ECONOMIC DECISION-MAKING ON NANO SIMPLY α^* ALPHA OPEN SET IN A ROUGH SET

by

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Real-life uses are more frequently confronted with huge quantities of data, particularly in the field of uncertain data. We present a new approach based on nano simply α^*open set. This new approach is a generalization of Pawlak's rough sets. The idea of separation axioms was introduced in this method, which was used to examine essential properties and preservation theorems. The relationships that exist between preservation and fundamental attributes theories was also explored. Developing and discussing their features has also been done by us. In the study, an application was used illustrate the relationship between the nano simply α^*open set was illustrated. This concept was explored utilizing the coarse set model, thereby acquiring new levels of precision. Furthermore, an exact suggestion was evaluated, which competitively rivals those of the Yao and Pawlak methodologies. To derive the outcomes, MATLAB software has been employed.

Key words: *nano topology, rough concepts, accuracy, nano simply α^* open set*

Introduction

Frequently, data regarding our surroundings is incorrect, lacking in detail, or unsure. Thivagar, *et al*. [1] they are introduced the concepts of $nano^{\alpha\text{-open}}$ sets, $nano^{\text{semi}}$, open, and $nano^{\text{pre}}$ open sets. Thivagar *et al*. [1] introduced was a nano space that is associated using a subse X of a cosmos, established according to the lower and upper approximations of X. Within this structure, the components are denoted as nano open sets. Additionally, they examined the concepts of nano closure and nano interior of a set. Nano topology is an advanced field that studies the structure of space at an extremely fine scale, focusing on concepts such as nano open sets, nano closure, and nano interior. This field has significant importance in practical applications, where understanding fine details is crucial in areas such as advanced materials and nano technology. For instance, insights from nano topology can improve electronics, medicine, and manufacturing by providing a framework for analyzing and optimizing precise systems. A framework for making decisions created by El Sayed *et al*. [2] introduced presenting a new approach and methods for decision-making in medical diagnosis using nano open sets. We let $\mathfrak{M} \subseteq \mathfrak{U}$ be nano subsets of. We denoted by $\text{int}^{\mathfrak{N}}(\mathfrak{M})$ and $cl^{n}(\mathfrak{M})$ for the $nano^{\text{interior}}$ and the $nano^{\text{closure}}$

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of $\mathfrak{M}$ set, as well as, and the *nano*alpha interior, *nano*alpha iclosure of the set $\mathfrak{M}$ is represented by $\text{int}^{\mathfrak{N}}(\mathfrak{M})$ and $cl^{n}(\mathfrak{M})$. Consequently profoundly influences the creation and execution of intelligent systems [3, 4]. Decision-making is vital in our everyday routines, with various applications such as [5, 6]. By providing a precise framework for system analysis and optimization, nano topology contributes to technological advancements and impacts various aspects of daily life. Nano topology there are several uses in different fields, [7]. Pawlak's [8], the foundation of rough set theory is rooted in the concept of forest chaos, which results from insufficient and incomplete information systems. A filter-based approach can be used to extract knowledge efficiently from a domain while preserving information quality and minimizing the need for knowledge, which is made possible by rough set attribute reduction. The domain of nano topology encompasses numerous results and concepts that can assist in uncovering the hidden information within data, which is relevant to real-world applications, [9-12]. Thus, nano topological techniques are consistent with static processing methods. It is important to note that this principle can also be applied to investigate further reduced classes, [13-15]. The authors in this study define the nano simply α^* open set, nano simply α^*normal, and nano alpha simply$^{\alpha}$ normal, (in short, $S^{\mathfrak{N}^*}-\alpha$ open set, $S^{\mathfrak{N}^*}-\alpha$ normal, and $\alpha S^{\mathfrak{N}}\alpha$ – normal). The complement of all (α-open "$\alpha^{\mathfrak{N}}o(\mathfrak{U})$", nano open "$o(\mathfrak{U})$", nano semi-open "$S^{\mathfrak{N}}o(\mathfrak{U})$" ", and nano pre-open "$Po(\mathfrak{U})$") sets are (nano α – closed, "$\alpha^{\mathfrak{N}}C(\mathfrak{U})$", nano closed "$C^{n}(\mathfrak{U})$", nano semi-closed "$S^{\mathfrak{N}}C(\mathfrak{U})$", and nano pre-closed "$P^{\mathfrak{N}}C(\mathfrak{U})$") sets. (The collections of nano open sets within the universe, $\mathfrak{A}$, specifically referred to as nano nanoalpha simply$^{\alpha}$ open sets, are indicated by ($S^{\mathfrak{N}}\alpha o(\mathfrak{A})$). In this study, nano topological concepts and their fundamental properties were examined to achieve a proposed accuracy. Additionally, it explores the properties of these attributes. the nano simply α^*open set as illustrated. Moreover, we introduced a suggested accuracy measure that relies on the nano simply α^*open set. In summary, our investigation presents crafted a novel model that has the ability to achieve significant accuracy and is a viable alternative to Pawlak and Yao's methods.

Preliminaries

Definition 1. [1] If ($T_{\mathcal{R}}(\mathfrak{X})$, $\mathfrak{A}$) is nano space in regard to $\mathfrak{X} \subset \mathfrak{A}$ and $\mathfrak{M} \subset \mathfrak{A}$ be nano subsets of $\mathfrak{A}$ then:

i. The nano interior of $\mathfrak{M}$ denoted by $\text{int}^{\mathfrak{N}}(\mathfrak{M})$. It is described since the combination It is the largest nano uncovered area within all nano open subsets. It is the largest nano uncovered area within all nano open subsets.
ii. The nano closure of denoted by $cl^{\mathfrak{N}}(\mathfrak{M})$, It is known since the meeting point of all nano closed sets that contain $\mathfrak{M}$. It embodies the tiniest nano enclosed collection ncompassing.

