ESTIMATION OF INVERSE ISHITA DISTRIBUTION WITH APPLICATION ON REAL DATA

by

Hanaa ABU-ZINADAH * **and Dalal ALJADANI**

Department of Mathematics and Statistics, College of Science,
University of Jeddah, Jeddah, Saudi Arabia

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In this article, a new lifetime distribution named "inverse Ishita" with one parameter for modelling lifetime data is presented as a good alternative to known one-parameter distributions. Moreover, two types of estimation: point estimation and interval estimation are used to estimate the unknown parameter. Furthermore, numerical simulation is conducted to evaluate the performance of estimates at different parameter values and different sample sizes. Ultimately, to illustrate the flexibility and efficiency of the distribution, it was applied to a set of data and compared to the Weibull and Shanker distributions. It was found that the inverse Ishita distribution was a better fit for the data than the other distributions.

Key words: *inverse Ishita distribution, maximum likelihood estimator, least squares estimator, weighted least squares estimator, percentile estimator*

Introduction

In recent years, researchers have shown great interest in one-parameter probability distributions to model many types of data in several different fields. Also, researchers study new one parameter distributions to find models better goodness of fit for data than the well-known distributions. Recently, one parameter distribution called *Ishita distribution* was proposed by [1]. Some of its properties, parameter estimation and applications were discussed. Also, there are many generalizations for this distribution see [2-5]. In this article, we will discuss one parameter distribution proposed by [6] which can be produced by using transformation , called *inverse Ishita distribution* (IID) for modelling lifetime data. Also, the probability density function (PDF) and cumulative distribution function (CDF) are defined:

$$f\left(x,\vartheta\right)=\frac{\vartheta^3}{x^4\left(\vartheta^3+2\right)}\left(x^2\vartheta+1\right)\mathrm{e}^{-\vartheta/x},\ \ x>0\ ,\ \ \vartheta>0 \tag{1}$$

$$F\left(x,\vartheta\right)=\left[1+\frac{\dfrac{\vartheta}{x}\left(\dfrac{\vartheta}{x}+2\right)}{(\vartheta^3+2)}\right]\mathrm{e}^{-\vartheta/x},\ \ x>0,\ \ \vartheta>0 \tag{2}$$

* Corresponding author, e-mail: hhabuznadah@uj.edu.sa

Through the following figs. 1 and 2, we show the behavior for the curves of PDF, CDF for IID at several different values of the parameter.

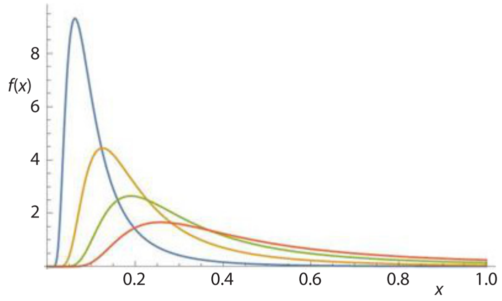

Figure 1. The PDF curves of the IID at θ = {0.25, 0.5, 0.75, 1}

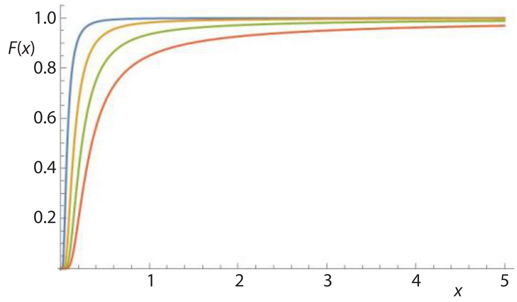

Figure 2. The CDF curves of the IID at θ = {0.2, 0.4, 0.6, 0.8}

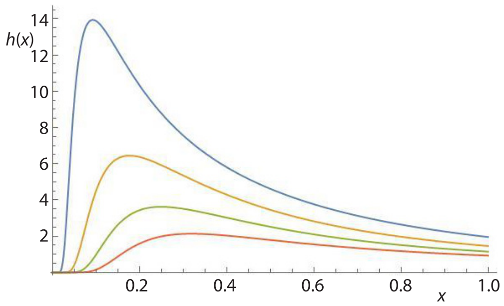

Figure 3. The HRF curves of the IID at θ = {0.25, 0.5, 0.75, 1}

Figure 3 shows the hazard rate function (HRF) at various parameter values. The general objective of this article is to estimate the distribution parameter using two types of estimation: point estimation and interval estimation. Also, application a set of data to illustrate the efficiency and flexibility of the distribution compared to other lifetime distributions. However, for more details about inverse distribution see [7-9]. On the other hand, different estimation methods are discussed by [10-12] for various distributions. As for the goodness of fit tests, Abu-Zinadah and Binkhamis have discussed [13]. Further, Abu-Zinadah and Alsumairi also studied it for [14].

