An extended conformable fractional order grey prediction model of per capita electricity consumption for daily life in China

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Abstract: Conformable fractional-order grey prediction models have attracted considerable attention due to their versatile modeling techniques. However, existing models often suffer from limitations in adaptability. To address this, this study proposes a new extended conformable fractional-order grey prediction model, namely the ECFGM(1,1) model. By integrating an adaptive weighting coefficient into the conformable fractional-order accumulation process, the model can effectively prioritize new information, thereby enhancing its rationality and adaptability. Moreover, the adjusted process can be tailored to either emphasize new information or adhere to traditional accumulation methods, which improves its adaptability. To verify the effectiveness of the ECFGM(1,1) model, ECFGM(1,1) is applied to two examples from the literature. The model evaluation results show that the ECFGM(1,1) model has higher fitting accuracy and predictive accuracy than the GM(1,1), CFGM(1,1), and NIPGM(1,1) models. Using the constructed ECFGM(1,1) for predictive analysis of the per capita electricity consumption for daily life in China, the results show that this model can capture the laws of its changes over time. Finally, per capita electricity consumption for daily life in China from 2022 to 2026 is predicted**.** The results show that by 2026, such consumption is estimated to reach 1165.35 KW.h.

Keywords: Fractional-order grey prediction model; New information priority; Particle swarm optimization algorithm; Per capita electricity consumption for daily life

1. Introduction

The fractional-order grey prediction model encompasses models based on fractional-order accumulation operations originating from Wu's fractional-order cumulative operation [1]. These operations replace the traditional first-order accumulation operation in grey prediction models, enhancing modeling selectivity and predictive performance. Consequently, numerous fractional-order grey prediction models have emerged from academia [2-4].

Among these models, the fractional order GM(1,1) is considered the fundamental model, with the GM(1,1) model serving as the mandatory algorithm to validate new fractional-order accumulation operations. Inspired by the work of Wu, various other fractional-order accumulation operations have been developed [5, 6]. Notably, the conformable fractional-order accumulation operation proposed by Ma et al. has gained popularity due to its simpler mechanism of modeling [7]. However, the conformable fractional-order accumulation operation exhibits shortcomings. For example, it lacks sufficient attention to new information, leading to inadequate model rationality [8]. To address this, this paper proposes modifications to the conformable fractional-order accumulation operation. Zhou et al. introduced the concept of the new information priority accumulation operation, demonstrating its effectiveness across numerous cases [9]. At its core, this approach involves assigning each value in a time series a hyper-parameter function that decreases with time.Furthermore, the new information priority accumulation operation not only boasts a simple modeling mechanism but can also be reduced to a regular first-order accumulation operation,making it valuable. This paper integrates the new information priority accumulation operation into the conformable fractional accumulation operation to enhance model performance. The resulting cumulative operation inherits the advantages of both operations and seamlessly degrades to them without loss. Consequently, the model based on this new summation operation has improved predictive performance.

The main contributions and content layout of this paper are as follows:

(1) An extended conformable fractional order accumulating generator operator is presented, and based on this, the ECFGM(1,1) model is constructed with a detailed solution process provided.

(2) The classical particle swarm optimization algorithm is used to streamline the solution process of the new model, which circumvents the complexities associated with traditional mathematical solution algorithms.

(3) The effectiveness of the ECFGM(1,1) model is verified through two real case studies from the literature. The model is also applied to predict and analyze the per capita electricity consumption for daily life in China.

Section 2 details the specific construction and solution process of the ECFGM(1,1) model, while section 3 validates its effectiveness using examples from the literature. Section 4 applies the ECFGM(1,1) model to predict and analyze the per capita electricity consumption for daily life in China. Finally, conclusions are drawn in section 5.

2. GM(1,1) model based on the extended conformable fractional order accumulated operation

2.1 The traditional GM(1,1) model

Grey system theory is a relatively important method for studying discrete data sequences with a small number of samples and incomplete information [1]. It fully develops and utilizes the explicit and implicit information in the existing data to study the future time distribution over a specific interval. The traditional GM(1,1) modeling process is as follows.

