DYNAMICS OF QUANTUM CORRELATIONS IN A TWO-PARAMETER CLASS SYSTEM UNDER QUANTUM NOISE

by

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In this paper, we investigate the time evolution of quantum correlations in a two-qubit system influenced by an optical channel. The system's dynamics are analyzed in terms of concurrence (CN), Bell non-locality (BN), and trace distance discord (TDD), which quantify entanglement, non-local correlations, and quantum discord, respectively. Our model explores the impact of key parameters such as the beam-splitter angle, θ, channel parameter, λ, and angular frequency, ω, on these quantum correlations. The results show that CN and BN exhibit oscillatory behavior with periodic revivals, but tend to decay more rapidly under noisy conditions. Conversely, TDD demonstrates greater robustness, persisting even when CN and BN collapse, indicating the survival of quantum correlations in separable states. Higher noise strength and angular frequencies induce faster oscillations and revivals across all measures, with systems prepared with stronger initial quantum correlations showing increased resilience. This study highlights the robustness of quantum discord in noisy environments and its potential role in quantum information processing, even in the absence of entanglement.

Key words: *two-qubit system, optical channel, concurrence, Bell non-locality, trace distance discord*

Introduction

An open quantum system can be described as a quantum system that is coupled to an external environment or reservoir, and which thus experiences state decoherence and other degeneration of quantum characteristics [1, 2]. Closed systems on the other hand develop unitarily, however, due to decoherence, dissipation and noise, open systems are subjected to non-unitary evolution [3, 4]. The interaction of the system with the environment makes it so that it is exposed to various types of quantum noise, for example, bit-flip, phase-flip, or amplitude damping, which damage the quantum information stored inside the system [5]. For this purpose, open quantum systems are supposed to be described by quantum channels, which are described by Kraus operators or master equations or both [6, 7]. Studying such systems is crucial for comprehension of quantum information procedures in actual noisy conditions as quantum computing, dots, and cryptography [8, 9].

Quantum entanglement can be described as the most stunning and paradoxical in the context of the contemporary views on the world founded on the non-classical correlationship between the quantum systems [10]. When two or more particles get in contact the state vector

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of one particle then depends on the state vector of the other even if they are separated by a large distance [11]. This distinctive attribute was initially revealed in the widely known EPR paradox in 1935 which rose question concerning the absolute completeness of quantum mechanics [12]. Einstein described entanglement as getting an eerie feeling because, by a measurement of one of the particles, something strange occurred at the other particle [13]. In response, Bell's theorem gave the mathematical relation describe how entangled particles are different from classical systems to yield correlation [14]. Aspect *et al.* [15] conducted experiments that agreed with the quantum mechanics. After that, entanglement has become a fundamental resource necessary for quantum information processing, including flying qubits such as quantum teleportation, quantum communication like quantum cryptography, and quantum computer such as quantum computation [16].

The BN describes a situation where the correlations of measurements made of entangled quantum systems cannot be accounted for by some hidden variable theory that is local, as described by Bell's inequalities [17]. In quantum laboratory these violations indicate that what is measured on one of the particles of a system may determine on what is measured in the other regardless the distance that might be separating them [18]. The BN dates back from Bell theorem postulated in 1964 up to the recent advances in quantum information theory where BN is relevant in quantum cryptography, entanglement authentication and quantum communication [19, 20]. Studies that include explorations of coherent quantum states and quantum interfaces have also reinvestigated the Bell type non-locality in the high dimensional quantum systems regarding their foundation of quantum mechanics and potential applications in novel quantum technologies [21].

In a two-qubit process analysis when correlating two qubits with optical elements including beam splitters, the time continuous process is governed by the time-dependent Kraus operators [22]. These optical elements lead to both decoherence and entanglement-generation of the two-parameter class of two-qubit states, which can be described in terms of generalized Bell states or parameterized computational basis states. In this case, Kraus operators specify the time evolution of the system [23]. To capture the evolution, we define tensor products of these operators for the two-qubit system. Further, the beam splitter (BS) angle causes time-dependent interference between the two modes generating superpose globalization of vacuum and single-photon state.