In addition, the HRF for IID is given:

$$h(x;\vartheta)=\frac{\vartheta^{3}\left(1+\vartheta x^{2}\right)\ \mathrm{e}^{-\vartheta/x}}{x^{2}\left[\left(\vartheta^{3}+2\right)x^{2}-\left[\left(\vartheta^{3}+2\right)x^{2}+\vartheta\left(\vartheta+2x\right)\right]e^{-\vartheta/x}\right]} \tag{3}$$

Maximum likelihood estimation method

In this section, we estimate the unknown parameter for the IID using: the MLE and approximate confidence interval.

Maximum likelihood estimator

Let $x_1, x_3, ..., x_3$ be a random sample of size m from IID, then the log-likelihood function is:

$$L=3m\ln\left(\vartheta\right)-m\ln\left(\vartheta^{3}+2\right)+\sum_{i=1}^{m}\ln\left(1+\vartheta x_{i}^{2}\right)-\sum_{i=1}^{m}\ln\left(x_{i}{}^{4}\right)-\vartheta\sum_{i=1}^{m}\frac{1}{x_{i}} \tag{4}$$

Taking the derivative of eq. (4) with respect ϑ, and equating this to zero, we have:

$$\frac{3m}{\vartheta}-\frac{3\vartheta^2 m}{\left(\vartheta^3+2\right)}+\sum_{i=1}^{m}\frac{x_i^2}{\left(1+\vartheta x_i^2\right)}-\sum_{i=1}^{m}\frac{1}{x_i}=0 \tag{5}$$

The MLE of ϑ, say $\hat{\vartheta}_{\mathrm{MLE}}$ can be found by numerically by solving eq. (5) using numerical iteration.

Approximate confidence interval

When the sample size is large, approximate methods can form confidence intervals. such as the maximum likelihood estimator has the property that when the sample size is large then:

$$\frac{\vartheta-\hat{\vartheta}}{\sqrt{var\left(\hat{\vartheta}\right)}}\sim N\left(0,1\right) \tag{6}$$

where $\mathrm{var}(\hat{\vartheta}) = I^{-1}(\hat{\vartheta})$:

$$I\left(\hat{\vartheta}\right)=-E\left(\frac{\partial^2 \log l}{\partial \vartheta^2}\right)|_{\vartheta=\hat{\vartheta}}=\frac{3m}{\hat{\vartheta}^2}+\frac{6\hat{\vartheta}m}{(\hat{\vartheta}^3+2)}-\frac{9\hat{\vartheta}^4 m}{(\hat{\vartheta}^3+2)^2}+\sum_{i=1}^{m}\frac{x_i^4}{\left(1+\hat{\vartheta}x_i^2\right)^2} \tag{7}$$

then, the $(1-\alpha)$% approximate confidence interval (ACI) for parameter ϑ is obtained:

$$\hat{\vartheta}\mp\frac{z_{\alpha/2}}{\sqrt{\frac{3m}{\hat{\vartheta}^2}+\frac{6\hat{\vartheta}m}{(\hat{\vartheta}^3+2)}-\frac{9\hat{\vartheta}^4 m}{(\hat{\vartheta}^3+2)^2}+\sum_{i=1}^{m}\frac{x_i^4}{\left(1+\hat{\vartheta}x_i^2\right)^2}}} \tag{8}$$

where the value of $z_{d/2}$ is the standard normal value.

Other estimation methods

In this section, we provide other estimation methods added to the maximum likelihood estimator introduced and discussed in the previous section and it is least squares, weighted least squares and percentiles.