First, first-order accumulation generation is conducted on the original data series

$X^{(0)} = \{x^{(0)}(i)\}_{i=1}^{n}$ to obtain the sequence $X^{(1)} = \{x^{(1)}(i)\}_{i=1}^{n}$ where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(k)$.

Then, the mean series is calculated:

$$z^{(1)}(k) \approx \frac{1}{2}(x^{(1)}(k) + x^{(1)}(k-1)), k = 2,3,...,n \tag{1}$$

Using this series, the first-order differential equation based on a single variable is established and used as the prediction model (i.e., the GM(1,1) model). The standard form of the grey differential equation is as follows:

$$x^{(0)}(k)+az^{(1)}(k)=b, k=2,3,....,n \tag{2}$$

The corresponding whitening differential equation is:

$$\frac{dx^{(1)}(t)}{dt}+ax^{(1)}(t)=b \tag{3}$$

where *a* and *b* are the development coefficient of the system and endogenous control greyscale, respectively.

The formula for estimating the parameter vector $\hat{\theta}$ can be written in the form:

$$\hat{\theta}=(a,b)^T=(B^TB)^{-1}B^TY \tag{4}$$

where

$$B=\begin{pmatrix} -z_\lambda^{(\alpha)}(2) & 1 \\ z_\lambda^{(\alpha)}(3) & 1 \\ \vdots & \vdots \\ z_\lambda^{(\alpha)}(n) & 1 \end{pmatrix}, Y=\begin{pmatrix} x_\lambda^{(\alpha)}(2)-x_\lambda^{(\alpha)}(1) \\ x_\lambda^{(\alpha)}(3)-x_\lambda^{(\alpha)}(2) \\ \vdots \\ x_\lambda^{(\alpha)}(n)-x_\lambda^{(\alpha)}(n-1) \end{pmatrix}, \tag{5}$$

The time-response function of the GM(1,1) model is:

$$\hat{x}^{(1)}(k)=(x^{(0)}(1)-\frac{b}{a})e^{-a(k-1)}+\frac{b}{a}, k=2,3,...,n. \tag{6}$$

Finally, through a single inverse accumulated generating operation (IAGO), the predicted values of the original series, $\hat{x}^{(0)}(k)$, can be obtained:

$$\hat{x}^{(0)}(k)=\hat{x}^{(1)}(k)-\hat{x}^{(1)}(k-1).k=2,3,...,n \tag{7}$$

2.2 The CFGM(1,1) model

Definition 2 [7] For the original sequence $X^{(0)}$, the sequence $X^{(\alpha)}$ generated by α -order conformable fractional (α -CFA) is

$$x^{(\alpha)}(k)=\nabla^{\alpha}x^{(0)}(k)=\begin{cases} \sum_{i=1}^{k}\frac{x^{(0)}(i)}{i^{[\alpha]-\alpha}} & 0<\alpha\le 1, \\ \sum_{i=1}^{k}x^{(\alpha-1)}(i) & \alpha>1, \end{cases} \tag{8}$$

while $\lceil\cdot\rceil$ denotes the ceil function, which returns the smallest integer not less than its argument.

On this basis, by utilizing the differential equation structure of GM(1,1), the CFGM(1,1) model can be constructed.

By combining the conformable fractional accumulating operator with GM(1,1), the CFGM(1,1) model can be constructed, and the final predicted sequence is as follows:

$$\hat{x}^{(0)}(k)=\begin{cases}\Delta^{\alpha}\hat{x}^{(\alpha)}(k)=k^{[\alpha]-\alpha}\Delta^{n}\hat{x}^{(\alpha)}(k) & \alpha\in(n,n+1]\\ k^{1-\alpha}(\hat{x}^{(\alpha)}(k)-\hat{x}^{(\alpha)}(k-1)) & \alpha\in(0,1]\end{cases},k=2,\ldots n, \tag{9}$$

2.3 The NIPGM(1,1) model

Definition 1 [9]: For the original sequence $X^{(0)}=\{x^{(0)}(i)\}_{i=1}^{n}$, the sequence $X^{(\lambda)}$ is generated by $\lambda-order$ new information priority accumulation, where

$$X^{(\lambda)}(k)=\sum_{i=1}^{k}\lambda^{k-i}x^{(0)}(i),k=1,2,\ldots,n. \tag{10}$$

λ is a nonlinear parameter in the interval $[0,1]$.