Definition 2. [1] If ($T_{\mathcal{R}}(X)$, $\mathfrak{A}$) is nano space, $\mathfrak{M} \subset \mathfrak{A}$ be nano subsets of $\mathfrak{A}$ is referred to as:

i. The *nano* alpha open set if, $\mathfrak{M} \subseteq int^{\mathfrak{N}}(cl^{\mathfrak{N}}(int^{\mathfrak{N}}(, \mathfrak{M})))$.
ii. The *nano* pre open is identified if $\mathcal{A} \subseteq int^{\mathfrak{N}}((cl^{\mathfrak{N}}(, \mathfrak{M})))$.
iii. Nano regular open if, $\mathfrak{M} = int^{\mathfrak{N}}((cl^{\mathfrak{N}}(, \mathfrak{M})))$.
iv. Simply open set if $V = \mathfrak{G} \cup \mathfrak{N}$ where $\mathfrak{G}$ is nano open and $\mathfrak{N}$ is nano nowhere dense.
v. The *nano* alpha interior ($int^{\alpha}(\mathfrak{M})$).

It is the merging of all nano alpha open sets that are encompassed within $\mathfrak{M}$.

vi. The *nano*alpha closure closure ($cl^{\alpha}(\mathfrak{M})$) It is where all nano alpha closed sets intersect, enclosing set $\mathfrak{M}$.

Definition 3. [16] A subset $\mathfrak{V}$ of a NTS ($\mathfrak{A}$, $\tau_R(\mathfrak{X})$) is nano β-open in $\mathfrak{A}$ if $\mathfrak{V} \subseteq cl^{N}(int^{N}(cl^{N}(\mathfrak{V})))$.

The set of all nano β-open sets of $\mathfrak{A}$ is denoted by $\beta^N o(\mathfrak{A})$.

Definition 4. [17] A subset E of a NTS $(U, \tau_R(X))$ is

$$nano\ b-open\ if\ A \subseteq ncl(nint(A)) \cup nint(ncl(A)).$$

Definition 5. A space $(\mathfrak{X}, \mathfrak{T})$. It is known as:

i. Normal space [18] for each $F_1, F_2 \in \tau^C$, $F_1 \cap F_2 = \phi$, then there exist $U, V \in \tau$, $U \cap V = \phi$, in a manner that $F_1 \subseteq U$ additionally, $F_2 \subseteq U$.
ii. α^{normal} space [19] if for each $F_1, F_2 \in \alpha c(X)$, $F_1 \cap F_2 = \phi$, then there exist $U, V \in \alpha o(X)$, $U \cap V = \phi$, such that $F_1 \subset U$ and $F_1 \subset U$.

Definition 6. [8] $(\mathfrak{U}, \mathfrak{R})$. It is a system for managing information. The $\mathfrak{S} \in \mathfrak{P}$, let "$x\mathfrak{R}$", be an after set defined by: $x\mathfrak{P} = \{y \in \mathfrak{U}: x\mathfrak{P}y\}$, $\mathfrak{S}$ is unnecessary in $\mathfrak{P}$ if $(\mathfrak{P})^{\mathcal{IND}} = (\mathfrak{P} - \{\mathfrak{F}\}^{\mathcal{IND}})$, but $(\mathfrak{P})^{\mathcal{IND}} \neq (\mathfrak{P} - \{\mathfrak{F}\}^{\mathcal{IND}})$, subsequently, $\mathfrak{E}$ *is is essential* in $\mathfrak{P}$ and $\mathfrak{P}^{\text{lover}}$,$\mathfrak{P}^{\text{upper}}$ approximations of $\mathfrak{X}$ as:

$$\underline{\mathfrak{P}}(\mathfrak{X}) = \cup\left\{\mathfrak{Y} \in \frac{\mathfrak{A}}{\mathfrak{P}}, \mathfrak{Y} \subseteq \mathfrak{X}\right\}, \bar{\mathfrak{P}}(\mathfrak{X}) = \cup\left\{Y \in \frac{U}{\mathfrak{P}}, Y \cap \mathfrak{X} \neq \varphi\right\}$$

Definition 7. [20] If $\mathfrak{R}$ is a two-way connection in the cosmos $\mathfrak{A}$. Subsequently, for every $\alpha \in \mathfrak{U}$, we suggest its *right neighborhood as:* $\mathcal{R}_r(\alpha) = \{\beta \in \mathfrak{U}: \alpha\mathfrak{R}\beta\}$.

Definition 8. [20] If $\mathcal{R}$ is a two-way connection in the universe $\mathfrak{U}$. subsequently, the *right*$^{\text{lover}}$ and *right*$^{\text{upper}}$ approximations of $\mathbb{A} \subseteq \mathfrak{U}$ are suggested, followed, by:

$$\underline{\mathfrak{L}}_r(\mathbb{A}) = \{\alpha \in \mathfrak{U}: \mathfrak{N}_r(\alpha) \subseteq \mathbb{A}\} \text{ and } \bar{\mathcal{U}}_r(\mathbb{A}) = \{\alpha \in \mathfrak{U}: \mathfrak{N}_r(\alpha) \cap \mathbb{A} \neq \varphi\}$$

Nano simply$^{\alpha}$-open set

In the upcoming part, we present various kinds of nano *simply*$^{\alpha}$-open set, *nano simply*$^{\alpha*}$-open set, nano *simply*$^{\alpha*}$-closed, and nano s*imply*$^{\alpha*}$, *limit*$^{\text{point}}$.

Definition 9. If $(T_{\mathcal{R}}(\mathfrak{X}), \mathfrak{U})$ is nano space and, $\varepsilon \subset \mathfrak{A}$ be nano subsets of $\mathfrak{U}$ is called nano *simply*$^{\alpha}$-open set if $\text{int}^{\alpha}(\alpha cl(\varepsilon)) \subseteq cl^{\alpha}(\text{int}(^{\alpha}\varepsilon))$.