These optical elements in turn bring about the evolution of the two parameter density matrix which contains coherence terms between the $|00\rangle$ and $|11\rangle$ basis set. These time-dependent Kraus operators altered by the BS afford this evolution characterizing the global state-transition by means of a quantum superposition of operations. This yields a final time evolved density matrix which results from the application of the Kraus operators to the initial state. This process reveals how the optical arrangements affect the entanglement, coherence trade off, manifested through the quantum discord defined by the trace distance or the BN. This shows that in entangling the states, the BS has shown that the optical elements can in fact force the evolution of correlations in quantum systems whenever there are environment impacts or channels of measurements present. Finally, we measure quantum correlations in the system using CN, BN, and TDD measures.

Physical model

In this work, we consider a two-qubit system subjected bit-flip and phase-flip noise quantum channel characterized by time-dependent Kraus operators. The evolution of the system is described using the Kraus operators that represent bit-flip and phase-flip operations.

The derivation utilizes the concept of the time evolution operator and the relation between the Hamiltonian and the Kraus representation in quantum mechanics [24]. Starting with the total Hamiltonian of the system, we have:

$$H_{\text{total}} = J\left(\sigma_x^{(1)}\sigma_x^{(2)} + \sigma_y^{(1)}\sigma_y^{(2)}\right) + \frac{\lambda}{2}\sum_{i=1}^{2}\sigma_z^{(i)} \tag{1}$$

where the first part is H_{int} interaction and the second is the noise part H_{noise}. The time evolution of the quantum state is governed by the unitary operator given by $U(t) = \mathrm{e}^{-iH_{\text{total}}t/\hbar}$ [25]. For small time intervals, this can be approximated using the first-order expansion:

$$U(t) \approx I - \frac{i}{\hbar}H_{\text{total}}t$$

which leads to:

$$U(t) \approx I - \frac{i}{\hbar}(H_{\text{int}} + H_{\text{noise}})t$$

The interaction Hamiltonian induces coherent operations on the qubits and for small (t), the effect of (H_{int}) can be analyzed [26]. We can write the action of (H_{int}) on the qubit states as:

$$H_{\text{int}}\left|\psi\right> = J(\sigma_x^{(1)}\sigma_x^{(2)} + \sigma_y^{(1)}\sigma_y^{(2)})\left|\psi\right>$$

This interaction can be viewed as leading to a state transformation represented by ($k_1(t)$) and ($k_2(t)$). The noise Hamiltonian induces bit-flip and phase-flip operations on the qubits while the action of (H_{noise}) can be described:

$$H_{\text{noise}}\left|\psi\right> = \frac{\lambda}{2}\sum_{i=1}^{2}\sigma_z^{(i)}\left|\psi\right>$$

which leads to decoherence in the system [27]. To derive the Kraus operators from this, we consider how the noise affects the state over time. The evolution under (H_{noise})can be written in terms of Pauli operators:

$$\left|\psi(t)\right> = \mathrm{e}^{-iH_{\text{noise}}t/\hbar}\left|\psi(0)\right>$$

To find the Kraus representation, we examine the operator-sum representation of the quantum channel:

$$\rho_f = \sum k_i \rho k_i^{\dagger}$$

where (k_i) are the Kraus operators. By identifying the contributions from both the inclusive Hamiltonian, we can express the Kraus operators with bit-flip and phase-flip operation can be written:

$$k_1(t) = \sqrt{\frac{1+\lambda}{2}}\sigma_x \qquad k_2(t) = \sqrt{\frac{1-\lambda}{2}}\sigma_y \tag{2}$$

where σ_x and σ_y are the Pauli matrices representing bit-flip and phase-flip operations, respectively. Next, we define the tensor products of these Kraus operators to account for the evolution of the two-qubit system:

$$k_{11}(t) = k_1(t)\otimes k_1(t) \qquad k_{12}(t) = k_1(t)\otimes k_2(t) \tag{3}$$

$$k_{21}(t) = k_2(t)\otimes k_1(t) \qquad k_{22}(t) = k_2(t)\otimes k_2(t) \tag{4}$$

