NANO SIMPLY ALPHA OPEN SET AND NOVEL NEIGHBORHOOD TECHNIQUES FOR ACCURATE SYMPTOM DETECTION IN MEDICAL APPLICATIONS

by

M. El SAYED *

Department of Mathematics, College of Science and Arts,
Najran University, Najran, Saudi Arabia

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This study introduces the nano simply alpha open set and proposes a new approximation space extending Pawlak's approximation. This new space includes the nano simply alpha lower and nano simply alpha upper approximations, denoted by specific notations, offering a refined framework for analyzing data. The $\zeta^n\alpha$-lower and $\zeta^n\alpha$-upper approximations for any set Also, we study nano $\zeta^n\alpha$-rough approximation. Those investigations look at the connections between various approximation types and their characteristics, proposing methods applicable to medical diagnosis and other decision-making fields. These methods provide deeper data insights, enhancing precision and reliability in complex problem-solving. We introduce a "general neighborhood" concept, expanding on Pawlak space with a general upper and lower approximation. A case study for chronic kidney disease demonstrates the effectiveness of these methods in identifying critical symptoms. Additionally, an algorithm a, supports application for any number of patients or decision-making issues.

Key words: *rough sets, nanotopology, general neighborhood, general approximation space*

Introduction

Pawlak's [1] methodology for handling rough sets was introduced to address problems involving inexact. However, its application is limited by the reliance on equivalence relations, as it requires a complete information system. Thivagar and Richard [2] we introduce the concept of nanotopology, which encompasses notions such as nanosets that are closed, nanosets that have a nanointerior, and nanosets that have a nanoclosure. To tackle complex applied problems, various generalizations have been proposed, including similarity relations [3], pre-order relations [4], reflexive relations [5], and topological approaches [6-10], most of which satisfy Pawlak's rough set properties. Several concepts have been introduced in the literature to address uncertainty, vagueness, and ambiguity, with Pawlak's rough set theory [1] playing a significant role, especially in areas artificial intelligence encompasses methods like inductive reasoning, automated categorization, pattern identification, and learning algorithms. The conventional rough set theory begins with an equivalence relation, and there are practical applications for this theory, as demonstrated in [11]. Many researchers have presented real-life applications using nanotopological concepts such as [12, 13]. In this research, we introduce a novel type of collections in nanotopological spaces, known as nano simply α-open sets. Addi-

* Author's e-mail: mebadria@nu.edu.sa

tionally, we propose new approximations based on these nano simply α-open sets, This study contributes to the generalization of approximation spaces by establishing an extended framework for Pawlak's rough set approximations, introducing the nano simply α-lower and nano simply α-upper approximations, along with the nano simply α-boundary region of a subset. This approach provides a broader basis for analyzing data with enhanced precision, facilitating decision-making in complex scenarios such as medical diagnoses.This study defines the nano simply alpha open set within a universe shaped by an equivalence relation and examines its properties, exploring connections with existing sets.

Preliminaries

Definition 1. [14] A subset $\mathfrak{B}$ of a space $(\mathfrak{X}, \mathfrak{T})$ it is called nowhere dense if $int(cl(B) = \varphi$.

Definition 2. [12] A subset $\mathfrak{B}$ of a space $(\mathfrak{X}, \mathfrak{T})$ it is called simply α-open set if $\mathfrak{B} = \mathfrak{G} \cup \mathcal{N}$, in which $\mathfrak{G}$ is α-open set and $\mathcal{N}$ is nowhere dense.

We denoted to the class of simply α-open sets of a universe set $\mathfrak{A}$ by $\zeta^n\alpha O(U\mathfrak{U})$ and the class of simply α- closed sets by $\zeta^n\alpha C(\mathfrak{U})$, in such a way that $\zeta^n\alpha C(\mathfrak{U}) = \zeta^n\alpha O(\mathfrak{A})$.

