefficient.

The trend of the lift coefficient variation of the downstream cylinder with the spacing ratio is shown in Fig. 12. When the spacing ratio is small, the lift coefficients of the four arrangements are basically the same, close to 0. Subsequently, there is a sudden increase, and the spacing ratios for the sudden increase in lift coefficient for the four arrangements are 4,2.8,2.4 and 2.6. in the intermediate region $2.4\text{-}2.8 \le S < 6$, the lift coefficient vary irregularly with the spacing ratio. When the spacing ratio exceeds 6, the lift coefficients of all four arrangements gradually decrease as the spacing ratio increases, and the variation stabilizes. When comparing Fig.11 to Fig.12, it can be observed that the lift coefficient of the downstream cylinder is generally larger than that of the upstream cylinder.

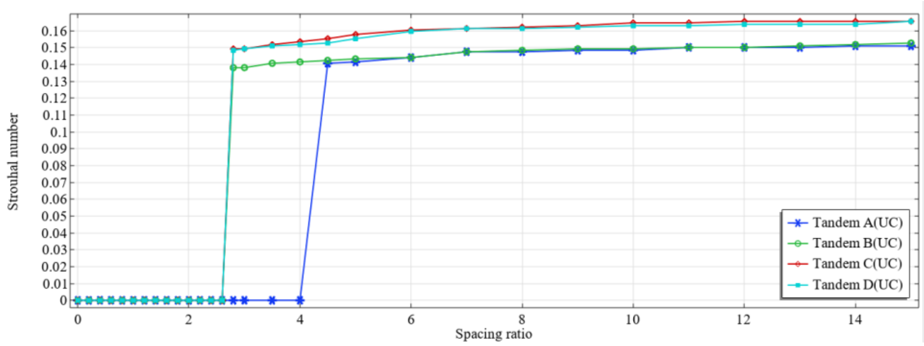


Fig.13 Strouhal number $S_t$ for upstream cylinder(UC) of different tandem arrangement with variation of $S$ at Re=200.

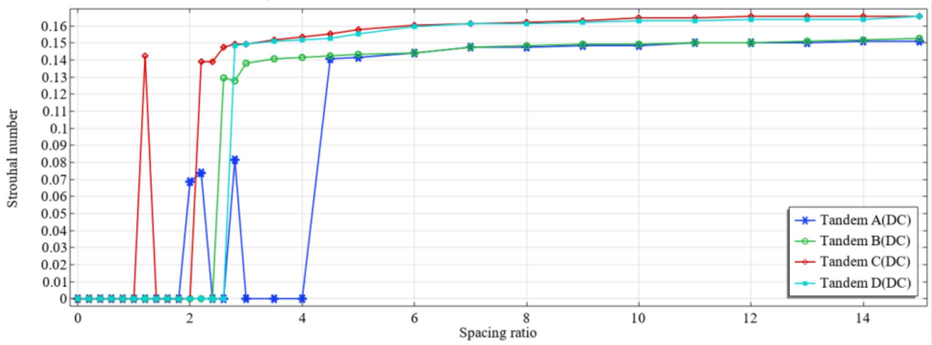


Fig.14 Strouhal number $S_t$ for downstream cylinder(DC) of different tandem arrangement with variation of $S$ at Re=200.

The variation of the Strouhal number of the upstream cylinder for all four arrangements with spacing ratio is shown in Fig.13, the Strouhal number of the four arrangements is 0 in the front region, indicating that there is no vortex street in the wake. As the spacing ratio increases, the Strouhal number is not 0, indicating the generation of a vortex. The spacing ratio of the vortex streets generated for the four arrangements are 4.5,2.8,2.8 and 2.8.Compared with the other three arrangements, the spacing ratio of vortex shedding generated by Arrangement A is the largest. With an increase in the spacing ratio, the Strouhal number shows a gradual increase until the spacing ratio exceeds 6, and then the Strouhal number stabilizes. The Strouhal number for arrangements A and B is lower than that for arrangements C and D, suggesting that the vortex shedding at the wake of arrangements A and B is more pronounced.

The variation of the Strouhal number of the downstream cylinder with the spacing ratio is shown in Fig.14. The wake characteristics of the downstream cylinder are basically similar to those of the upstream cylinder. Due to the influence of the wake from the upstream cylinders, the spacing ratios differ when the constant vortex shedding is generated for the four arrangements, which are 4.5,2.6,2.2 and 2.8. And intermittent vortex shedding occurs in Arrangements A and C. In Arrangement A, it happens at spacing ratios of 2.2, 2.4, and 2.8, while in Arrangement C, it occurs at a spacing ratio of 1.2.