Least squares estimation method

Suppose that $Y_1, Y_2,..., Y_m$, are the order statistics of the random sample from any probability distribution. The mean and the variance of the i^{th} order statistic are given:

$$E\left[F\left(Y_i\right)\right]=\frac{i}{\left(m+1\right)} \text{ and } V\left[F\left(Y_i\right)\right]=\frac{i\left(m-i+1\right)}{\left(m+1\right)^2\left(m+2\right)}$$

We can get the least squares estimator of parameter ϑ of inverse Ishita by minimizing:

$$\sum_{\boldsymbol{i}=1}^{\boldsymbol{m}}\left(\left(\frac{\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}^{\ 2}}+\dfrac{2\vartheta}{x_{(i)}}}{(\vartheta^3+2)}\right)e^{-\frac{\vartheta}{x_{(i)}}}-\frac{\boldsymbol{i}}{\boldsymbol{m}+1}\right)^2 \tag{9}$$

with respect to ϑ. Differentiating eq. (9) with respect to the parameter ϑ, gives:

$$\sum_{i=1}^{m}\left[\frac{\left(\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}{}^2}+\dfrac{2\vartheta}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{\vartheta^3+2}-\frac{i}{m+1}\right]\left[\frac{\left(3\vartheta^2+\dfrac{2\vartheta}{x_{(i)}{}^2}+\dfrac{2}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{\vartheta^3+2}-\right.$$
$$\left.-\frac{3\vartheta^2\left(\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}{}^2}+\dfrac{2\vartheta}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{\left(\vartheta^3+2\right)^2}-\frac{\left(\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}{}^2}+\dfrac{2\vartheta}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{x_{(i)}\left(\vartheta^3+2\right)}\right]=0 \tag{10}$$

Then, the LSE of ϑ, say $\hat{\vartheta}_{\mathrm{LSE}}$ can be found numerically by solving eq. (10).

Weighted least squares estimation method

We can obtain the estimator for the weighted least squares by minimizing:

$$\sum_{i=1}^{m}\left[F\left(Y_i\right)-\frac{i}{m+1}\right]^2$$

with respect to the unknown parameter. Where:

$$w_i=\frac{1}{V\left[F\left(Y_i\right)\right]}=\frac{\left(m+1\right)^2\left(m+2\right)}{i\left(m-i+1\right)}$$

Thus, the weighted least squares estimator for the parameter ϑ of inverse Ishita can be found by minimizing:

$$\sum_{i=1}^{m}w_i\left[\left(\frac{\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}{}^2}+\dfrac{2\vartheta}{x_{(i)}}}{\vartheta^3+2}\right)\mathrm{e}^{-\vartheta/x_{(i)}}-\frac{i}{m+1}\right]^2 \tag{11}$$

with respect to ϑ. Differentiating eq. (11) with respect to the parameter ϑ, gives:

$$\sum_{i=1}^{m}w_i\left[\frac{\left(\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}{}^2}+\dfrac{2\vartheta}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{\vartheta^3+2}-\frac{i}{m+1}\right]\left[\frac{\left(3\vartheta^2+\dfrac{2\vartheta}{x_{(i)}{}^2}+\dfrac{2}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{\vartheta^3+2}-\right.$$
$$\left.-\frac{3\vartheta^2\left(\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}{}^2}+\dfrac{2\vartheta}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{\left(\vartheta^3+2\right)^2}-\frac{\left(\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}{}^2}+\dfrac{2\vartheta}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{x_{(i)}\left(\vartheta^3+2\right)}\right]=0 \tag{12}$$

Thus, the WLSE of ϑ, say $\hat{\vartheta}_{\text{WLSE}}$ cannot be found analytically, then can be found using numerical methods.

