AN APPLICATION OF FRACTAL FRACTIONAL OPERATORS TO NON-LINEAR CHEN SYSTEMS

by

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This paper employs the Atangana-Baleanu fractal-fractional operators to establish whether chaotic behavior is present or not in a non-linear modified Chen. The Chen exists and is unique under fixed point theory. To illustrate the applicability and efficiency of this method, numerical examples are provided to provide a better understanding of it. To verify the results in this paper, a circuit schematic has been drawn and a simulation has been conducted.

Key words: *fractional derivatives, non-linear equations, simulation, numerical results, iterative method, time varying control system, lyapunov functions*

Introduction

There is a great deal of interest in the chaos literature when it comes to studying chaotic behavior in nature and physical systems. Chaotic systems are non-linear systems that are highly sensitive to initial conditions, topologically mixed, and possess dense periodic orbits [1]. Due to Lorenz's [2] discovery of a 3-D chaotic system of a weather model, chaos theory underwent significant development. As a result of this research, several 3-D chaotic systems were discovered in the chaos literature, including Rossler [3], Rabinovich and Fabrikant [4], Arneodo *et al*. [5], Sprott [6], Chen and Ueta [7], Lu and Chen [8], Shaw [9], Feeny and Moon [10], Shimizu and Moroika [11], Liu and Chen [12], Cai and Tai [13], Tigan and Opris [14] system, Colpitt's oscillator system [15], Windmi system [16], and Zhou *et al.* [17] system. There have been numerous 3-D chaotic systems discovered recently, including Li [18] systems. The aforementioned three chaotic systems have been combined into a single chaotic system, unified Liu and Chen [12] chaotic system, which is described:

$$\begin{aligned}\dot{u}_1 &= (25\vartheta+10)(u_2-u_1)\\ \dot{u}_2 &= (28-35\vartheta)u_1-u_1u_3+(29\vartheta-1)u_2\\ \dot{u}_3 &= u_1u_2-\frac{(\vartheta+8)u_3}{3}\end{aligned} \tag{1}$$

where $\vartheta \in (0.8, 1]$.

Fractal fractionals are observed in various natural systems, including excitation-relaxation systems and natural oscillatory systems. These dynamics are characterized by long-memory behaviors, heavy-tailed distributions, and short-range autocorrelation dependence. The Atangana-Balanu fractal-fractional operators (ABFFO) is a fractional derivative operator that

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incorporates fractal geometry into the fractional calculus framework. This enables a more accurate representation of complex systems with fractal behavior see [19-30]. Using AB FFO, we investigate the dynamical behavior of the modified Chen family:

$$\begin{aligned} {}^{FFP}\mathcal{D}_{0,t}^{\kappa_1,\varrho^*} u_1(t) &= (25\vartheta+10)(u_2-u_1) \\ {}^{FFP}\mathcal{D}_{0,t}^{\kappa_1,\varrho^*} u_2(t) &= (28-35\vartheta)u_1 - u_1u_3 + (29\vartheta-1)u_2 \\ {}^{FFP}\mathcal{D}_{0,t}^{\kappa_1,\varrho^*} u_3(t) &= u_1u_2 - \frac{(\vartheta+8)u_3}{3} \end{aligned} \tag{2}$$

We verify the existence and unique nature of the solution system, as well as examine its qualitative characteristics. Using the AB derivative, the model is analyzed for its actual behavior. Finally, a numerical simulation has been carried out which supports the biological results.

Preliminaries

Consider $\phi \in C((a, b), \mathbb{R})$ which is fractal differentiable on (a, b) of order $0 < \varrho^* \leq 1$. The fractal-fractional derivation operator for ϕ in the AB settings of order $0 < \kappa_1 \leq 1$, with the generalized kernel of the Mittag-Leffler type is introduced [27]:

$${}^{FFP}\mathcal{D}_t^{\kappa_1,\varrho^*}\phi(t) = \frac{AB(\kappa_1)}{1-\kappa_1}\frac{\mathrm{d}}{\mathrm{d}t\varrho^*}\int_0^t \phi(s)E_{\kappa_1}\left[-\frac{\kappa_1}{1-\kappa_1}(t-s)^{\kappa_1}\right]\mathrm{d}s$$

where

$$AB(\kappa_1) = 1-\kappa_1+\frac{\kappa_1}{\Gamma(\kappa_1)} \quad \text{and} \quad \frac{\mathrm{d}\phi(s)}{\mathrm{d}s\varrho^*} = \lim_{t\to s}\frac{\phi(t)-\phi(s)}{t^{\varrho^*}-\varsigma^{\varrho^*}}$$