## 4. Conclusions

In this paper, the numerical method was used to analyze the flow characteristics of two tandem teardrop cylinders under various Reynolds numbers and spacing ratios. The drag, lift coefficients and Strouhal numbers of the upstream and downstream cylinders in four arrangements are calculated for

Reynolds numbers ranging from 50 to 200 and spacing ratios from 0 to 15. From the calculation results, the main findings are summarized below:

(1) At $50 \le \mathrm{Re} \le 200$ and $S = 4$, the influence of the Reynolds number on the drag and lift coefficients for two teardrop cylinders in four arrangements can be divided into three regions. In the front and rear regions, the variation is regular. It is irregular in the intermediate region, the irregular regions in the four arrangements are: 140~ 170,125 ~ 140,80 ~ 125 and 125 ~ 140.The Strouhal number increases suddenly from 0 and then increases slowly.

(2) For upstream cylinders, the drag coefficient decreases as the Reynolds number increases, while the lift coefficient initially decreases gradually, then experiences a sudden increase before stabilizing. On the other hand, for downstream cylinders, the drag coefficient first decreases with increasing Reynolds number, then suddenly increases before gradually decreasing. The lift coefficient for downstream cylinders decreases slowly at first and stabilizes after a sudden increase.The Strouhal number of both the upstream and downstream cylinders is similar.

(3) In the four arrangements, as the Reynolds number increase,Arrangements A and B exhibit higher drag coefficient compare to Arrangements C and D for upstream cylinders.Moreover, Arrangements B and C show lower lift coefficients than Arrangements A and D in both the front and rear regions.The lift coefficients are the same for all four arrangements in front region,while in the rear region,Arrangements C and D have higher lift coefficient than Arrangements A and B for upstream cylinders.The variations in lift coefficients for downstream cylinders among the four arrangements are minimal.With increasing Reynolds number, Arrangement C generates the earliest vortex shedding and the highest frequency of vortex shedding.

(4) As the spacing ratio increases at Re=200, the drag and lift coefficients for two teardrop cylinders in four arrangements can also be divided into three regions: $0 \le S < 2.4\text{-}2.8$, $2.4\text{-}2.8 \le S < 6$ and $S \ge 6$.In the intermediate region of $2.4\text{-}2.8 \le S < 6$,the drag and lift coefficients vary irregularly and experience a sudden increase. The constant vortex shedding is observed depending on the spacing ratio.

(5) The influence of the spacing ratio differs between the upstream and downstream cylinders when compared to the Reynolds number. Initially, the drag coefficient decreases gradually, then experiences a sudden increase before eventually stabilizing. The drag coefficient tends to increase with the spacing ratio, with a sudden increase at a specific spacing ratio. The lift coefficient for both cylinders remains consistent as the spacing ratio increases in the front region. In the rear region, the upstream cylinder stabilizes while the downstream cylinder decreases gradually. An irregular intermediate region is present, similar to the one influenced by the Reynolds number. Both cylinders exhibit continuous vortex shedding at a specific spacing ratio.

(6) In the four arrangements, there is a consistent trend in the spacing ratio of drag and lift coefficients. Arrangements C and D exhibit smaller drag coefficients compared to Arrangements A and B in the upstream cylinders. In the downstream cylinder, Arrangement A shows a smaller drag coefficient in the front region, while Arrangements C and D have smaller values in the rear region. The lift coefficient of the four arrangements is similar in the front region. In the rear region, Arrangements A and C exhibit the maximum and minimum lift coefficients in the upstream cylinder, respectively, while they have the same value in the downstream cylinders. Additionally, intermittent vortex shedding is observed in Arrangements A and C before transitioning to a constant state.

The two tandem teardrop cylinders experience a sudden significant change in the values of drag, lift coefficients and Strouhal number in the intermediate regions as Reynolds numbers and spacing ratios varies,the trend is similar to that observed in two tandem circular cylinders[5,7].The drag coefficient of two tandem teardrop cylinders is smaller [2, 5-7]. The higher Reynolds number leads to the formation of unsteady wakes and better wake characteristics in low spacing ratios[19]. Overall, the teardrop cylinders demonstrate superior flow characteristics compared to the circular cylinders in tandem arrangement.

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**Nomenclature**

| | | | |
|---|---|---|---|
| Re | Reynolds number | $C_D$ | Drag coefficient |
| $S$ | Spacing ratios | $C_L$ | Lift coefficient |
| $\rho$ | Density (kg/m$^3$) | $\overline{C}_D$ | Mean drag coefficient |
| u | Velocity vector | $C_L'$ | Root-mean-square value of lift coefficient |
| $p$ | Pressure (Pa) | $S_t$ | Strouhal number |
| $I$ | Identity matrix | $L$ | Surface-to-surface distance (m) |
| $f$ | Volume force vector | $D$ | Diameter (m) |
| $U_\infty$ | Fluid velocity (m/s) | $h$ | Height (m) |
| $D_h$ | Hydraulic diameter (m) | UC | Upstream cylinder |
| $f_t$ | Vortex shedding frequency (Hz) | DC | Downstream cylinder |

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