By combining the new information priority accumulating operator with GM(1,1), the NIPGM(1,1) model can be constructed, and the final predicted sequence is as follows:

$$\hat{x}^{(0)}(k)=\hat{x}^{(1)}(k)-\lambda\hat{x}^{(1)}(k-1).k=2,3,\ldots,n \tag{11}$$

2.4 ECFGM(1,1) model

To enhance the information extraction capability of the grey accumulating operator, inspired by Definitions 1 and 2, this paper proposes a definition of an extended conformable fractional-order accumulating generator operator.

Definition 3: If the original non-negative time series is $X^{(0)}=\{x^{(0)}(i)\}_{i=1}^{n}$, then its extended conformable fractional accumulated operation can be expressed as

$$x_{\lambda}^{(\alpha)}(k)=\nabla_{\lambda}^{\alpha}x^{(0)}(k)=\begin{cases}\sum_{i=1}^{k}\frac{x^{(0)}(i)}{i^{[\alpha]-\alpha}}\lambda^{k-i} & 0<\alpha\leq 1,\\ \sum_{i=1}^{k}x_{\lambda}^{(\alpha-1)}(i)\lambda^{k-i} & \alpha>1,\end{cases} \tag{12}$$

In the equation provided, α and λ represent the conformable fractional-order accumulation operator and the new information priority accumulation operator, respectively, It is evident that when $\lambda=1$, Equation (1) simplifies to the conformable fractional-order accumulation operation. When $\alpha=1$, Equation (1) reduces to the new information priority accumulation operation. Finally, when both $\lambda=1$ and $\alpha=1$, Equation (1) transforms into a first-order accumulation operation. Notably, the extended conformable accumulation operation assigns greater weight to new information, aligning with the principle of prioritizing new information. It is easy to obtain that the inverse of Equation (12) is

$$x^{(0)}(k)=(x_{\lambda}^{(\alpha)}(k)-\lambda x_{\lambda}^{(\alpha)}(k-1))(k^{[\alpha]-\alpha}). \tag{13}$$

Based on Equation (12) and the modeling steps of the GM(1,1) model [9], one can give the expression for the GM(1,1) model based on the extended conformable fractional order summation operation (ECFGM(1,1)), that is,

$$\frac{dx_{\lambda}^{(\alpha)}(t)}{dt}+a\cdot x_{\lambda}^{(\alpha)}(t)=b \qquad (14)$$

where a and b are the structural parameters of the model, usually obtained by the least squares method.

The discrete formula of Equation (14) can be obtained as follows:

$$x_{\lambda}^{(\alpha)}(k)-x_{\lambda}^{(\alpha)}(k-1)+a\cdot z_{\lambda}^{(\alpha)}(k)=b, \qquad (15)$$

where $z_{\lambda}^{(\alpha)}(k)$ is called the background value, which is obtained by performing an integral operation on Equation (14) over the interval $[k-1,k]$. Usually,

$$z_{\lambda}^{(\alpha)}(k)\approx\frac{1}{2}\left(x_{\lambda}^{(\alpha)}(k)+x_{\lambda}^{(\alpha)}(k-1)\right). \qquad (16)$$

Based on Equation (16) and the least squares method, one can give a parameter estimator for the model, that is,

$$\begin{pmatrix}\widehat{a}\\ \widehat{b}\end{pmatrix}=(B^{T}B)^{-1}B^{T}Y \qquad (17)$$

where

$$B=\begin{pmatrix}-z_{\lambda}^{(\alpha)}(2) & 1\\ z_{\lambda}^{(\alpha)}(3) & 1\\ \vdots & \vdots\\ z_{\lambda}^{(\alpha)}(n) & 1\end{pmatrix},Y=\begin{pmatrix}x_{\lambda}^{(\alpha)}(2)-x_{\lambda}^{(\alpha)}(1)\\ x_{\lambda}^{(\alpha)}(3)-x_{\lambda}^{(\alpha)}(2)\\ \vdots\\ x_{\lambda}^{(\alpha)}(n)-x_{\lambda}^{(\alpha)}(n-1)\end{pmatrix}, \qquad (18)$$

and n denotes the sample size.