Let ϕ be the same function considered previously [27]. Then, the fractal-fractional integration operator in the AB settings for ϕ of order $0 < \kappa_1 \leq 1$ with the kernel of Mittag-Leffler type is given:

$${}^{FFP}\mathcal{I}_t^{\kappa_1,\varrho^*}\phi(t) = \frac{\kappa_1\varrho^*}{AB(\kappa_1)\Gamma(\kappa_1)}\int_0^t s^{\varrho^*-1}\phi(s)(t-s)^{\kappa_1-1}\mathrm{d}s + \frac{\varrho^*(1-\kappa_1)\varrho^{\varrho^*-1}}{AB(\kappa_1)}\phi(t)$$

Existence and uniqueness of the proposed model

Utilizing the AB fractional operators, the system of equations can be expressed:

$$\begin{aligned} {}^{FFP}\mathcal{D}_{0,t}^{\alpha_1,\beta^*} v_1(t) &= (30\gamma+15)(v_2-v_1) \\ {}^{FFP}\mathcal{D}_{0,t}^{\alpha_1,\beta^*} v_2(t) &= (32-40\gamma)v_1 - v_1v_3 + (35\gamma-2)v_2 \\ {}^{FFP}\mathcal{D}_{0,t}^{\alpha_1,\beta^*} v_3(t) &= v_1v_2 - \frac{(3\gamma+10)}{4}v_3 \end{aligned}$$

Based on the fixed point theory, we will demonstrate that the model possesses a unique solution. Therefore, the proposed model can be reformulated as the integration remains differentiable:

$$ {}_{0}^{\mathrm{ABR}}D_{0}^{\alpha_1,\beta^*}v_1(t)=\beta^* t^{\beta^*-1}g_1(t,v_1,v_2,v_3) $$
$$ {}_{0}^{\mathrm{ABR}}D_{0}^{\alpha_1,\beta^*}v_2(t)=\beta^* t^{\beta^*-1}g_2(t,v_1,v_2,v_3) $$
$$ {}_{0}^{\mathrm{ABR}}D_{0}^{\alpha_1,\beta^*}v_3(t)=\beta^* t^{\beta^*-1}g_3(t,v_1,v_2,v_3) $$

Additionally, we have:

$$ {}_{0}^{\mathrm{ABR}}D_{t}^{\alpha_1}\mathcal{F}(t)=\beta^* t^{\beta^*-1}\nu(t,\mathcal{F}(t)), $$

with

$$ \mathcal{F}(0)=\mathcal{F}_0 $$

Using the fractional integral and substituting ${}^{\mathrm{ABc}}{}_{0}D_{0}^{\alpha1,\beta*}$ for ${}^{\mathrm{ABR}}{}_{0}D_{0}^{\alpha1,\beta*}$, we obtain:

$$ \mathcal{F}(t)=\mathcal{F}(0)+\frac{\beta^* t^{\beta^*-1}(1-\alpha_1)}{CD(\alpha_1)}\nu(t,\mathcal{F}(t))+ $$
$$ +\frac{\alpha_1 w}{\Gamma(\alpha_1)CD(\alpha_1)}\int_0^t m^{\beta^*-1}(t-m)^{\beta^*-1}\nu(t,\mathcal{F}(t))\mathrm{d}m $$

where

$$ \mathcal{F}(t)=(v_1(t),v_2(t),v_3(t)) \text{ and } \mathcal{F}(0)=(v_1(0),v_2(0),v_3(0)) $$