To account for the influence of optical elements on the system, we define time-dependent parameters for the BS angle $\mathcal{Q}(t) = \theta\cos(\omega t)$. The operation of the BS on an incoming state can be expressed as a superposition of the input states. For a two-mode input state $|\psi\rangle$, the action of the BS can be written:

$$\mathrm{BS}\left|\psi\right\rangle = \cos(\mathcal{Q}(t))\left|0\right\rangle_A\left|0\right\rangle_B + \sin(\mathcal{Q}(t))\left|1\right\rangle_A\left|1\right\rangle_B + \mathcal{Y} \tag{5}$$

where ($|0\rangle$) and ($|1\rangle$) represent the vacuum and single-photon states in each mode, respectively. The cross-terms $\mathcal{Y}$ represent the entangled states generated by the BS, which can be written:

$$\left|\mathcal{Y}\right\rangle = \sin(\mathcal{Q}(t))\left|0\right\rangle_A\left|1\right\rangle_B + \cos(\mathcal{Q}(t))\left|1\right\rangle_A\left|0\right\rangle_B \tag{6}$$

Besides, the initial density matrix ($\rho(\alpha, \beta)$) of a two-qubit system can be written:

$$\rho(\alpha,\beta) = \begin{pmatrix} \cos^2\left(\dfrac{\alpha}{2}\right) & 0 & 0 & e^{-i\beta}\sin(\alpha)\cos(\alpha) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ e^{\mathrm{I\beta}}\sin(\alpha)\cos(\alpha) & 0 & 0 & \sin^2\left(\dfrac{\alpha}{2}\right) \end{pmatrix} \tag{7}$$

where (α) is a real parameter angle, while (β) – the phase angle.

Finally, the time evolution of the final density matrix is computed:

$$\rho_f^{\mathrm{BS}}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} k_{ij}^{\mathrm{BS}}(t)\rho(\alpha,\beta)(k_{ij}^{\mathrm{BS}}(t))^\dagger, \text{ with } k_{ij}^{\mathrm{BS}}(t) = \mathrm{BS}(t)k_{ij}(t)\mathrm{BS}(t)^\dagger \tag{8}$$

The aforementioned matrix has the structure:

$$\rho_f^{\mathrm{BS}}(t) = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{12}^* & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{13}^* & \rho_{23}^* & \rho_{33} & \rho_{34} \\ \rho_{14}^* & \rho_{24}^* & \rho_{34}^* & \rho_{44} \end{pmatrix} \tag{9}$$

where the entries are defined:

$$\rho_{11} = \frac{1}{2}\sin^2\left(\frac{\alpha}{2}\right)(X_\alpha^2 + X_b^2)\left(X_\alpha^2(\cos(4\theta(t)) + 1) + 2X_b^2\right)$$

$$\rho_{12} = \frac{1}{2}X_\alpha^2\sin^2\left(\frac{\alpha}{2}\right)(X_\alpha^2 + X_b^2)\sin(4\theta(t))$$

$$\rho_{13} = -\frac{1}{4}e^{ib}X_\alpha^2\sin(2\alpha)(X_\alpha^2 - X_b^2)\sin(4\theta(t))$$

$$\rho_{14} = \frac{1}{4}\mathrm{e}^{ib}\sin(2\alpha)(X_\alpha^2 - X_b^2)\left(X_\alpha^2(\cos(4\theta(t)) + 1) - 2X_b^2\right)$$

$$\rho_{22} = X_\alpha^2\sin^2\left(\frac{\alpha}{2}\right)(X_\alpha^2 + X_b^2)\sin^2(2\theta(t))$$