Percentiles method

Assume the unknown parameter ϑ of IID can be estimated via the percentile method by equating the sample percentile points with the population percentile points. First, when parameter is unknown. Since:

$$F\left(x;\vartheta\right)=\frac{\left(\vartheta^3+2+\dfrac{\vartheta^2}{x^2}+\dfrac{2\vartheta}{x}\right)\mathrm{e}^{-\vartheta/x}}{\vartheta^3+2} \tag{13}$$

therefore

$$\ln\left[F\left(x;\vartheta\right)\right]=\ln\left[\frac{\left(\vartheta^3+2+\dfrac{\vartheta^2}{x^2}+\dfrac{2\vartheta}{x}\right)\mathrm{e}^{-\frac{\vartheta}{x}}}{\vartheta^3+2}\right] \tag{14}$$

Let $X_{(i)}$ is the i^{th} order statistic, $X_{(1)} < X_{(2)} < ... < X_{(m)}$. If p_i is the some estimate of $F[(x_i); \vartheta]$, then estimate of ϑ, can be obtained by minimizing:

$$\sum_{i=1}^{m}\left(ln\left[p_i\right]-\ln\left[\frac{\left(\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}^{\;2}}+\dfrac{2\vartheta}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{\vartheta^3+2}\right]\right)^2 \tag{15}$$

with respect to ϑ. Differentiating eq. (15) with respect to the parameter ϑ, gives:

$$\sum_{i=1}^{m}\left\{\ln[pi]-\ln\left[\frac{\left(\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}^{\;2}}+\dfrac{2\vartheta}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{\vartheta^3+2}\right]\right\}\cdot$$

$$\cdot\left\{\frac{\left(\vartheta^3+2\right)\mathrm{e}^{-\vartheta/x_{(i)}}\left[\dfrac{\left(3\vartheta^2+\dfrac{2\vartheta}{x_{(i)}^{\;2}}+\dfrac{2}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{\vartheta^3+2}-\dfrac{3\vartheta^2\left(\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}^{\;2}}+\dfrac{2\vartheta}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{\left(\vartheta^3+2\right)^2}-\dfrac{\left(\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}^{\;2}}+\dfrac{2\vartheta}{x_{(i)}}\right)\mathrm{e}^{-\vartheta/x_{(i)}}}{x_{(i)}^{\left(\vartheta^3+2\right)}}\right]}{\left(\vartheta^3+2+\dfrac{\vartheta^2}{x_{(i)}^{\;2}}+\dfrac{2\vartheta}{x_{(i)}}\right)}\right\}=0 \tag{16}$$

Then, the PCE of ϑ, say $\hat{\vartheta}_{\text{PCE}}$ can be obtained numerically by solving eq. (16).

Bootstrap confidence interval

The bootstrap was published by [15] and is a non-parametric technique that we use to estimate variance and find approximate confidence intervals for parameters. Although the method is non-parametric, it can be used to make inferences about parameters in parametric and non-parametric models, in this paper we discuss the parametric methods. The steps used to create confidence intervals bootstrap-p and bootstrap-t are:
- Compute the MLE in eq. (5).
- To obtain the bootstrap sample, we substitute m, $\hat{\vartheta}$ in IID eq. and denoted by $x^* = \{x_1^*, x_2^*, ..., x_m^*\}$.
- From bootstrap sample compute estimate of bootstrap $\hat{\vartheta}^*$.
- Repeat Steps 2 and 3 N times we obtain the sample $\hat{\vartheta}_1^*, \hat{\vartheta}_2^*, \cdots, \hat{\vartheta}_N^*$.
- The bootstrap estimate is:

$$\sum_{i=1}^{N} \frac{\hat{\theta}_i^*}{N} \tag{17}$$

Percentile bootstrap confidence interval

To obtain the percentile bootstrap, we arrange the sample $\hat{\vartheta}_1^*, \hat{\vartheta}_2^*, \cdots, \hat{\vartheta}_N^*$ in ascending order $\hat{\vartheta}_{(1)}^* \leq \hat{\vartheta}_{(2)}^* \leq ... \leq \hat{\vartheta}_{(N)}^*$. Suppose that the CDF of ordered sample is defined by the following distribution $\varnothing(y) = P[\hat{\vartheta}_{(i)}^* \leq y]$. The approximate $(1 - \alpha)\%$ boot-p confidence interval for ϑ is calculated:

$$\left[\hat{\vartheta}^*_{\text{boot-p}\left(\frac{\alpha}{2}\right)}, \hat{\vartheta}^*_{\text{boot-p}\left(1-\frac{\alpha}{2}\right)}\right] \tag{18}$$

where $\hat{\vartheta}^*_{\text{boot-p}} = \varnothing^{-1}(y)$.