The time response function of the model is

$$\widehat{x}_{\lambda}^{(\alpha)}(k)=(x_{\lambda}^{(\alpha)}(1)-\frac{\widehat{b}}{\widehat{a}})e^{-\widehat{a}(k-1)}+\frac{\widehat{b}}{\widehat{a}},k=1,\cdots,n, \qquad (19)$$

which is the key formula used to obtain the model's prediction results. Once the time response series is obtained using Equation (8), one can obtain the final prediction of the model using Equation (3).

From the modeling process, it is evident that the hyper-parameters α and λ impede the model's solving process. Traditionally, addressing this issue involves treating it as a planning problem [10]. However, conventional mathematical methods for solving such programming problems are often time-consuming. In this study, we propose employing the classical particle swarm optimization algorithm (PSO) [11-12] to rapidly solve the programming model and obtain the optimal parameters.

In this paper, the mean absolute percentage error [13] is adopted as the loss function for the planning model. This choice allows us to efficiently optimize the parameters and refine the model's predictive accuracy.

$$\underset{\alpha,\lambda}{MAPE} = \frac{1}{n-1}\sum_{k=2}^{n}\left|\frac{x^{(0)}(k)-\widehat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\%, \tag{20}$$

$$s.t.\begin{cases} \alpha > 0, 0 < \lambda \le 1, \\ \begin{pmatrix} \widehat{a} \\ \widehat{b} \end{pmatrix} = (B^T B)^{-1} B^T Y, \\ B = \begin{pmatrix} -z_\lambda^{(\alpha)}(2) & 1 \\ z_\lambda^{(\alpha)}(3) & 1 \\ \vdots & \vdots \\ z_\lambda^{(\alpha)}(n) & 1 \end{pmatrix}, Y = \begin{pmatrix} x_\lambda^{(\alpha)}(2) - x_\lambda^{(\alpha)}(1) \\ x_\lambda^{(\alpha)}(3) - x_\lambda^{(\alpha)}(2) \\ \vdots \\ x_\lambda^{(\alpha)}(n) - x_\lambda^{(\alpha)}(n-1) \end{pmatrix}, \\ \widehat{x}_\lambda^{(\alpha)}(k) = (x_\lambda^{(\alpha)}(1) - \frac{\widehat{b}}{\widehat{a}})e^{-\widehat{a}(k-1)} + \frac{\widehat{b}}{\widehat{a}}, \\ \widehat{x}^{(0)}(k) = (\widehat{x}_\lambda^{(\alpha)}(k) - \lambda \widehat{x}_\lambda^{(\alpha)}(k-1))(k^{[\alpha]-\alpha}). \end{cases}$$

Indeed, once the optimal parameters of the model are obtained through the particle swarm optimization algorithm, obtaining the final prediction results becomes straightforward. With the optimized parameters in hand, the model is equipped to generate accurate predictions for the target variable. This streamlined approach enhances the efficiency and effectiveness of the prediction process, allowing reliable forecasting outcomes based on the developed model.

3 Validation of the ECFGM(1,1) model

3.1 Evaluation criteria

Before actual application, this section presents two examples to illustrate the effectiveness of the proposed model compared with three competing models: GM(1,1) [1], CFGM(1,1) [7], and NIPGM(1,1) [9]. Furthermore, to assess the predictive accuracy of these grey models, the mean absolute percentage error (MAPE) and root mean square error (RMSE) are used as metrics to evaluate the predictive performance. The definition of RMSE is as follows:

$$RMSE = \sqrt{\frac{1}{n-1}\sum_{k=2}^{n}(x^{(0)}(k) - \hat{x}^{(0)}(k))^2} \tag{21}$$

Lewis gives the criteria used to evaluate MAPE [14], as shown in Table 1.

Table 1: The criteria for MAPE

MAPE	<10	10–20	20–50	>50
Forecasting ability	Excellent	Good	Reasonable	Weak

3.2 Example verification

Example 1: Predicting the annual per capita electricity consumption in China

The data involved in this case are the annual per capita electricity consumption in China from 2000 to 2015, as described previously [15]. According to the modeling scale of this previous study, data from 2000–2010 are used to train the model, to predict the per capita electricity consumption in China in 2011–2015, and to compare it with the actual values to test the predictive ability of the model.