$$\rho_{23} = -\frac{1}{2}e^{ib}X_\alpha^2\sin(2\alpha)(X_\alpha^2 - X_b^2)\sin^2(2\theta(t))$$

$$\rho_{24} = \frac{1}{4}e^{ib}X_\alpha^2\sin(2\alpha)(X_\alpha^2 - X_b^2)\sin(4\theta(t))$$

$$\rho_{33} = 4X_\alpha^2\cos^2\left(\frac{\alpha}{2}\right)(X_\alpha^2 + X_b^2)\sin^2(\theta(t))\cos^2(\theta(t))$$

$$\rho_{34} = -\frac{1}{2}X_\alpha^2\cos^2\left(\frac{\alpha}{2}\right)(X_\alpha^2 + X_b^2)\sin(4\theta(t))$$

$$\rho_{44} = \frac{1}{2}\cos^2\left(\frac{\alpha}{2}\right)(X_\alpha^2 + X_b^2)\left(X_\alpha^2(\cos(4\theta(t))+1) + 2X_b^2\right)$$

with

$$X_a = \sqrt{\frac{\lambda+1}{2}} \text{ and } X_b = \sqrt{\frac{\lambda-1}{2}}$$

This framework provides a comprehensive understanding of the dynamics of the two-qubit system under the influence of both quantum noise and optical elements.

Quantum measurements

Concurrence

The CN is a measure of quantum entanglement for two-qubit states and is defined. For a given two-qubit density matrix (ρ), the CN can be computed using [28]:

$$CN = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{10}$$

where λ_1 are the square roots of the eigenvalues of the matrix

$$R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

with (ρ^*) being the complex conjugate of (ρ) in the computational basis, and (σ_y) is the Pauli-Y matrix.

Bell non-locality

The key aspect of quantum system that sets them apart is their ability to violate Bell-like inequalities, which is quantified by BN. This non-locality is fundamentally different from classical correlations, as it cannot be explained by any local hidden variable theory. To quantify BN for the density matrix (ρ_{ab}), we define it:

$$BN := \max_{\xi}\left[B(\rho_{ab}, \xi)\right] \tag{11}$$

In normalized form, the BN can be expressed [29]:

$$BN := \max\left[0, \frac{B_{\text{CHSH}} - 2}{\mathcal{B}_{\max} - 2}\right] \tag{12}$$

For a specific two-qubit (X)-shaped density matrix, the expression for (B_{CHSH}) becomes:

$$B_{\text{CHSH}} = 2\max\left\{b_1\mathcal{B}_{\max}, b_2\right\}, \tag{13}$$

where

$$\left(b_1 = \sqrt{|\varrho_{14}|^2 + |\varrho_{23}|^2}\right) \text{ and } \left(b_2 = \sqrt{4(|\varrho_{14}| + |\varrho_{23}|)^2 + \mathcal{C}_{33}^2}\right)$$

with

$$(\mathcal{B}_{\max} = 2\sqrt{2})$$

Therefore, the normalized BN function lies within the range $0 \le BN \le 1$, given that (B_{CHSH}) must not exceed ($\mathcal{B}_{\max}$).

Trace measure of geometric discord

Paula and Oliveira [30] used the trace norm (also known as the 1 norm) as a trustworthy geometric notation of quantum discord. Analysis has been done to extract the expressions of TDD for any arbitrary two-qubit X states and Bell-diagonal states [31]. The TDD for a two-qubit state ρ_{AB} can be defined:

$$TDD = \frac{1}{2}\min_{\eta\in\varphi}\left\|\rho_{AB} - \varrho_{AB}\right\|_1 \tag{14}$$

where

$$\left\|\rho_{AB} - \varrho_{AB}\right\|_1 = \varrho_{AB}\sqrt{(\rho_{AB} - \varrho_{AB})^{\dagger}(\rho_{AB} - \varrho_{AB})}$$

defines the trace distance. The TDD measure for the two-qubit state ρ_{AB} has the form:

$$TDD = \frac{1}{2}\sqrt{\frac{\mathcal{Q}_{11}^2\mathcal{Q}_{\max}^2 - \mathcal{Q}_{22}^2\mathcal{Q}_{\min}^2}{\mathcal{Q}_{\max}^2 - \mathcal{Q}_{\min}^2 + \mathcal{Q}_{11}^2 - \mathcal{Q}_{22}^2}} \tag{15}$$

where

$$\mathcal{Q}_{\min}^2 = \min\{\mathcal{Q}_{11}^2, \mathcal{Q}_{33}^2\}, \text{ with } \mathcal{Q}_{\max}^2 = \max(\mathcal{Q}_{33}^2, \mathcal{Q}_{22}^2 + \mathcal{Q}_{30}^2)$$

along with

$$(\mathcal{Q}_{11} = 2(|\rho_{23}|),\ \ \mathcal{Q}_{22} = 2(|\rho_{23}|),\ \ \mathcal{Q}_{33} = -2(\rho_{22} + \rho_{33}) \text{ and } \mathcal{Q}_{30} = 2(\rho_{11} + \rho_{22}) - 1)$$

Results and discussion

In this section, we provide the results obtained for the two-qubit state of two parameter class when influenced by an optical channel and quantum noise.

The time dynamics of the two-qubit quantum correlations displayed in the plots represent CN, BN, and TDD under the influence of an optical channel is shown in fig. 1. Each of these measures evolves with time for different initial conditions parameterized by (θ), with ($\theta = \pi$), ($\pi/2$), and ($\pi/4$), indicating different levels of entanglement or coherence in the initial state. At time ($t = 0$), all the quantum correlations (CN, BN, and TDD) begin with relatively high values, showcasing strong initial entanglement, non-locality and quantum discord. Over time, the CN exhibits oscillatory decay, with periods of revival, which suggests periodic entanglement loss and recovery under the channel influence, governed by the optical interaction parameters ($\lambda = 1$), ($\omega = 0.1$), ($a = 1$). Similarly, BN also shows an oscillatory pattern, though it tends to sustain non-local correlations for longer periods than CN, indicating a delayed loss of BN compared to entanglement. The TDD, which quantifies quantum correlations even in separable states, displays a smoother and less oscillatory evolution than CN and BN, generally decaying slower and stabilizing over time. The observed dynamics indicate that TDD is more robust against channel-induced decoherence, whereas entanglement and non-locality experience fre-