Definition 3. [14] Let $\mathfrak{A}$ be considered a universal set, and $\mathfrak{K}$ be binary relation. Then the lower and upper approximations are defined:

$$\underline{L}\mathcal{A} = \cup\{\mathfrak{G}, \mathfrak{G} \subseteq A, \mathfrak{G} \text{ is open set}\},\ \overline{L}\mathcal{A} = \cap\{\mathfrak{F}, \mathfrak{F} \supseteq \mathcal{A}, \mathfrak{F} \text{ is closed set}\}.$$

Definition 4. [1] If $(\mathfrak{U}, \mathfrak{K})$ is an information system then we can define $\mathfrak{K}$-lower and $\mathfrak{N}$-upper approximation of X:

$$\underline{\mathfrak{K}}(X) = \cup\{Y \in \mathfrak{A} | Y \subseteq X\},\ \overline{\mathfrak{K}}(X) = \cup\{Y \in U | Y \cap X = \varphi\}$$

Definition 5. [15] For the pair $(\mathfrak{A}, \mathfrak{K})$ where $\mathfrak{U}$ is a universe set and $\mathfrak{K}$ be a binary relation, let $x\mathfrak{K}$ be an after set defined by : $x\mathfrak{K} = \{y \in \mathfrak{U} : x\mathfrak{K}y\}$.

Definition 6. [16]. A subset $\mathfrak{B}$ of a space $(\mathfrak{X}, \mathfrak{T})$ it is called α-open set if $B \subseteq int(cl(int(\mathrm{B})))$.

Proposition 1. [1]. If $(\mathfrak{U}, \mathfrak{K})$ is an information system and $\mathfrak{D}, \mathfrak{P} \subseteq \mathfrak{U}$. Then:

$$1.\ \underline{\mathfrak{K}}(\mathfrak{D}) \subseteq \mathfrak{D} \subseteq \overline{\mathfrak{K}}(\mathfrak{D})$$

$$2.\ \underline{\mathfrak{K}}(\varphi) = \overline{\mathfrak{K}}(\varphi) = \varphi \text{ and } \underline{\mathfrak{K}}(\mathfrak{U}) = \overline{\mathfrak{K}}(\mathfrak{U}) = \mathfrak{U}$$

$$3.\ \overline{\mathfrak{K}}(\mathfrak{D} \cup \mathfrak{P}) = \overline{\mathfrak{K}}(\mathfrak{D}) \cup \overline{\mathfrak{K}}(\mathfrak{P}) \text{ and } \underline{\mathfrak{K}}(\mathfrak{D} \cap \mathfrak{P}) = \underline{\mathfrak{K}}(\mathfrak{D}) \cap \underline{\mathfrak{K}}(\mathfrak{P})$$

$$4.\ \text{If } \mathfrak{D} \subseteq \mathfrak{P} \text{ then } \underline{\mathfrak{K}}(\mathfrak{D}) \subseteq \underline{\mathfrak{K}}(\mathfrak{P}) \text{ and } \overline{\mathfrak{K}}(\mathfrak{D}) \subseteq \overline{\mathfrak{K}}(\mathfrak{P})$$

$$5.\ \underline{\mathfrak{K}}(\mathfrak{D} \cup \mathfrak{P}) \supseteq \underline{\mathfrak{K}}(\mathfrak{D}) \cup \underline{\mathfrak{K}}(\mathfrak{P}) \text{ and } \overline{\mathfrak{K}}(\mathfrak{D} \cap \mathfrak{P}) \supseteq \overline{\mathfrak{K}}(\mathfrak{D}) \cap \overline{\mathfrak{K}}(\mathfrak{P})$$

$$6.\ \overline{\mathfrak{K}}(-\mathfrak{D}) = -\underline{\mathfrak{K}}(\mathfrak{D}) \text{ and } \underline{\mathfrak{K}}(-\mathfrak{D}) = -\overline{\mathfrak{K}}(\mathfrak{D})$$

$$7.\ \overline{\mathfrak{K}}(\overline{\mathfrak{K}}(\mathfrak{D}) = \underline{\mathfrak{K}}\left(\overline{\mathfrak{K}}(\mathfrak{D})\right) = \overline{\mathfrak{K}}(\mathfrak{D}), \text{ and } \underline{\mathfrak{K}}(\overline{\mathfrak{K}}(\mathfrak{D})) = \overline{\mathfrak{K}}\left(\underline{\mathfrak{K}}(\mathfrak{D})\right) = \underline{\mathfrak{K}}(\mathfrak{D})$$

where $-\mathfrak{D}$ is the complement $\mathfrak{D}$.