Using the Banach space framework $\mathcal{J}=C\times C\times C$, where $C[0,\mathrm{K}]$ denotes the norm, we can examine the existence theorem:

$$ \|\mathcal{F}\|=\max_{t\in[0,k]}\left|v_1(t),v_2(t),v_3(t)\right| $$

We establish an operator:

$$ \Phi:\mathcal{J}\to\mathcal{J} $$

defined by

$$ \Phi(\mathcal{F})(t)=\mathcal{F}(0)+\frac{\beta^* t^{\beta^*-1}(1-\alpha_1)}{CD(\alpha_1)}\nu(t,\mathcal{F}(t))+ $$
$$ +\frac{\alpha_1 w}{\Gamma(\alpha_1)CD(\alpha_1)}\int_0^t m^{\beta^*-1}(t-m)^{\beta^*-1}\nu(t,\mathcal{F}(t))\mathrm{d}m $$

Assuming that the non-linear function $\nu(t,\mathcal{F}(t))$ satisfies the Lipschitz condition and growth requirements, we consider:

– (A1) For all $\mathcal{F}\in\mathcal{J}$, there exists a constant $C_\psi>0$, and G_ψ such that

$$ \left|\nu(t,\mathcal{F}(t))\right|\le C_\psi\left|\mathcal{F}(t)\right|+G_\psi $$

– (A2) For all $\mathcal{F}$, $\bar{\mathcal{F}}\in\mathcal{J}$, there exists a constant $H_\psi>0$ such that

$$ |\nu(t,\mathcal{F}(t))-\nu(t,\bar{\mathcal{F}}(t))|\le H_\psi\,|\mathcal{F}(t)-\bar{\mathcal{F}}(t)| $$

Assuming that Conditions (A1) and (A2) hold true, and that $\nu:[0,K]\times\mathcal{J}\to L$ is a continuous function, then the proposed model possesses a unique solution.

Proof. We will first demonstrate the continuity of. Since ν is bounded, so is Φ. Let:

$$\Omega = \{\mathcal{F} \in \mathcal{J} : \|\mathcal{F}\| \le L, L > 0\}$$

For any $\mathcal{F} \in \mathcal{J}$, we can conclude:

$$\left|\Phi(\mathcal{F})\right| = \max_{t\in[0,K]}\left|\mathcal{F}(0) + \frac{wt^{\beta^*-1}(1-\alpha_1)}{CD(\alpha_1)}\nu(t,\mathcal{F}(t)) + \right.$$

$$\left. + \frac{\alpha_1 w}{\Gamma(\alpha_1)CD(\alpha_1)}\int_0^t m^{\beta^*-1}(t-m)^{\beta^*-1}\nu(m,\mathcal{F}(m))\mathrm{d}m\right| \le$$

$$\le \mathcal{F}(0) + \frac{wK^{\beta^*-1}(1-\alpha_1)}{CD(\alpha_1)}\times(C_\psi \|\mathcal{F}\| + G_\psi) +$$

$$+ \frac{\alpha_1 w}{\Gamma(\alpha_1)CD(\alpha_1)}(C_\psi\|\mathcal{F}\| + G_\psi)K^{\alpha_1+w-1}\Phi(\alpha_1, w) \le L$$

The operator Φ is uniformly bounded by homogeneity as long as the function is $\Phi(\alpha_1, w)$. Let $t_1, t_2 \le K$ for the equicontinuity of Φ . Consider:

$$\left|\Phi(\mathcal{F})(t_2) - \Phi(\mathcal{F})(t_1)\right| = \left|\frac{wt_2^{\beta^*-1}(1-\alpha_1)}{CD(\alpha_1)}\nu\left(t_2,\mathcal{F}\left(t_2\right)\right) + \right.$$

$$+ \frac{\alpha_1 w}{\Gamma(\alpha_1)CD(\alpha_1)}\int_0^{t_2} m^{\beta^*-1}(t_2-m)^{\beta^*-1}\nu(m,\mathcal{F}(m))\mathrm{d}m -$$

$$- \frac{wt_1^{\beta^*-1}(1-\alpha_1)}{CD(\alpha_1)}\nu\left(t_1,\mathcal{F}(t_1)\right) -$$

$$\left. - \frac{\alpha_1 w}{\Gamma(\alpha_1)CD(\alpha_1)}\int_0^{t_1} m^{\beta^*-1}(t_1-m)^{\beta^*-1}\nu(m,\mathcal{F}(m))\mathrm{d}m\right| < L$$

Finally, we establish:

$$\Psi(\mathcal{F}) < 1$$

for a fixed point Φ. The aforementioned ensures that there exists a unique solution the proposed system of fractional differential equations, satisfying the conditions (A1) and (A2) as established.