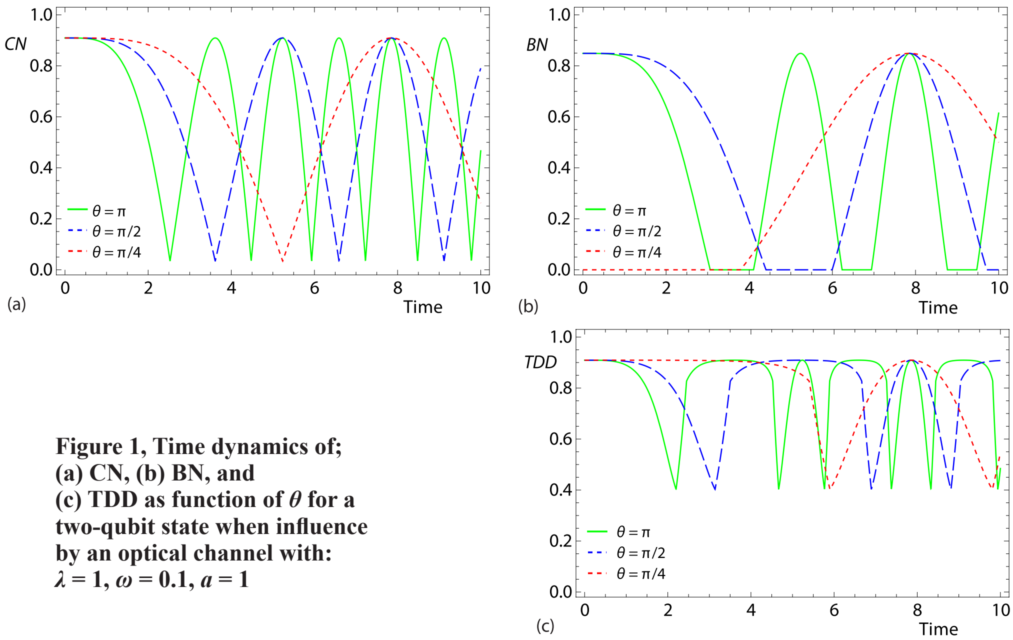

Figure 1, Time dynamics of; (a) CN, (b) BN, and (c) TDD as function of θ for a two-qubit state when influence by an optical channel with: $\lambda = 1$, $\omega = 0.1$, $a = 1$

quent collapse and revival cycles. Physically, this behavior reflects how different quantum resources-entanglement, non-locality, and discord, respond variably to the optical channel, with entanglement and non-locality being more sensitive to environmental interactions, while discord remains more resilient, persisting even when other correlations diminish. The oscillations are characteristic of periodic interactions between the qubits and the optical channel, modulated by the interaction strength and decay rates. The final levels show that while CN and BN eventually drop significantly or vanish in some cases, TDD retains a non-zero value, implying that certain quantum correlations persist even when entanglement is completely lost. This interplay between different quantum correlations gives insight into the resilience of quantum coherence and the potential for using discord in quantum information tasks even in noisy environments.

Figure 2 illustrates the time evolution of quantum correlations, specifically CN, BN, and TDD, for a two-qubit system influenced by an optical channel. The system is evaluated under three different values of the noise parameter (λ) (0.1, 0.5, and 1.0), revealing distinct dynamical behavior across the measures. In fig. 2(a), for ($\lambda = 0.1$), the CN remains near zero throughout the time evolution, indicating an absence of significant entanglement between the qubits. As (λ) increases to 0.5 and 1.0, the CN begins to exhibit periodic oscillations, with larger (λ) values resulting in higher amplitudes and more frequent oscillations. This suggests that higher noise levels, in this context, lead to a transient increase in entanglement. Similarly, in fig. 2(b), BN follows a comparable trend. For ($\lambda = 1.0$), the system exhibits pronounced oscillations of BN, starting from its maximum value and periodically decaying and reviving. For ($\lambda = 0.5$), the oscillations are less frequent and lower in amplitude, while for ($\lambda = 0.1$), BN is nearly absent, indicating a lack of significant non-local correlations in the system. Figure 2(c) displays the time dynamics of TDD, which shows a more robust behavior compared to CN and BN. Even for ($\lambda = 0.1$), TDD remains non-zero, and for ($\lambda = 0.5$) and ($\lambda = 1.0$), the discord exhibits sustained oscillations, reflecting the persistence of quantum correlations despite the influence of noise. Notably, TDD does not completely vanish, even at points where CN and BN