The iterative process of the proposed model is depicted in Figure 1. This figure illustrates that the loss function of the proposed model rapidly reaches equilibrium, indicating high computational efficiency. Table 2 provides additional parameter details, while Table 3 presents the results and parameter details of the four models in this case.According to Table 3, both the MAPE and the RMSE values of the proposed model outperform those of the competing models in both the training set and the test set, affirming its effectiveness. Notably, this can be attributed to the fact that the other three models are special forms of the proposed method. Moreover, the MAPE of the advanced model GPMB(1,1,2) proposed in the literature is 2.97, which is inferior to the MAPE of the proposed model. This underscores the significant benefits that the new accumulation operation can bring to the GM(1,1) model.

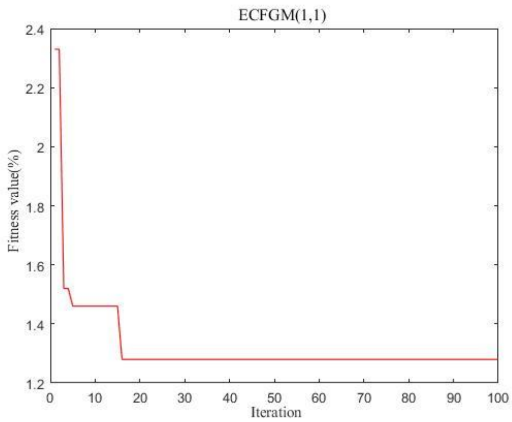

Figure 1: Track of searching for the optimal power index by PSO.

Table 2: Parameter values for four grey models.

	ECFGM(1,1)	NIPGM(1,1)	CFGM(1,1)	GM(1,1)
α	0.18	1	0.71	1
λ	0.74	0.89	1	1

Table 3: Simulated and predicted results by different grey models.

Year	Actual values	ECFGM(1,1)		NIPGM(1,1)		CFGM(1,1)		GM(1,1)	
		Value	Error (%)	Value	Error (%)	Value	Error (%)	Value	Error (%)
2000	1066.9	--	--	--	--	-	--	--	--
2001	1157.6	1128.76	2.49	1156.76	0.07	1098.26	5.13	1231.25	6.36
2002	1286	1285.35	0.05	1330.83	3.49	1313.72	2.16	1368.97	6.45
2003	1477	1475.27	0.12	1514.90	2.57	1518.82	2.83	1522.08	3.05
2004	1695.2	1695.51	0.02	1709.54	0.85	1723.48	1.67	1692.32	0.17
2005	1913	1932.22	1.00	1915.38	0.12	1932.75	1.03	1881.61	1.64
2006	2180.6	2175.23	0.25	2133.04	2.18	2149.85	1.41	2092.06	4.06
2007	2482.2	2418.89	2.55	2363.21	4.79	2377.11	4.23	2326.05	6.29
2008	2607.6	2660.47	2.03	2606.61	0.04	2616.45	0.34	2586.21	0.82
2009	2781.7	2898.88	4.21	2863.99	2.96	2869.56	3.16	2875.47	3.37
2010	3134.8	3133.77	0.03	3136.17	0.04	3138.03	0.10	3197.09	1.99
MAPE			1.28		1.71		2.21		3.42
RMSE			46.66		51.82		51.99		77.90
2011	3497	3365.20	3.77	3423.99	2.09	3423.38	2.11	3554.67	1.65
2012	3684.2	3593.33	2.47	3728.35	1.20	3727.16	1.17	3952.26	7.28
2013	3993	3818.40	4.37	4050.20	1.43	4050.94	1.45	4394.31	10.05
2014	4132.9	4040.62	2.23	4390.55	6.23	4396.34	6.37	4885.80	18.22
2015	4321	4260.20	1.41	4750.45	9.94	4765.04	10.28	5432.26	25.72
MAPE			2.85		4.70		4.82		12.58
RMSE			116.9		228.63		235.45		638.43