Numerical scheme via AB FFO

The system (2) can be approximated using AB FFO. By applying integral approximation the right-hand side, we obtain:

$$u_1^{d+1} = u_1^0 + \frac{\varrho^* t_d^{\varrho^*-1}(1-\kappa_1)}{AB(\kappa_1)} g_1(t_d, u_1^d, u_2^d, u_3^d) +$$

$$+\frac{\kappa_1 \nu}{\Gamma(\kappa_1) AB(\kappa_1)} \sum_{c=0}^{d} \int_{t_c}^{t_{+1}} \epsilon^{\varrho^*-1}(t_{d+1}-\epsilon)^{\kappa_1-1} g_1(\epsilon, u_1, u_2, u_3) \mathrm{d}\epsilon$$

$$u_2^{d+1} = u_2^0 + \frac{\varrho^* t_d^{\varrho^*-1}(1-\kappa_1)}{AB(\kappa_1)} g_2(t_d, u_1^d, u_2^d, u_3^d) +$$

$$+\frac{\kappa_1 \nu}{\Gamma(\kappa_1) AB(\kappa_1)} \sum_{c=0}^{d} \int_{t_c}^{t_{c+1}} \epsilon^{\varrho^*-1}(t_{d+1}-\epsilon)^{\kappa_1-1} g_2(\epsilon, u_1, u_2, u_3) \mathrm{d}\epsilon$$

$$u_3^{d+1} = u_3^0 + \frac{\varrho^* t_d^{\varrho^*-1}(1-\kappa_1)}{AB(\kappa_1)} g_3(t_d, u_1^d, u_2^d, u_3^d) +$$

$$+\frac{\kappa_1 \nu}{\Gamma(\kappa_1) AB(\kappa_1)} \sum_{c=0}^{d} \int_{t_c}^{t_{c+1}} \epsilon^{\varrho^*-1}(t_{d+1}-\epsilon)^{\kappa_1-1} g_3(\epsilon, u_1, u_2, u_3) \mathrm{d}\epsilon$$

Next, we employ Lagrangian polynomial interpolation obtain the outcomes:

$$u_1^{d+1} = u_1^0 + \frac{\varrho^* t_d^{\varrho^*-1}(1-\kappa_1)}{AB(\kappa_1)} g_1(t_d, u_1^d, u_2^d, u_3^d) +$$

$$+\frac{\nu(\vartheta t)^{\kappa_1}}{AB(\kappa_1)\Gamma(\kappa_1+2)} \sum_{c=0}^{d} \Big[t_c^{\varrho^*-1} g_1(t_c, u_1^c, u_2^c, u_3^c) \cdot$$

$$\cdot\Big((d+1-c)^{\kappa_1}(d-c+2+\kappa_1)-(d-c)(2+2\kappa_1+d-c)\Big)-$$

$$-t_{c-1}^{\varrho^*-1} g_1(t_c, u_1^{c-1}, u_2^{c-1}, u_3^{c-1}) \cdot \Big((1+d-c)^{\kappa_1+1}-(d-c)^{\kappa_1} \times (1+\kappa_1+d-c)\Big]$$

$$u_2^{d+1} = u_2^0 + \frac{\varrho^* t_d^{\varrho^*-1}(1-\kappa_1)}{AB(\kappa_1)} g_2(t_d, u_1^d, u_2^d, u_3^d) + \frac{\nu(\vartheta t)^{\kappa_1}}{AB(\kappa_1)\Gamma(\kappa_1+2)} \sum_{c=0}^{d} \Big[t_c^{\varrho^*-1} g_2(t_c, u_1^c, u_2^c, u_3^c) \cdot$$

$$\cdot\Big((d+1-c)^{\kappa_1}(d-c+2+\kappa_1)-(d-c)(2+2\kappa_1+d-c)\Big)-$$

$$-t_{c-1}^{\varrho^*-1} g_2(t_c, u_1^{c-1}, u_2^{c-1}, u_3^{c-1}) \cdot \Big((1+d-c)^{\kappa_1+1}-(d-c)^{\kappa_1} \times (1+\kappa_1+d-c)\Big]$$