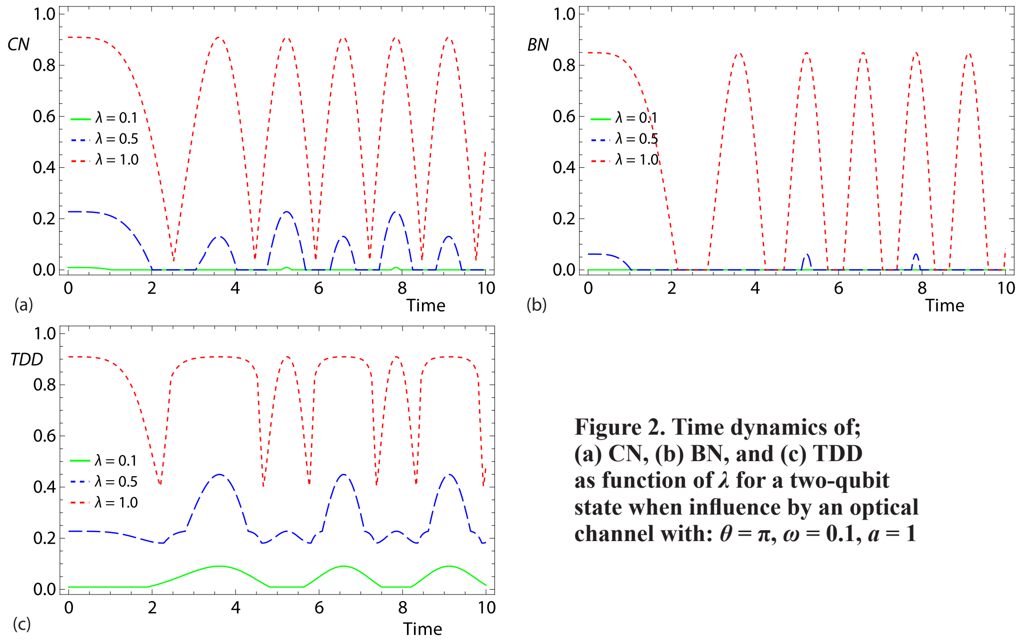

Figure 2. Time dynamics of; (a) CN, (b) BN, and (c) TDD as function of λ for a two-qubit state when influence by an optical channel with: $\theta = \pi$, $\omega = 0.1$, $a = 1$

reach zero, highlighting the fact that discord is less sensitive to decoherence and may capture quantum correlations that are not reflected in entanglement or non-locality. Besides, as the noise parameter (λ) increases, all measures of quantum correlations exhibit more pronounced oscillatory behavior, with greater amplitudes and frequencies observed for higher (λ). TDD, however, demonstrates greater resilience compared to CN and BN, underscoring its robustness in capturing quantum correlations under noisy conditions.

Figure 3 shows the time dynamics of three quantum correlation measures: CN, BN, and TDD, for a two-qubit system under the influence of an optical channel, as a function of the angular frequency (ω). The parameters used are ($\theta = \pi$), ($\lambda = 1$), ($\omega = 0.1$, 1.5, 2.0), and ($a = 1$). In this panel, the dynamics of entanglement are shown for different values of (ω). At lower frequencies ($\omega = 0.1$), green solid line – 1, the entanglement exhibits slower oscillations, with relatively longer periods of high entanglement before decaying to zero and reviving. As (ω) increases to 1.5 (blue dashed line – 2) and 2.0 (red dotted line – 3), the oscillations become faster, with more frequent revivals and collapses. Higher (ω) leads to quicker oscillations, indicating that the rate at which the quantum system evolves is governed by the frequency of the optical channel. The behavior of BN closely mirrors that of CN. For lower frequencies ($\omega = 0.1$), BN decays more slowly and revives after a significant time interval. As (ω) increases to 1.5 and 2.0, the non-locality oscillates more frequently. The periods of maximal BN align with those of CN, suggesting that the system's entanglement and non-local correlations are strongly coupled. At higher frequencies, the rapid oscillations reflect faster decoherence and revival of non-locality. For all values of (ω), the TDD exhibits periodic oscillations, similar to those observed for CN and BN. At ($\omega = 0.1$), the oscillations are smoother, with a slow decay followed by revivals. For ($\omega = 1.5$) and ($\omega = 2.0$), the discord oscillates more rapidly, similar to the trends observed for CN and BN. However, discord tends to retain a higher baseline value, indicating that even in regions where entanglement or BN collapses, quantum correlations (as measured by discord) still persist. The plot shows that increasing the angular frequency (ω)