Definition 10. If $(T_{\mathcal{R}}(\mathfrak{X}), \mathfrak{U})$ is nano space and, $\mathfrak{M} \subset \mathfrak{U}$ be nano subsets of $\mathfrak{A}$ is called nano simply$^{\alpha*}$-open (briefly, $\mathfrak{S}^{\mathfrak{N}^*}\alpha$-open) set if $\mathfrak{M} \in \{\mathfrak{X}, \varphi, \mathfrak{Q} \cup \mathfrak{K}: \mathfrak{Q}\}$ is a nano proper α-open set, and $\mathfrak{K}$ is a nano no-where dense set}. The all-encompassing family nano simply$^{\alpha*}$-open set of set $\mathfrak{X}$ is represented by $\mathfrak{S}^{\mathfrak{N}^*}\alpha o(\mathfrak{U})$. The op posite of nano simply$^{\alpha*}$-open set is considered to be nano simply$^{\alpha*}$-closed (for short, $\mathfrak{S}^{\mathfrak{N}^*}\alpha$-closed, $\mathfrak{S}^{\mathfrak{N}^*}\alpha c(\mathfrak{U})$) set.

The diagram illustrates the connections among, nano **simply**$^{\alpha*}$ open sets and some other types of nano open sets.

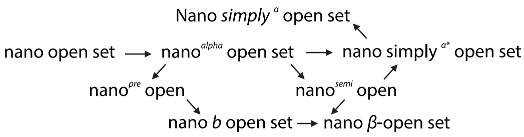

Figure 1. Demonstrate the nano simply α* open set

The aforementioned figure's implications can not be reversed, as demonstrated by following example.

Example 1. Let = $\{b, c, d, a\}$, $\mathfrak{U}/\mathcal{R} = \{\{a\}, \{c\}, \{b, d\}\}$, $\mathfrak{X} = \{a, b\}$, then the nano topology with respect to $\mathfrak{X}$ is $\tau_{\mathcal{R}}(\mathfrak{X}) = \{\mathfrak{U}, \varphi, \{a, c, d\}, \{c, d\}, \{a\}\}$.

If $\mathfrak{Z} = \{c\}$ is nano simply alpha open set *i.e.* $\{c\} \in S^{\mathfrak{N}}\alpha o(\mathfrak{U})$, but not simply* alpha open set *i.e.* $\{c\} \notin S^{\mathfrak{N}^*}\alpha o(\mathfrak{U})$:

i. If $\mathfrak{Y} = \{a, c\}$ is simply* alpha open set *i.e.* $\{a, c\} \in S^{\mathfrak{N}^*}\alpha o(\mathfrak{U})$ but not nano alpha open set *i.e.* $\{c\} \notin \alpha\overset{\mathfrak{N}}{o}(\mathfrak{U})$.

ii. *If* $\mathfrak{Y} = \{a, b\}$ *is nano b-open sets i.e.* $\{a, b\} \in b\overset{\mathfrak{N}}{o}(\mathfrak{U})$ but not nano semi open set *i.e.* $\{a, b\} \notin S\overset{\mathfrak{N}}{o}(\mathfrak{U})$.

iii. If $\mathfrak{Q} = \{b, c\}$ is nano β-open set *i.e.* $\{b, c\} \in \beta\overset{\mathfrak{N}}{o}(\mathfrak{U})$ but not nano b- open sets *i.e.* $\{b, c\} \notin b\overset{\mathfrak{N}}{o}(\mathfrak{U})$.

iv. If $\mathfrak{Y} = \{a, c\}$ is nano b-open sets but not nano pre open set *i.e.* $\{a, b\} \in P\overset{\mathfrak{N}}{o}(\mathfrak{U})$.

v. If $S = \{a, d\}$ is nano pre open set *i.e.* $S = \{a, d\} \in P\overset{\mathfrak{N}}{o}(\mathfrak{U})$ but not nano alpha open set *i.e.* $\{a, d\} \in \alpha\overset{\mathfrak{N}}{o}(\mathfrak{U})$.

Remark 1. If $(T_{\mathcal{R}}(\mathfrak{X}), \mathfrak{A})$ is nano space, the class of nano *simply*$^{\alpha}$-open sets "$\mathfrak{S}^{\mathfrak{N}}\alpha o(\mathfrak{U})$", of $\mathfrak{U}$ and the class of nano *simply* α closed sets of $\mathfrak{A}$ "$\mathfrak{S}^{\mathfrak{N}^*}\alpha o(\mathfrak{U})$".

Definition 11. If $(T_{\mathcal{R}}(\mathfrak{X}), \mathfrak{U})$ is nano topological space and, $\mathcal{P} \in \mathfrak{U}$, $\mathcal{P}$ is nano simply$^{*\alpha}$ *limit*$^{\text{point}}$ (in short, $\mathfrak{S}^{\mathfrak{N}}\alpha$-*limit*$^{\text{point}}$ of $\mathfrak{Z}$ if for each nano $\mathfrak{S}^{\mathfrak{N}^*}\alpha$-open set G including $\mathcal{P}$ encompasses a point of $\mathfrak{E}$ apart from $\mathcal{P}$. The collection of all $\mathfrak{S}^{\mathfrak{N}^*}\alpha$-*limit*$^{\text{point}}$ of $\mathfrak{E}$ is referred to as nano *simply*$^{\alpha*}$-resulting collection of $\mathfrak{Z}$ is denoted by $(\mathfrak{S}^{\mathfrak{N}^*}\alpha o(\mathfrak{Z}))$. For every nano subsets $\mathfrak{Z} \subseteq \mathfrak{U}$.