$$u_3^{d+1} = u_3^0 + \frac{\varrho^* t_d^{\varrho^*-1}(1-\kappa_1)}{AB(\kappa_1)} g_3(t_d, u_1^d, u_2^d, u_3^d) +$$

$$+\frac{\nu(\vartheta t)^{\kappa_1}}{AB(\kappa_1)\Gamma(\kappa_1+2)} \sum_{c=0}^{d} \Big[t_c^{\varrho^*-1} g_3(t_c, u_1^c, u_2^c, u_3^c) \cdot$$

$$\cdot\Big((d+1-c)^{\kappa_1}(d-c+2+\kappa_1)-(d-c)(2+2\kappa_1+d-c)\Big)-$$

$$-t_{c-1}^{\varrho^*-1} g_3(t_c, u_1^{c-1}, u_2^{c-1}, u_3^{c-1}) \cdot \Big((1+d-c)^{\kappa_1+1}-(d-c)^{\kappa_1} \times (1+\kappa_1+d-c)\Big]$$

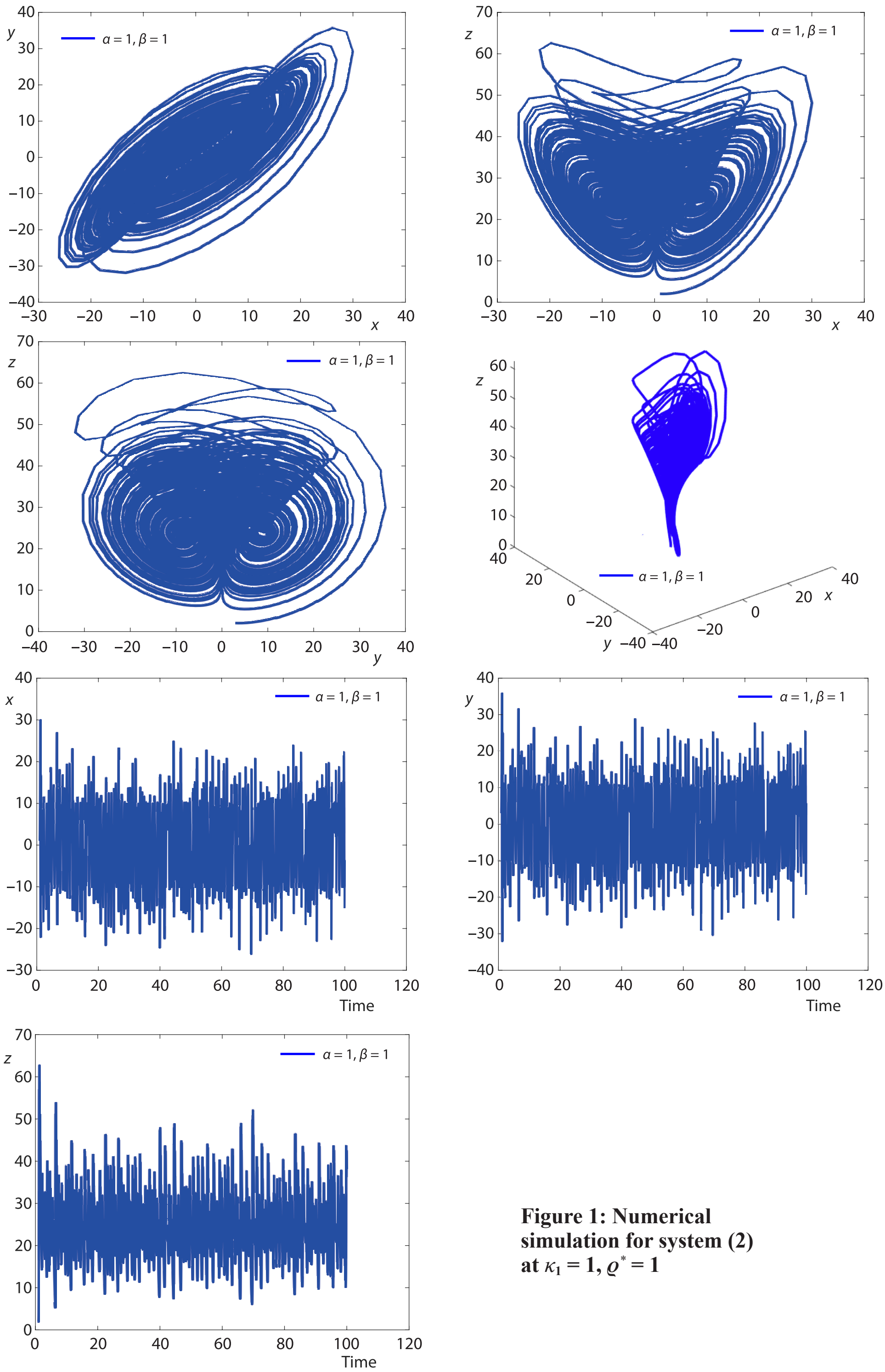

Figure 1: Numerical simulation for system (2) at $\kappa_1 = 1$, $\varrho^* = 1$

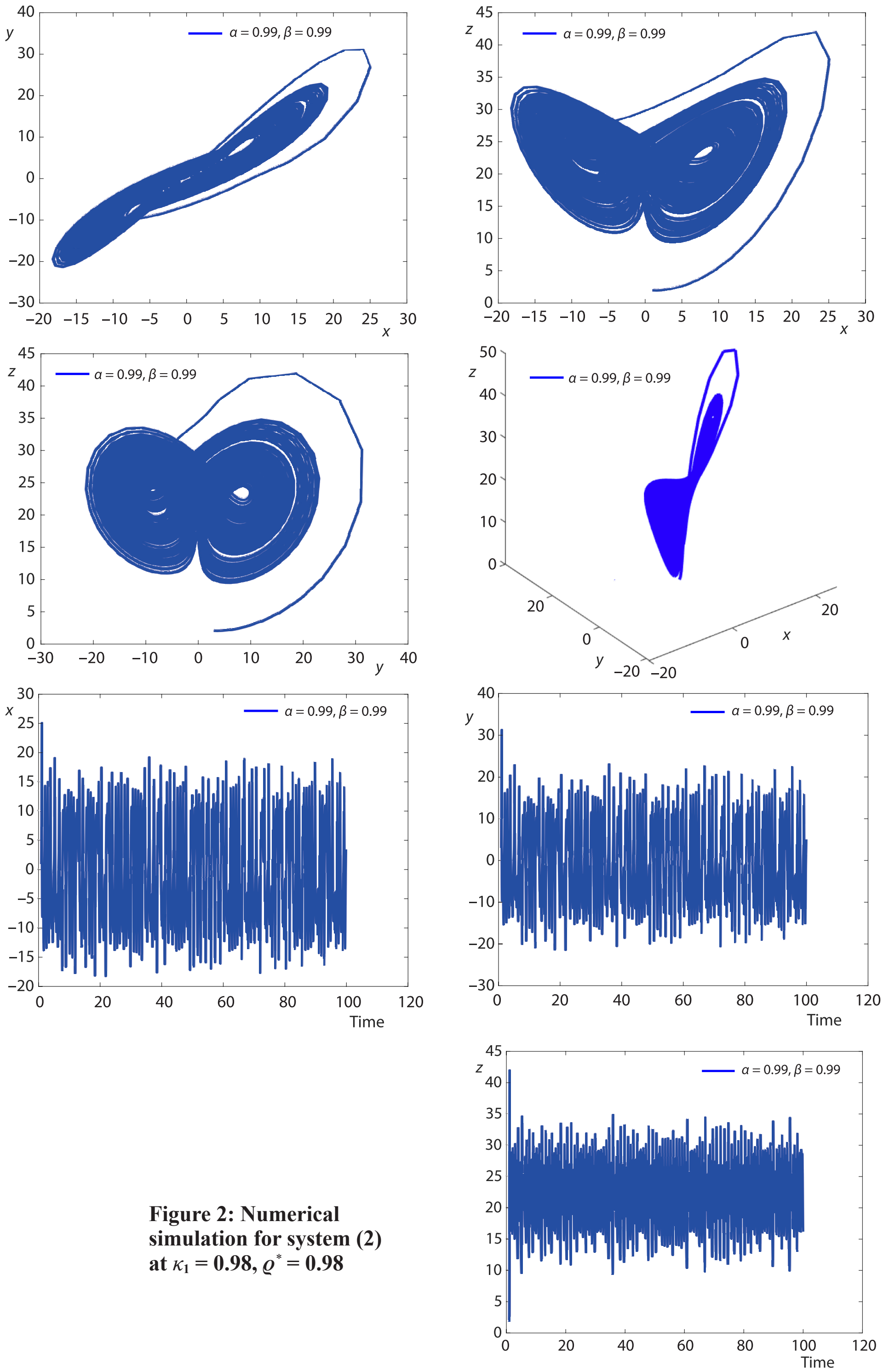

Figure 2: Numerical simulation for system (2) at $\kappa_1 = 0.98$, $\varrho^* = 0.98$