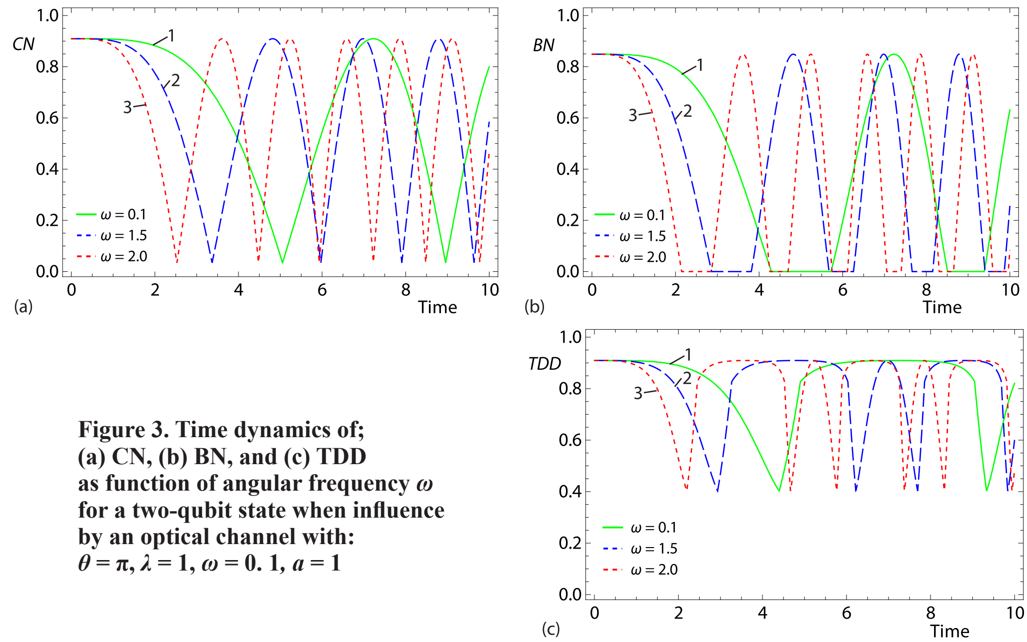

Figure 3. Time dynamics of; (a) CN, (b) BN, and (c) TDD as function of angular frequency ω for a two-qubit state when influence by an optical channel with: $\theta = \pi$, $\lambda = 1$, $\omega = 0.1$, $a = 1$

of the optical channel leads to faster oscillations of the quantum correlations, with all three measures, CN, BN, and TDD, exhibiting a higher frequency of revival and decay cycles. This highlights the role of (ω) in controlling the dynamics of quantum correlations in the system.

Conclusions

In this work, we have explored the time dynamics of quantum correlations in a two-qubit system subjected to an optical channel, characterized by CN, BN, and TDD. Our analysis, using a physical model influenced by key parameters such as (θ), noise strength (λ), angular frequency (ω), and the initial state parameter, a, reveals distinct behaviors across these quantum measures, providing a deeper understanding of how quantum correlations evolve under environmental decoherence.

The results demonstrate that CN, which measures entanglement, and BN, which reflects the presence of non-classical correlations, exhibit oscillatory decay with periodic revival, driven by the interaction between the qubits and the optical channel. Both CN and BN are more sensitive to noise and tend to diminish rapidly, especially under lower noise conditions or smaller initial quantum correlations. However, TDD proves to be more robust, displaying a slower decay and persisting even when CN and BN collapse, indicating that quantum correlations, in the form of discord, survive decoherence for much longer durations. Furthermore, we observe that increasing noise strength λ and angular frequency (ω) leads to more rapid oscillations and faster revival and collapse cycles for all three quantum measures. In particular, systems initialized with stronger quantum correlations, larger (a). exhibit greater resilience to the environmental noise, with more pronounced and frequent oscillations compared to systems with weaker initial correlations. Our findings suggest that while entanglement and non-locality may be quickly lost under noisy conditions, quantum discord persists as a valuable resource for quantum information processing, even in scenarios where entanglement is absent. This highlights the potential utility of discord in quantum systems operating in realistic, noisy environments, where it may continue to play a critical role in quantum communication, computing, and cryptographic tasks. These insights into the dynamic behavior of quantum correlations under environmental decoherence provide a valuable framework for optimizing quantum protocols in noisy systems and could be further explored in the context of different quantum channels and multi-qubit systems.

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