Nano simply$^{\alpha*}$ normal

We talked about various forms of normality and regularity stemming from nano *simply*$^{\alpha*}$ set and nano *simply*$^{\alpha}$ open set.

Definition 12. In regards to $(T_{\mathcal{R}}(X), \mathfrak{U})$ is nano space:

i. Nano alpha simply$^{\alpha*}$-normal space

$$\forall F_1, F_2 \in \alpha^{\mathfrak{N}}c(\mathfrak{U}),\ \ F_1 \cap F_2 = \phi \to \exists U, V \in \mathfrak{S}^{\mathfrak{N}^*}\alpha o(\mathfrak{U})$$

such that $U \cap V = \phi$ and $F_1 \subseteq U$ and $F_2 \subseteq V$.

ii. Nano simply$^{\alpha*}$-normal space,

$$'F_1, F_2 \in \mathfrak{S}^{\mathfrak{N}^*}\alpha c(\mathfrak{U}), F_1 \cap F_2 = \phi \to \exists U, V \in \mathfrak{S}^{\mathfrak{N}^*}\alpha o(\mathfrak{U}$$

such that $U \cap V = \phi$ and $F_1 \subseteq U$ and $F_2 \subseteq V$.

Remark 2. The fig. 2 illustrates the correlation between the recently introduced forms of normality discussed in the prior definition, nano alpha simply$^{\alpha}$ normal space includes entirely nano simply$^{\alpha*}$ normal, nano alpha normal and simply$^{\alpha*}$ normal.

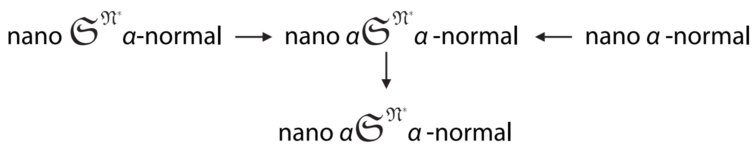

Figure 2. Demonstration of certain connections between topological ideas

The following *Example 2* demonstrates the impacts shown in the fig. 2 does not have to be able to be reversed.

Example 2. Permit

$$\mathfrak{U} = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}\},\ \ \frac{\mathfrak{U}}{\mathcal{R}} = \{\{\mathfrak{a}\}, \{\mathfrak{c}\}, \{\mathfrak{b}, \mathfrak{d}\}\},\ \ \mathfrak{L} = \{\mathfrak{a}, \mathfrak{c}\}$$

then the nano topology concerning $\mathfrak{L}$ is $\tau_{\mathcal{R}}(\mathfrak{L}) = \{\mathfrak{A}, \phi, \{a, c\}\}$, then $(\tau_{\mathcal{R}}(\mathfrak{L}), \mathfrak{U})$ is $\alpha\mathfrak{S}^{\mathfrak{N}}\alpha$-**normal** but not nano $\alpha\mathfrak{S}^{\mathfrak{N}^*}\alpha$ normal, since there exist disjoint nano alpha open sets, $\{\mathbf{b}\}$, $\{\mathbf{d}\} \in \boldsymbol{\alpha}^{\mathfrak{N}}\mathbf{c}(\mathfrak{U})$ however, there exists no disjoint nano **simply**$^{*\alpha}$ open sets encompassing those.

Example 3. Let = $\{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d},\}$, $\mathfrak{U}/\mathcal{R} = \{\{\mathfrak{b}\}, \{\mathfrak{d}\}, \{\mathfrak{a}, \mathfrak{c}\}\}$, $\mathfrak{L} = \{\mathfrak{b}, \mathfrak{d}\}$, then the nano topology with respect to $\mathfrak{L}$ is $\tau_{\mathcal{R}}(\mathfrak{L}) = \{\mathfrak{A}, \phi, \{\mathfrak{b}\}, \{\mathfrak{d}\}, \{\mathfrak{b}, \mathfrak{d}\}\}$ then $(T_{\mathcal{R}}(\mathfrak{L}), \mathfrak{U})$ is $\alpha\mathfrak{S}^{\mathfrak{N}^*}\alpha$-normal but not nano $\alpha\mathfrak{S}^{\mathfrak{N}^*}\alpha$ normal, since, for each disjoint **simply**$^{\alpha*}$ closed sets,$\{\mathfrak{a}, \mathfrak{b}\}$, $\{\mathfrak{c}, \mathfrak{d}\} \in \mathfrak{S}^{\mathfrak{N}^*}$ $\alpha c(\mathfrak{U})$ however, there exists no disjoint nano **simply**$^{\alpha*}$open sets encompassing those.

Proposition 1. If $(T_{\mathcal{R}}(\mathfrak{L}), \mathfrak{U})$ is nano space:

i. Each nano alpha **simply**$^{\alpha*}$ normal space is nano alpha **simply**$^{\alpha*}$ normal space.
ii. Each nano **simply**$^{\alpha*}$ normal space is nano alpha **simply**$^{\alpha*}$ normal space. σ

Proof:

i. Let $\mathfrak{F}_1$, $\mathfrak{F}_2 \in \alpha^{\mathfrak{N}}c\ (\mathfrak{U})$, $\mathfrak{F}_1 \cap \mathfrak{F}_2 = \phi$, since $(T_{\mathcal{R}}(\mathfrak{L}), \mathfrak{U})$ is nano **simply**$^{\alpha*}$ normal space, at that point, there is a disjoint nano **simply**$^{\alpha*}$ open set U, $V \in \mathfrak{S}^{\mathfrak{N}^*}\alpha O(\mathfrak{U})$ such that $\mathfrak{F}_1 \subseteq U$ and $\mathfrak{F}_2 \subseteq V$ and since every **simply**$^{\alpha*}$ open set is nano **simply**$^{\alpha*}$ open set *i.e.* U, $V \in \mathfrak{S}^{\mathfrak{N}^*}\alpha O(\mathfrak{U})$, then $\mathfrak{F}_1 \subseteq U$ and $\mathfrak{F}_2 \subseteq V$. Consequently, $(T_{\mathcal{R}}(\mathfrak{L}), \mathfrak{U})$ is nano alpha **simply**$^{\alpha*}$ normal space.
ii. Permit $\mathfrak{F}_1$, $\mathfrak{F}_2 \in \alpha c\ (X)$, $\mathfrak{F}_1 \cap \mathfrak{F}_2 = \phi$, since $(T_{\mathcal{R}}(X), \mathfrak{U})$ is nano **simply**$^{\alpha*}$ normal space, at that point, there is disjoint nano simply$^{\alpha*}$ open set U, $V \in \mathfrak{S}^{\mathfrak{N}^*}\alpha O(\mathfrak{U})$, since every nano alpha closed set is nano **simply**$^{\alpha*}$ closed set then there is U, $V \in \mathfrak{S}^{\mathfrak{N}^*}\alpha O(\mathfrak{U})$ and $\mathfrak{F}_1 \subseteq U$ and $\mathfrak{F}_2 \subseteq V$. Thus $(T_{\mathcal{R}}(X), \mathfrak{U})$ is nano alpha **simply**$^{\alpha*}$ normal space.

Utilization

Definition 13. Regarding the system of information $(\mathfrak{A}, \mathfrak{N}, \mathfrak{S}^{\mathfrak{N}^*}\alpha o(\mathfrak{U}))$ is a $\mathfrak{S}^{\mathfrak{N}^*}\alpha$ approximation space related to relation $\mathfrak{N}$ on a given set $\mathfrak{A}$ and $\sigma \subseteq \mathfrak{U}$ nano simply$^{\alpha*}$-lower and nano simply$^{\alpha*}$-upper are defined:

$$\underset{-S^*\alpha}{\mathfrak{B}}\ \sigma = \cup\left\{\mathcal{G}, \mathcal{G} \in \mathfrak{S}^{\mathfrak{N}^*}\alpha o\left(\mathfrak{U}\right), \mathcal{G} \subseteq \sigma\right\},\ \overset{-S^*\alpha}{\mathfrak{B}}\left(\mathcal{E}\right) = \cap\left\{\mathfrak{F}, \mathfrak{F} \in \mathfrak{S}^{\mathfrak{N}^*}\alpha c\left(\mathfrak{U}\right), \mathfrak{F} \supseteq \sigma\right\}$$

in a similar manner. The precision of t he nano **simply**$^{\alpha*}$-lower as well as nano **simply**$^{\alpha*}$ -upper approximations of σ in $(\mathfrak{A}, \mathfrak{N}, \mathfrak{S}^{\mathfrak{N}^*}\alpha o(\mathfrak{U}))$ is specified:

$$\mu\left(\mathcal{E}\right) = \left|\frac{\underset{-S^*\alpha}{\mathfrak{B}}\ \sigma}{\overset{-S^*\alpha}{\mathfrak{B}}\ \sigma}\right|$$

in which |.| indicates the size of the set.

Definition 14. For every $\mathfrak{Z} \subset E$, is explained by

$$wR_{\mathfrak{Z}}z = \frac{\sum_{I \in B}\left|i\left(w\right) - i\left(z\right)\right|}{\left|\mathfrak{Z}\right|} \le \alpha$$

where $|B|$ is the size of B, α symbolizes any quantity and wherein α is computed by a specialist in the field, such as. Permit $\mathfrak{Z} = \{\varepsilon_2\}$, $|B| = 1$

$$wR_1 z \leftrightarrow \frac{\left|i\left(w\right) - i\left(z\right)\right|}{1 \le \alpha},\ wR_1 z \leftrightarrow \frac{\left|i\left(w\right) - i\left(z\right)\right|}{1 \le \alpha}$$

Data gathering

Utilizing the information from the market's securities business [5], the method is applied in the place where it is carried out $Z = \{y_1, y_2, ..., y_{10}\}$ indicates 10 companies that are publicly traded, the characteristics of companies $b = \{b_1, b_2, \ldots, b_8\}$ as wel as $D = \{d\}$ = {decision of investment}.

The coding is demonstrated by transforming a value from 0 to 1 as:

$$S_{\text{new}} = (S_{\text{old}} - S_{\min})/(S_{\max} - S_{\min})$$

Partitioning the interval [0, 1] divided into three parts, the result of tab. 1 discretionary use of the ε – means clustering is illustrated in the tab. 2.

Table 1. Enterprise informations

Z	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	d
y_1	9697.23	1.36	0.31	9565.70	73.93	172.95	9697.23	1.36	1
y_2	2830.22	60.01	5.01	1731.24	144.99	256.40	2830.22	60.01	2
y_3	2807.22	1.88	0.04	2753.61	19.54	24.73	2807.22	1.88	0
y_4	1578.80	0.19	0,25	1575.66	56.80	78.09	1578.80	0.19	0
y_5	1525.40	2.63	0.89	1483.68	54.93	56.26	1525.40	2.63	1
y_6	1107.52	1.52	0.34	1089.49	32.23	63.99	1107.52	1.52	0
y_7	1100.50	1.21	0.99	1089.54	72.23	670.57	1100.50	1.21	0
y_8	870.05	–1.43	0.69	884.13	–7.16	96.73	870.05	–1.43	0
y_9	795.94	7.32	1.28	734.84	40.09	50.26	795.94	7.32	1
y_{10}	789.92	10.23	3.32	707.31	21.78	64.33	789.92	10.23	1