Bootstrap-t confidence interval

For arrange the sample in ascending order $\hat{\vartheta}_1^*, \hat{\vartheta}_2^*, \cdots, \hat{\vartheta}_N^*$, we built the order statistic Z_1, Z_2, ..., Z_N where:

$$Z_i = \frac{\hat{\vartheta}_{(i)}^* - \hat{\vartheta}}{\sqrt{var\left(\hat{\vartheta}\right)}} \tag{19}$$

Suppose that the CDF of ordered sample Z_1, Z_2, ..., Z_N is defined by the following distribution $\varnothing(y) = P(Z_i \leq y)$. The approximate $(1 - \alpha)\%$ boot-t confidence interval for ϑ is calculated:

$$\left[\hat{\vartheta}^*_{\text{boot-t}\left(\frac{\alpha}{2}\right)}, \hat{\vartheta}^*_{\text{boot-t}\left(1-\frac{\alpha}{2}\right)}\right] \tag{20}$$

where

$$\hat{\vartheta}^*_{\text{boot-t}} = \hat{\vartheta}^* + \sqrt{var\left(\hat{\vartheta}\right)}\varnothing^{-1}\left(y\right) \tag{21}$$

Numerical simulation

In this section, a Monte Carlo simulation is studied for different sample sizes to compare different estimators. The simulation is conducted by using Mathematica (V.12.1) and has been repeated N = 10000 times with different sample sizes m = 30, 50, 100 while choosing (ϑ = 0.3, 0.5, 0.6). The performance of the resulting estimator of the parameter has been considered in terms of their absolute relative bias (ARBias) and relative RMSE (RRMSE), where:

$$ARBias\left(\hat{\vartheta}\right)=\left|\frac{\hat{\vartheta}-\vartheta}{\vartheta}\right| \tag{22}$$

and

$$RRMSE\left(\hat{\vartheta}\right)=\frac{\sqrt{MSE\left(\hat{\vartheta}-\vartheta\right)}}{\vartheta} \tag{23}$$

On other hand, we compare the performance of approximate confidence interval and two types of bootstraps using the AIL (average interval length) and CP (coverage probability) with nominal value 0.95. Results for the simulation study are given in tabs. 1 and 2, respectively.

From tabs. 1 and 2 show the results of different estimation methods for a parameter:

– We need numerical methods to solve the non-linear equations for the different estimators: MLE, LSE, WLSE, and PCE.
– For the point estimate, the MLE is considered a good among other estimators in terms of RRMSE, while the WLSE is a best for the parameter in terms of ARBias.
– The ARBias and RRMSE of the estimates8 are decrease when sample size are increases in almost of time.
– For interval estimate, the ACI is better than PBCI and BTCI for both average interval length and coverage probability.

As the sample size increases, the average interval length decreases, and coverage probability increases to approach the nominal value.

Applications

In this section, we employed two data sets to reveal using the IID as a good lifetime model by comparing it with Weibull distribution and Shanker distribution. The parameter is estimated by using the maximum likelihood. Mathematica (V.12.1) is used for computation. The goodness of fit tests is also used to compare the models:

$$AIC\left(\text{Akaike information criterion}\right)=-2l\left(\hat{\vartheta}\right)+2k \tag{24}$$

$$BIC\left(\text{Bayesian information criterion}\right)=-2l\left(\hat{\vartheta}\right)+k\log\left(m\right) \tag{25}$$

$$CAIC\left(\text{consistent Akaike information criterion}\right)=-2l\left(\hat{\vartheta}\right)+\frac{2km}{m-k-1} \tag{26}$$

$$HQIC\left(\text{Hannan-Quinn information criterion}\right)=-2l\left(\hat{\vartheta}\right)+2k\log\left[\log\left(m\right)\right] \tag{27}$$

where $l(\hat{\vartheta})$ is the log likelihood function, k – the number of estimated parameters, and m – the sample size. About other information for these measures see [16]. The smallest value of these measures determines the model that better fits the data.