Example 2: Predicting natural gas consumption in China

The second dataset pertains to China's natural gas consumption from 2003 to 2013, as described previously [16]. In this example, data from 2003 to 2009 are used for model training. The trained model is then used to predict the values in 2010–2013 and compare them with the actual values. The iterative process of the proposed model is illustrated in Figure 3. Similarly to Case 1, Figure 3 demonstrates that the proposed model achieves convergence in a very short time, indicating its computational efficiency. Table 5 presents the results and parameter details of the four models in this case, and Table 4 provides additional parameter details. Referring to the results in Table 4, it is evident that the proposed model, along with NIPGM(1,1) and CFGM(1,1), significantly outperforms the traditional GM(1,1) model in terms of test set performance. Notably, among the three models, the proposed model exhibits the best performance, highlighting the efficacy of combining the two accumulation operations. This suggests the soundness of the integration of the two accumulation operations.

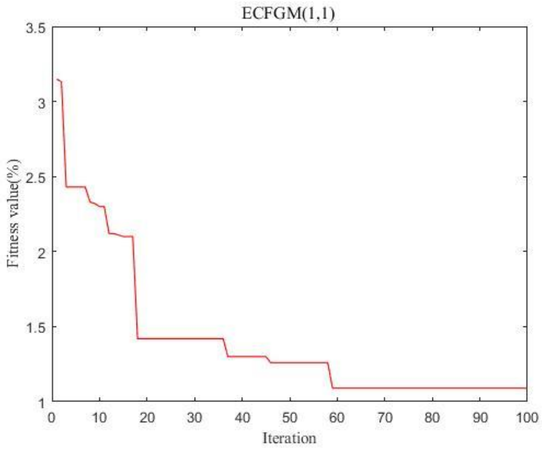

Figure 2: Track of searching for the optimal power index by PSO.

Table 4: Parameter values for four grey models.

	ECFGM(1,1)	NIPGM(1,1)	CFGM(1,1)	GM(1,1)
α	1.82	1	0.73	1
λ	0.55	0.75	1	1

Table 5: Simulated and predicted results by different grey models.

Year	Actual values	ECFGM(1,1)		NIPGM(1,1)		CFGM(1,1)		GM(1,1)	
		Value	Error (%)	Value	Error (%)	Value	Error (%)	Value	Error (%)
2003	35	--	--	--	--	--	--	--	--
2004	41.5	41.46	0.09	41.5	0.00	41.50	0.00	43.7	5.30
2005	49.3	49.24	0.12	50.3	2.03	50.28	1.98	50.3	2.03
2006	58.6	59.33	1.24	59.3	1.19	58.98	0.65	57.9	1.19
2007	69.2	69.00	0.29	68.4	1.16	67.98	1.77	66.7	3.61
2008	80.3	78.26	2.54	77.6	3.36	77.47	3.52	76.9	4.23
2009	85.2	87.13	2.26	87.1	2.23	87.62	2.84	88.6	3.99
MAPE			1.09		1.66		1.79		3.39
RMSE			1.19		1.47		1.66		2.44
2010	94.8	95.60	0.85	96.6	1.90	98.54	3.94	102.2	7.81
2011	103.1	103.69	0.58	106.3	3.10	110.34	7.02	117.5	13.97
2012	107.2	111.41	3.93	116.2	8.40	123.13	14.86	135.4	26.31
2013	119.3	118.77	0.44	126.3	5.87	137.03	14.87	155.9	30.68
MAPE			1.45		4.82		10.17		19.69
RMSE			2.18		5.99		12.59		24.48

4. Predicting per capita electricity consumption for daily life in China

Given the development of the economy and the rise in living standards, the significance of electricity in the lives of Chinese people has grown substantially. With enhanced power infrastructure in rural regions, increased disposable income, and widespread adoption of various electrical appliances, electricity has become indispensable for daily life. Moreover, the surge in the usage of new energy vehicles among Chinese people has further elevated electricity demand. Consequently, accurate prediction of per capita electricity consumption for daily life is of paramount importance for electricity providers and related agencies to formulate effective electricity production plans. For this purpose, empirical data on per capita electricity consumption for daily life in China, sourced from China's National Bureau of Statistics (https://www.stats.gov.cn/), are utilized (as listed in Table 7). These data span the period of 2005–2017 and are divided into two sets: one for model construction and the other for assessing model accuracy. Specifically, data from 2005 to 2017 are employed to develop the four prediction models, while the remaining data are reserved for evaluating the accuracy of these models. In a manner akin to Cases 1 and 2, the parameters for these models are computed and presented in Table 6. The trajectories of λ and α using PSO are illustrated in Figure 3.