Table 2. Table's discretion 1

Z	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	d
$\mathfrak{w}_1$	a_3	a_1	a_1	a_3	a_2	a_3	a_3	a_1	a_1
$\mathfrak{w}_2$	a_2	a_3	a_3	a_2	a_3	a_3	a_2	a_3	a_2
$\mathfrak{w}_3$	a_2	a_1	a_1	a_2	a_1	a_1	a_2	a_1	a_0
$\mathfrak{w}_4$	a_1	a_1	a_1	a_2	a_1	a_1	a_1	a_1	a_0
$\mathfrak{w}_5$	a_1	a_2	a_2	a_1	a_1	a_1	a_1	a_2	a_1
$\mathfrak{w}_6$	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_0
$\mathfrak{w}_7$	a_1	a_1	a_2	a_1	a_2	a_3	a_1	a_1	a_0
$\mathfrak{w}_8$	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_0
$\mathfrak{w}_9$	a_1	a_2	a_2	a_1	a_1	a_1	a_1	a_2	a_1
$\mathfrak{w}_{10}$	a_1	a_2	a_3	a_1	a_1	a_1	a_1	a_2	a_1

Furthermore, tab. 3 is acquired after symmetry is removed; the items are:

$$Z = \{\mathfrak{w}_1, \mathfrak{w}_2, \ldots, \mathfrak{w}_8\}$$

which represent a total of eight companies that were mentioned. The characteristics are $B = \{B_1, B_2, \ldots, B_6\}$, as depicted in tab. 3.

Table 3. The evaluation of tab. 2

Z	B_1	B_2	B_3	B_4	B_5	B_6	d
$\mathfrak{w}_1$	a_3	a_1	a_1	a_3	a_2	a_3	a_1
$\mathfrak{w}_2$	a_2	a_3	a_3	a_2	a_3	a_3	a_2
$\mathfrak{w}_3$	a_2	a_1	a_1	a_2	a_1	a_1	a_0
$\mathfrak{w}_4$	a_1	a_1	a_1	a_2	a_1	a_1	a_0
$\mathfrak{w}_5$	a_1	a_2	a_2	a_1	a_1	a_1	a_1
$\mathfrak{w}_6$	a_1	a_1	a_1	a_1	a_1	a_1	a_0
$\mathfrak{w}_7$	a_1	a_1	a_2	a_1	a_2	a_3	a_0
$\mathfrak{w}_8$	a_1	a_2	a_3	a_1	a_1	a_1	a_1

Upon removal of B_1, the elements we had were $\mathfrak{w}_4$ and $\mathfrak{w}_3$ which are identical, and as B_3 was eliminated, resulting in $\mathfrak{w}_5$ and $\mathfrak{w}_8$ which are identical. In the same manner, deleting B_4,we acquired $\mathfrak{w}_4$ and $\mathfrak{w}_6$ which are identical. It became apparent that

$$IND^{(3)} \neq IND^{(3-\{31\})},\ IND^{(3)} \neq IND^{(3-\{33\})},\ \text{and}\ IND^{(3)} \neq IND^{(3-\{34\})}$$

Then B_1, B_3, and B_4 are crucial. Additionally, eliminating B_2, B_5, and B_6, we had

$$IND^{(B)} = IND^{(B-\{B2\})},\ IND^{(3)} = IND^{(3-\{35\})},\ IND^{(3)} = IND^{(3-\{36\})}$$

subsequently, B_2, B_5, and also B_6 they were considered unnecessary, as depicted in tab. 4.

Table 4. Eliminating characteristics

	Removal characteristics						
Elementary sets number	None	B_1	B_2	B_3	B_4	B_5	B_6
	8	7	8	7	7	8	8

Then, $\{B_1, B_3, B_4\}$ were called the core, and B_2, B_5, and B_6 attributes that were deemed unnecessary. Now, we will examine the data presented in tab. 5.

Table 5. Data related to work

U	B_2	B_5	B_6	d
$\mathfrak{w}_1$	a_1	a_2	a_3	a_1
$\mathfrak{w}_2$	a_3	a_3	a_3	a_2
$\mathfrak{w}_3$	a_1	a_1	a_1	a_0
$\mathfrak{w}_4$	a_1	a_1	a_1	a_0
$\mathfrak{w}_5$	a_1	a_1	a_1	a_1
$\mathfrak{w}_6$	a_1	a_1	a_1	a_0
$\mathfrak{w}_7$	a_1	a_2	a_3	a_0
$\mathfrak{w}_8$	a_2	a_1	a_1	a_1

Following the categorization of the tab. 5, we obtained the last tab. 6.

Table 6. Knowledge categorization

h	U/C	B_2	B_5	B_6
h_1	$\{\mathfrak{w}_1, \mathfrak{w}_7\}$	a_1	a_2	a_3
h_2	$\{\mathfrak{w}_2\}$	a_3	a_3	a_3
h_3	$\{\mathfrak{w}_3, \mathfrak{w}_4, \mathfrak{w}_6\}$	a_1	a_1	a_1
h_4	$\{\mathfrak{w}_5, \mathfrak{w}_8\}$	a_2	a_1	a_1

At the present time, we have presented the data. Of tab. 6 to achieve precision in models.

One of the features of Case:

Let $C = \{B_2\}$, $|C| =1$, $xR_1y \leftrightarrow |i(x) - i(y)|/1 \leq \alpha$, as illustrated in tab. 7.

Table 7. Resemblance regarding the features B_2

B_2	h_1	h_2	h_3	h_4
h_1	a_0	a_2	a_0	a_1
h_2	a_2	a_0	a_2	a_1
h_3	a_0	a_2	a_0	a_1
h_4	a_1	a_1	a_1	a_0

Discussing 1: whenever $\alpha \leq 0$, We acquired the system of intellect that ensues: $xR_1y = \{(h_1, h_1), (h_1, h_3), (h_2, h_2), (h_3, h_1), (h_3, h_3), (h_4, h_4)\}$, then $h_1R_1 = \{h_1, h_3\}$, $h_2R_1 = \{h_2\}$, $h_3R_1 = \{h_1, h_3\}$, $h_4R_1=\{h_4\}$. The $(x)R_1 = \{\{h_1, h_3\}, \{h_2\}, \{h_4\}\}$ was referred to as a class.