Data set 1. These data are survival times for a group of head and neck cancer patients treated with two types of radiotherapy and chemotherapy see [17]:

12.2, 23.56, 23.74, 25.87, 31.98, 37, 41.35, 47.38, 55.46, 58.36,63.47, 68.46, 78.26, 74.47, 81.43, 84, 92, 94, 110, 112, 119, 127, 130,133, 140, 146, 155, 159, 173, 179, 194, 195, 209, 249, 281, 319, 339, 432, 469, 519, 633, 725, 817, 1776.

Table 1. The ML estimate, ARBias, RRMSE values of parameter θ

m	Method		$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.6$
30	MLE	$\hat{\theta}$	0.302774	0.504396	0.604327
		ARBias	0.00924745	0.00879175	0.00721109
		RRMSE	0.10589	0.101143	0.0976432
	LSE	$\hat{\theta}$	0.306063	0.502757	0.602476
		ARBias	0.0202108	0.00551372	0.00412643
		RRMSE	1.42527	0.108122	0.105143
	WLSE	$\hat{\theta}$	0.301654	0.502656	0.602317
		ARBias	0.00551478	0.00531218	0.00386107
		RRMSE	0.111098	0.10505	0.102167
	PCE	$\hat{\theta}$	0.285547	0.477501	0.573198
		ARBias	0.0481752	0.0449982	0.0446699
		RRMSE	0.127646	0.124833	0.122178
50	MLE	$\hat{\theta}$	0.302088	0.502311	0.602929
		ARBias	0.00695935	0.00462226	0.00488155
		RRMSE	0.0830398	0.0779884	0.0761408
	LSE	$\hat{\theta}$	0.301408	0.501161	0.601927
		ARBias	0.00469188	0.00232213	0.00321118
		RRMSE	0.0892386	0.0834827	0.0817443
	WLSE	$\hat{\theta}$	0.301336	0.50116	0.601829
		ARBias	0.00445172	0.00231976	0.00304901
		RRMSE	0.0864648	0.080903	0.0790569
	PCE	$\hat{\theta}$	0.289505	0.482543	0.579234
		ARBias	0.0349844	0.0349141	0.0346101
		RRMSE	0.100778	0.0983719	0.096075
100	MLE	$\hat{\theta}$	0.300749	0.500865	0.601382
		ARBias	0.00249726	0.00172947	0.00230394
		RRMSE	0.0571517	0.0550082	0.0536239
	LSE	$\hat{\theta}$	0.300333	0.500431	0.600817
		ARBias	0.00110855	0.0008629	0.00136173
		RRMSE	0.061802	0.0592052	0.0575907
	WLSE	$\hat{\theta}$	0.300361	0.500462	0.60083
		ARBias	0.00120308	0.000923434	0.00138393
		RRMSE	0.0596702	0.0572735	0.0556408
	PCE	$\hat{\theta}$	0.292698	0.487512	0.58611
		ARBias	0.0243412	0.0249761	0.02315
		RRMSE	0.0711411	0.0703672	0.0681027

Table 2. The ACI, PBCI, and BTCI for parameter θ

N	θ	Average interval length			Coverage probability		
		ACI	PBCI	BTCI	ACI	PBCI	BTCI
30	0.3	0.1184	0.1189	0.1253	66.0648	66.7506	66.4073
	0.5	0.1992	0.1974	0.2039	67.2368	68.1664	68.1257
	0.6	0.2222	0.2222	0.2293	67.8577	68.4345	68.3637
50	0.3	0.084	0.084	0.0864	72.6117	72.973	72.8728
	0.5	0.1652	0.1631	0.1659	74.0496	74.6464	74.664
	0.6	0.1876	0.1882	0.191	74.6623	74.9067	74.9442
100	0.3	0.066	0.0662	0.0673	79.8719	79.9819	79.8984
	0.5	0.1052	0.1055	0.1065	80.6119	80.6847	80.696
	0.6	0.1224	0.1252	0.1265	81.0233	80.7533	80.74

Table 3 displays the values of MLE, measures AIC, BIC, CAIC and HQIC for inverse Ishita, Weibull and Shanker distributions. Figiure 4 shows the curves of empirical distribution and estimated CDF of inverse Ishita, Weibull and Shanker distributions. The results of MLE and confidence intervals are in tab. 4.