Furthermore, the simulated and predicted results are detailed in Table 7. Remarkably, the proposed method also exhibits a notable convergence rate in this case, affirming its feasibility. As shown in Table 7, while the training set MAPE of all models reached a "good" level, the MAPE and RMSE of the proposed method were still superior to those of all other models. However, a peculiar observation was made in the test set: both the NIPGM(1,1) and the CFGM(1,1) models exhibit lower test set results than the GM(1,1) model. This discrepancy can be attributed to the fact that both models underestimate the actual values during training, resulting in poorer test set performance. In summary, the ECFGM(1,1) model demonstrates superior performance in both fitting and prediction stages when compared with the comparative models. The 3D bar chart of APE shown in Figure 4 reveals relatively small relative errors at each time point, indicating the efficacy of the proposed method in simulating and predicting time series changes. Additionally, Figure 5 illustrates that the simulation and prediction trends of ECFGM(1,1) closely align with the actual data.

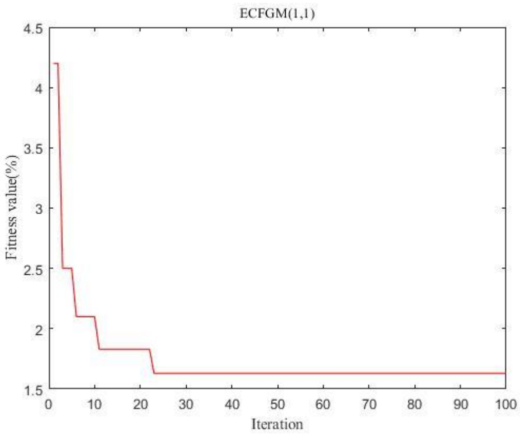

Figure 3: Track of searching for the optimal power index by PSO.

Table 6: Parameter values for four grey models.

	ECFGM(1,1)	NIPGM(1,1)	CFGM(1,1)	GM(1,1)
α	0.6	1	0.68	1
λ	0.1	0.79	1	1

Table 7: Simulated and predicted results by different grey models.

Year	Actual values	ECFGM(1,1)		NIPGM(1,1)		CFGM(1,1)		GM(1,1)	
		Value	Error (%)	Value	Error (%)	Value	Error (%)	Value	Error (%)
2005	184	--	--	--	--	--	--	--	--
2006	221	220.94	0.03	221.12	0.05	221.00	0.00	254.93	15.35
2007	256	262.87	2.68	257.15	0.45	260.93	1.93	276.29	7.93
2008	308	298.86	2.97	292.73	4.96	296.61	3.70	299.45	2.78
2009	332	331.75	0.08	327.87	1.24	330.25	0.53	324.55	2.25
2010	366	363.05	0.81	362.56	0.94	362.90	0.85	351.74	3.89
2011	383	393.78	2.81	396.82	3.61	395.18	3.18	381.22	0.46
2012	418	424.72	1.61	430.65	3.03	427.49	2.27	413.17	1.15
2013	459	456.55	0.53	464.06	1.10	460.10	0.24	447.80	2.44
2014	513	489.89	4.50	497.04	3.11	493.22	3.86	485.33	5.39
2015	523	525.41	0.46	529.62	1.26	527.02	0.77	526.00	0.57
2016	548	563.78	2.88	561.78	2.51	561.64	2.49	570.08	4.03
2017	607	605.75	0.21	593.54	2.22	597.18	1.62	617.86	1.79
MAPE			1.63		2.04		1.78		4.00
RMSE			9.56		10.45		9.56		16.90
2018	650	652.17	0.33	624.90	3.86	633.77	2.50	669.64	3.02
2019	717	703.99	1.81	655.87	8.53	671.49	6.35	725.76	1.22
2020	756	762.30	0.83	686.45	9.20	710.44	6.03	786.58	4.05
2021	808	828.35	2.52	716.65	11.3	750.70	7.09	852.50	5.51
MAPE			1.38		8.22		5.49		3.45
RMSE			8.44		55.39		38.34		21.58