Subsequently, $SR_1 = \{\{h_1, h_3\}, \{h_2\}, \{h_4\}\}$ since $\mathfrak{U} = \{a, b, c, d\}$, $\mathfrak{U}/\mathcal{R} = \{\{h_1, h_3\}, \{h_2\}, \{h_4\}\}$, $\mathfrak{X} = \{h_2, h_4\}$ the nano topology concerning $\mathfrak{X}$ is $\tau_{\mathcal{R}}(\mathfrak{X}) = \{\mathfrak{U}, \phi, \{h_2\}, \{h_4\}, \{h_2, h_4\}\}$.

We obtain precise results for every subset of U, B is a subset of U.

Outcomes 1: The accuracy provid in tabs. 8 and 9 is accurate.

The nano simply$^{\alpha*}$ technique and Yao, Pawlak's methods enabled us to obtain all the accuracy in tabs. 8 and 9 and the proposed method:

$$\mu_{11} = \frac{\operatorname{int}(H)}{cl(H)}, \quad \mu_{12} = \frac{L(H)}{U(H)}, \text{ and } \mu_{13} = \frac{B_{-S^*\alpha}(H)}{B^{-S^*\alpha}(H)}$$

as demonstrated in tabs. 8 and 9, correspondingly. The accuracy displayed in tab. 8 was achieved by using Pawlak and Yao techniques.

Table 8. Accuracies for B_2

$P(\mathfrak{U})$	Yao's model		Accuracy, μ_{11}	Pawlak's model		Accuracy, μ_{12}
	int(H)	$cl(H)$		$L(H)$	$U(H)$	
∅	∅	∅	0	∅	∅	0
$\{h_1\}$	∅	$\{h_1, h_3\}$	0	∅	$\{h_1, h_3\}$	0
$\{h_2\}$	$\{h_2\}$	$\{h_1, h_2, h_3\}$	1/3	$\{h_2\}$	$\{h_2\}$	1
$\{h_3\}$	∅	$\{h_1, h_3\}$	0	∅	$\{h_1, h_3\}$	0
$\{h_4\}$	$\{h_4\}$	$\{\boldsymbol{h_3}, \boldsymbol{h_4}, \boldsymbol{h_1}\}$	1/3	$\{\boldsymbol{h_4}\}$	$\{h_4\}$	1
$\{h_1, h_2\}$	$\{h_2\}$	$\{h_1, h_2, h_3\}$	1/3	$\{h_2\}$	$\{h_1, h_2, h_3\}$	1/3
$\{h_1, h_3\}$	∅	$\{h_1, h_3\}$	0	$\{h_1, h_3\}$	$\{h_1, h_3\}$	1
$\{h_1, h_4\}$	$\{h_4\}$	$\{h_1, h_3, h_4\}$	1/3	$\{h_4\}$	$\{h_1, h_3, h_4\}$	1/3
$\{h_2, h_3\}$	$\{h_2\}$	$\{h_1, h_3, h_2\}$	1/3	$\{h_2\}$	$\{h_1, h_2, h_3\}$	1/3
$\{h_4, h_2\}$	$\{h_2, h_4\}$	h	0.5	$\{h_2, h_4\}$	$\{h_2, h_4\}$	1
$\{h_3, h_4\}$	$\{h_4\}$	$\{\boldsymbol{h_3}, \boldsymbol{h_4}, \boldsymbol{h_1}\}$	1/3	$\{h_4\}$	$\{h_4, h_3, h_1\}$	1/3
$\{h_3, h_1, h_2\}$	$\{h_2\}$	$\{h_2, h_1, h_3\}$	1/3	$\{h_1, h_3, h_2\}$	$\{h_3, h_2, h_1\}$	1
$\{h_1, h_2, h_4\}$	$\{h_4, h_2\}$	h	0.5	$\{h_2, h_4\}$	h	1/2
$\{h_2, h_3, h_4\}$	$\{h_4\}$	h	0.25	$\{h_4, h_2\}$	h	1/2
$\{h_3, h_4, h_1\}$	$\{h_2, h_4\}$	$\{h_4, h_3, h_1\}$	0.5	$\{h_1, h_3, h_4\}$	$\{h_3, h_4, h_1\}$	1
h	h	h	1	h	h	1

The accuracy was determined according to the data provided in tab. 9 via nano simply$^{\alpha*}$ open set.

The precision of Yao and Pawlak it is demonstrated in tab. 8, and the recommended Precision it is demonstrated in tab. 9. Our proposed accuracy is superior to those of Yao and Pawlak in one aspect, making it clear.

Conclusion

According to the current research, the accuracy The effectiveness of the data collection system relies on the optimal attributes of life information. The best accuracy can be achieved by simply$^{\alpha*}$ open set method. In the business securities information system market, the simply$^{\alpha*}$ open set method offered the highest accuracy compared to other attributes when