Table 3. The values for ML estimates and goodness of fit measures of data Set 1

Model	ML estimates		Statistics			
	$\hat{\theta}$	$\hat{\beta}$	AIC	BIC	HQIC	CAIC
Inverse Ishita	76.7013	–	561.155	562.939	561.816	561.25
Weibull	216.116	0.94088	567.683	571.252	569.007	567.976
Shanker	0.00894963	–	642.501	644.285	643.163	642.597

Table 4. The MLE and 95%confidence interval of the parameter

$\hat{\theta}$	76.7013	ACI	(54.0384, 99.3642)
$\hat{\theta}_{\text{LSE}}$	82.5569	PBCI	(58.6436, 104.765)
$\hat{\theta}_{\text{WLSE}}$	80.18	BTCI	(62.8953, 115.034)
$\hat{\theta}_{\text{PCE}}$	63.465-		
$\hat{\theta}_{\text{Boot}}$	78.4675		

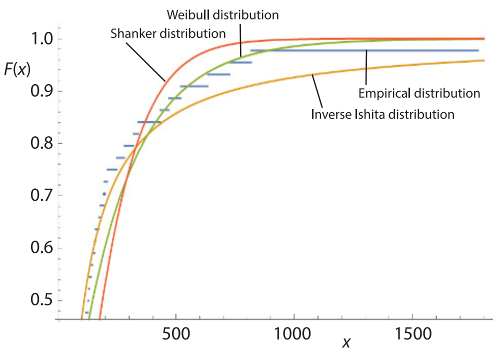

Figure 4. Empirical distribution and estimated CDF for the data of survival times for patients with head and neck cancer

Data set 2. These data represent the lengths of time it takes for 20 different components to fail. and its values are [18]:

70.175, 8.851, 2.968, 9.763, 57.637, 57.337, 9.773,48.442, 6.662, 37.386, 79.333, 85.283, 8.608, 6.56, 54.145, 4.229, 7.11, 10.578, 30.112, 19.136.

Table 5 displays the values of MLE, measures AIC, BIC, CAIC, and HQIC for inverse Ishita, Weibull-Shanker distributions. Figure 5 shows the curves of empirical distribution and estimated CDF of inverse Ishita, Weibull-Shanker distributions. The results of MLE and confidence intervals are in tab. 6.

Table 5. The values for ML estimates and goodness of fit measures of data Set 2

Model	ML estimates		Statistics			
	$\hat{\theta}$	$\hat{\beta}$	AIC	BIC	HQIC	CAIC
Inverse Ishita	11.333	–	179.176	180.172	179.371	179.399
Weibull	31.6069	1.0754	180.815	182.806	181.203	181.52
Shanker	0.065185	–	210.997	211.993	211.191	211.219

Table 6. The MLE and 95% confidence interval of the parameter

$\hat{\theta}$	11.333	ACI	(6.4002, 16.2658)
$\hat{\theta}_{\text{LSE}}$	10.9317	PBCI	(7.6836, 18.2708)
$\hat{\theta}_{\text{WLSE}}$	10.713	BTCI	(8.8974, 22.5766)
$\hat{\theta}_{\text{PCE}}$	9.9449		
$\hat{\theta}_{\text{Boot}}$	11.8851		

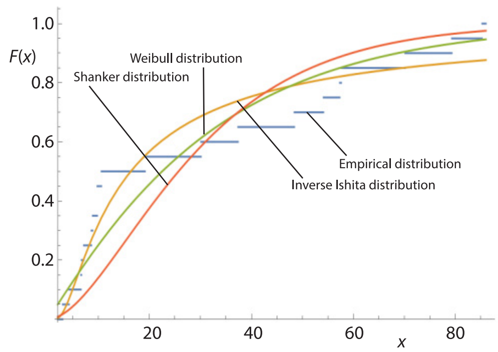

Figure 5. Empirical distribution and estimated CDF for the data of times for different components to fail

Conclusion

In this article, we adopted IID. As one of new one parameter distribution for modelling lifetime data with scarce information. As well, two types of estimation were used: point estimation and interval estimation estimate the unknown parameter. Finally, a set of data is applied to assess the efficiency of the distribution compared with specific models. According to the results, the IID shows a better fit for all the data instead of the known distributions we cared about it here.

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