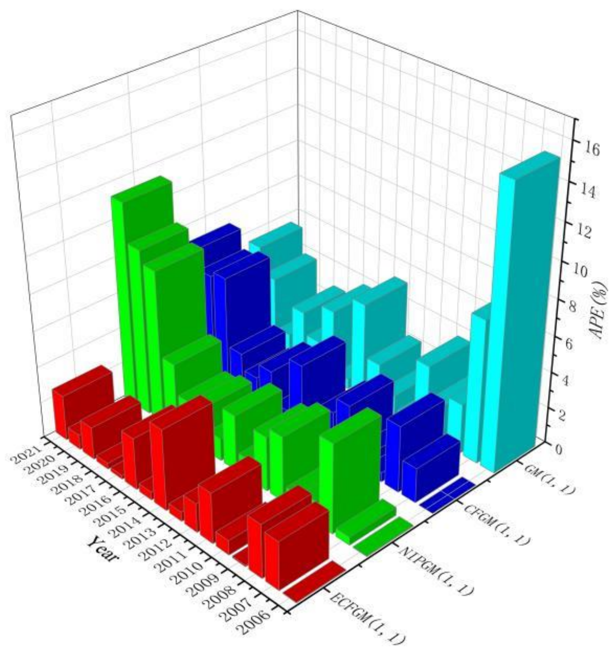

Figure 4: 3D bar chart of APE.

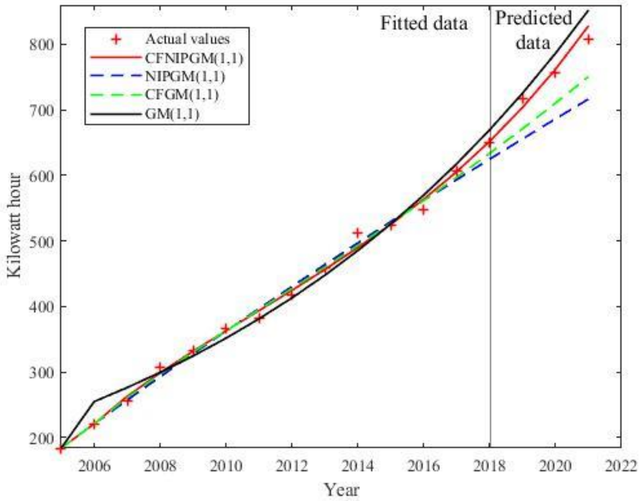

Figure 5: Trend of per capita electricity consumption for daily life in China.

Adhering to the preceding modeling process, the future trend of development of per capita consumption in China can now be predicted. By applying the ECFGM(1,1) model, the predicted outcomes for per capita consumption in China from 2022 to 2026 are derived, as presented in Table 8. The predicted results suggest that with the continued proliferation of electrical products in rural parts of China and the growing adoption of new energy vehicles, per capita consumption for daily life in China will continue to exhibit an upward trajectory. This suggests growing consumption of electricity, reflecting the ongoing increase in living standards and the evolving energy landscape in China.

Table 8: Prediction results of per capita electricity consumption for daily life in China.

Year	2022	2023	2024	2025	2026
Predicted Value	868.41	933.44	1004.19	1081.27	1165.35

5. Conclusion

This study introduces an innovative and efficient fractional grey model, which represents a significant advancement over the classical GM(1,1) model. A novel approach is employed, integrating the PSO algorithm to determine two optimal parameters crucial to the newly developed accumulation generating operator. Through comprehensive analysis across three numerical examples spanning diverse application fields, the efficacy of the proposed ECFGM(1,1) prediction model is demonstrated. Specifically, it substantially outperforms not only the classical GM(1,1) model but also two recently investigated improved GM(1,1) models, namely the CFGM(1,1) and NIPGM(1,1) models, in terms of both fitting accuracy and predictive precision. Future research should focus on further exploration of the potential of the proposed accumulation algorithm. Specifically, efforts should be directed at integrating it with other non-homogeneous grey models, with the aim of validating its practical modeling efficacy across a wider array of scenarios and applications. Such endeavors will contribute to advancing the field of grey modeling and enhancing its applicability across various domains.

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