Table 9. Precision of data via nano *simply*$^{\alpha^*}$

$P(H)$	$int(H)$	$cl(int(H))$	$int(cl(int\ (H)))$	$\alpha o(H)$	$\mathfrak{S}^{\mathfrak{N}}\alpha o(H)$	$\mathfrak{S}^{\mathfrak{N}}\alpha c(H)$	$B_{-S^*}\alpha(H)$	$B^{-S*}\alpha(H)$	μ_{13}
$\boldsymbol{\varnothing}$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\boldsymbol{\varnothing}$	$\boldsymbol{\varnothing}$	$\varnothing$	$\varnothing$	0
$\{\boldsymbol{h_1}\}$	$\varnothing$	$\varnothing$	$\varnothing$	–	–	$\{h_1\}$	$\varnothing$	$\{h_1\}$	0
$\{\boldsymbol{h_2}\}$	$\{h_2\}$	$\{h_1, h_2, h_3\}$	$\{\boldsymbol{h_2}\}$	$\{h_2\}$	$\{\boldsymbol{h_2}\}$	$\{\boldsymbol{h_2}\}$	$\{h_2\}$	$\{h_2\}$	1
$\{\boldsymbol{h_3}\}$	$\varnothing$	$\varnothing$	$\varnothing$	–	–	$\{\boldsymbol{h_3}\}$	$\varnothing$	$\{h_3\}$	0
$\{\boldsymbol{h_4}\}$	$\{h_4\}$	$\{h_1, h_3, h_4\}$	$\{\boldsymbol{h_4}\}$	$\{\boldsymbol{h_4}\}$	$\{\boldsymbol{h_4}\}$	–	$\{h_4\}$	$\{h_4\}$	1
$\{\boldsymbol{h_1}, \boldsymbol{h_2}\}$	$\{h_2\}$	$\{h_1, h_3, h_2\}$	$\{h_2\}$	–	$\{\boldsymbol{h_1}, \boldsymbol{h_2}\}$	$\{\boldsymbol{h_1}, \boldsymbol{h_2}\}$	$\{h_2, h_1\}$	$\{\boldsymbol{h_1}, \boldsymbol{h_2}\}$	1
$\{\boldsymbol{h_1}, \boldsymbol{h_3}\}$	$\varnothing$	$\varnothing$	$\varnothing$	–	–	$\{\boldsymbol{h_1}, \boldsymbol{h_3}\}$	$\varnothing$	$\{h_1, h_3\}$	0
$\{\boldsymbol{h_1}, \boldsymbol{h_4}\}$	$\{h_4\}$	$\{h_1, h_3, h_4\}$	$\{h_4\}$	–	$\{\boldsymbol{h_1}, \boldsymbol{h_4}\}$	$\{\boldsymbol{h_1}, \boldsymbol{h_4}\}$	$\{\boldsymbol{h_1}, \boldsymbol{h_4}\}$	$\{h_1, h_4\}$	1
$\{\boldsymbol{h_2}, \boldsymbol{h_3}\}$	$\{h_2\}$	$\{h_1, h_3, h_2\}$	$\{h_2\}$	–	$\{\boldsymbol{h_2}, \boldsymbol{h_3}\}$	$\{\boldsymbol{h_2}, \boldsymbol{h_3}\}$	$\{h_2, h_3\}$	$\{\boldsymbol{h_2}, \boldsymbol{h_3}\}$	1
$\{\boldsymbol{h_2}, \boldsymbol{h_4}\}$	$\{h_2, h_4\}$	h	h	$\{h_2, h_4\}$	$\{\boldsymbol{h_2}, \boldsymbol{h_4}\}$	–	$\{h_2, h_4\}$	h	1/2
$\{\boldsymbol{h_3}, \boldsymbol{h_4}\}$	$\{h_4\}$	$\{h_1, h_3, h_4\}$	$\{h_4\}$	–	$\{\boldsymbol{h_3}, \boldsymbol{h_4}\}$	$\{\boldsymbol{h_3}, \boldsymbol{h_4}\}$	$\{\boldsymbol{h_3}, \boldsymbol{h_4}\}$	$\{h_3, h_4\}$	1
$\{\boldsymbol{h_1}, \boldsymbol{h_2}, \boldsymbol{h_3}\}$	$\{h_2\}$	$\{h_1, h_3, h_2\}$	$\{h_2\}$	–	–	$\{\boldsymbol{h_1}, \boldsymbol{h_2}, \boldsymbol{h_3}\}$	$\{h_1, h_3, h_2\}$	h	3/4
$\{\boldsymbol{h_1}, \boldsymbol{h_2}, \boldsymbol{h_4}\}$	$\{h_2, h_4\}$	h	h	$\{h_1, h_4, h_2\}$	$\{\boldsymbol{h_1}, \boldsymbol{h_2}, \boldsymbol{h_4}\}$	–	$\{\boldsymbol{h_1}, \boldsymbol{h_2}, \boldsymbol{h_4}\}$	h	3/4
$\{\boldsymbol{h_2}, \boldsymbol{h_3}, \boldsymbol{h_4}\}$	h	h	h	$\{h_3, h_4, h_2\}$	$\{\boldsymbol{h_2}, \boldsymbol{h_3}, \boldsymbol{h_4}\}$	–	$\{h_4, h_3, h_2\}$	h	3/4
$\{\boldsymbol{h_1}, \boldsymbol{h_3}, \boldsymbol{h_4}\}$	$\{h_4\}$	$\{h_1, h_3, h_4\}$	$\{h_4\}$	–	$\{\boldsymbol{h_1}, \boldsymbol{h_3}, \boldsymbol{h_4}\}$	$\{\boldsymbol{h_1}, \boldsymbol{h_3}, \boldsymbol{h_4}\}$	$\{h_1, h_3, h_4\}$	$\{h_1, h_3, h_4\}$	1
h	h	h	h	h	h	h	h	h	1

introducing an application. It can also be applied in various other fields of study. Furthermore, these findings encourage us to confidently explore these ideas for potential application in various advanced topological domains. The rough set methods were introduced as classification or decision rules derived from a collection of past instances. Moreover, our method offered a fresh perspective on the issue of attribute reduction, as well as proposed additional semantic characteristics retained through attribute reduction. As a result, our approach allows the decision-maker to have greater flexibility in selecting what is most appropriate for them. We also acquired a suggested level of precision that relies on the simply $^{\alpha^*}$ open.The accuracy of the set was determined to be superior to the accuracies of Yao and Pawlak. In upcoming years, according to certain topological research.

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