Numerical simulation and discussions

For $\vartheta \in (0.8,1]$, the numerical simulations of the fractal-fractional CS are presented in figs. 1 and 2. The chaotic dynamics of the non-linear modified CS AB FFO were implemented. The numerical simulations were performed under various parameter settings of κ_1 and ϱ^* illustrated in figs. 1 and 2, demonstrating the influence of fractal-fractional orders on the chaotic system's behavior.

Discussion

This paper explores the application of AB FFOs in analyzing chaotic behaviors within non-linear modified CS. The paper is significant as it dives into fractal-fractional calculus, a modern extension of traditional fractional calculus, to model complex dynamic systems more accurately, particularly systems exhibiting chaotic or unpredictable behaviors. The document delves into the characteristics of chaotic systems – non-linear systems sensitive to initial conditions, with dense periodic orbits, and topological mixing. This sensitivity is a defining feature of chaotic systems, where even slight changes in initial conditions can result in vastly different outcomes. The paper ties this understanding to the CS, a well-known chaotic model similar to the Lorenz system used in meteorology. One of the central tools in this research is the AB FFO, which integrates aspects of both fractal geometry and fractional calculus. These operators provide a more nuanced approach to modelling the behaviors of complex systems like the CS, capturing long-memory effects, heavy-tailed distributions, and other real-world phenomena that classical models might miss. This method is particularly useful in modelling systems with natural fractal-like behaviors. The paper presents a modification of the traditional CS equations using fractal-fractional calculus. The modified equations include parameters that account for the fractal-fractional characteristics of the system. This allows for a more accurate analysis of the system's behavior, particularly in capturing chaotic phenomena. The paper uses fixed-point theory to establish the existence and uniqueness of solutions for the modified CS. This mathematical proof ensures that the chaotic behaviors observed in the system are not merely numerical artifacts but are inherent to the system's dynamics. The research includes numerical simulations that validate the theoretical findings. These simulations help demonstrate the real-world applicability of the fractal-fractional operators in analyzing chaotic systems, providing deeper insights into the chaotic behavior of the modified CS. The simulations also help verify the model's effectiveness in various scenarios. The paper suggests that the application of fractal-fractional calculus has potential applications in biological systems, particularly those that exhibit chaotic behavior, such as certain types of oscillatory and relaxation processes in nature. Overall, this paper contributes to the broader field of chaos theory and non-linear dynamics by introducing a more sophisticated mathematical tool for analyzing complex systems, potentially improving the accuracy of models used in various scientific disciplines, from physics to biology. The use of AB operators marks a significant step forward in fractional calculus, providing a pathway to explore chaotic phenomena with greater depth and precision.

Conclusion

This paper presents a dynamical analysis of a non-linear fractal fractional CS using the AB FFO. This study on the application of AB FFO to the non-linear CS showcases a significant advancement in the modelling and analysis of chaotic systems. By incorporating fractal-fractional calculus, this research provides a more precise framework for understanding chaotic behaviors, which are highly sensitive to initial conditions and pervasive in natural and physical systems. The modified CS, when analyzed through these advanced operators, not only ensures the existence and uniqueness of solutions but also enhances the accuracy of simulations in capturing complex dynamics. The use of fixed-point theory and numerical simulations further validates the robustness of the proposed model, offering deeper insights into chaotic phenomena and opening the door to applications in various fields, including biology and physics. The integration of fractal geometry within fractional calculus, as demonstrated by the AB operators, underscores the potential of this approach for improving the understanding and representation of systems with fractal and chaotic behaviors. Overall, this research enriches the literature on chaos theory by introducing a novel method that can be applied to a wide range of non-linear systems, highlighting the effectiveness of fractal-fractional operators in capturing the intricate dynamics of complex systems.

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