Nano simply α-open set

In this section we introduce new types of approximations named simply α-lower approximation and simply α-upper approximation for any subset A of space $(U, \zeta^n\alpha O(U))$.

Definition 7. A nanosubset ψ of a of nanospace $(\mathfrak{U}, \mathfrak{T}_R(\mathfrak{X}))$ it goes by the name of nano simply α-open set:

$$\alpha^n int\left(\alpha^n cl\left(\psi\right)\right) \subseteq \alpha^n cl\left(\alpha^n int\left(\psi\right)\right)$$

We denoted to the class of simply α-open sets of a universe set $\mathfrak{X}$ by $\zeta^n\alpha O(U)$. The class of simply α-closed sets by $\zeta^n\alpha C(U)$, such that $\zeta^n\alpha C(U) = \zeta^n\alpha O(U)$.

Definition 8. If $(U, \zeta^n\alpha O(U))$ is a $\zeta^n\alpha$-approximation space and for every nanosub set $A \subseteq U$. Then the nano *simply*$_\alpha$-lower and nano *simply*$_\alpha$-upper approximations defined:

$$\underset{-S\alpha}{\Psi}\left(\mathcal{A}\right) = \cup\left\{G : G \in \overset{n}{\zeta}\alpha\, O\left(U\right), G \subseteq \mathcal{A}\right\},\ \underset{S\alpha}{\overline{\Psi}}\left(\mathcal{A}\right) = \cap\left\{\mathfrak{F}, \mathfrak{F} \in \overset{n}{\zeta}\alpha\, C\left(U\right), \mathfrak{F} \supseteq \mathcal{A}\right\},$$

respectively. The accuracy of approximation of A in $(U, \zeta^n\alpha O(U))$ by

$$\mu\left(\mathcal{A}\right) = \frac{\left|\underset{-S\alpha}{B}\left(\mathcal{A}\right)\right|}{\left|\overset{-S\alpha}{B}\left(\mathcal{A}\right)\right|}$$

where $|.|$ is the cardinality of the set, $\Psi^{-s\alpha}(\mathcal{A}) \neq \varphi$. The nano simply α-border area of the set $\mathcal{A}$ (briefly $\zeta^n\alpha b(\mathcal{A})$ is

$$\overset{n}{\zeta}\alpha\, b\left(\mathcal{A}\right) = \overset{-S\alpha}{\Psi}\left(\mathcal{A}\right) - \underset{-S\alpha}{\Psi}\left(\mathcal{A}\right)$$

Remark 1. If the nano *simply*$_\alpha$-lower and nano *simply*$_\alpha$-upper approximations are identical $(\Psi_{-s\alpha}(A) = \Psi^{-s\alpha}(A))$, then the set A is definable or nano simply α-exact. Otherwise A is undefinable or nano simply α-roughly in U.

Remark 2. We can show that $o \leq \mu(A)$. If $\mu(A) = 1$, then A is definable in U. If $\mu(A)$. Then A is undefinable in U.

Definition 9. If $(\mathfrak{A}, \zeta^n\alpha O(\mathfrak{A})$ is a $\zeta^n\alpha$-approximation space and for every nanosubset $\mathcal{A} \subseteq \mathfrak{A}$.

Then:

1. If $\Psi_{-s\alpha}(\mathcal{A}) \neq \varphi$, and $B^{-s\alpha}(\mathcal{A}) \neq \mathfrak{A}$. Subsequently, $\mathcal{A}$ it is called nano simply alpha roughly (in short, $\zeta^n\alpha$-roughly) definable in $(\mathfrak{A}, \zeta^n\alpha O(\mathfrak{A}))$.
2. If $\Psi_{-s\alpha}(\mathcal{A}) \neq \varphi$, and $\Psi^{-s\alpha}(\mathcal{A}) = \mathfrak{A}$. Subsequently, $\mathcal{A}$ it is called nano simply alpha externally (in short, $\zeta^n\alpha$-undefinable in $(\mathfrak{A}, \zeta^n\alpha O(\mathfrak{A}))$.
3. If $\Psi_{-s\alpha}(\mathcal{A}) = \varphi$, and $\Psi^{-s\alpha}(\mathcal{A}) \neq \mathfrak{A}$. Subsequently, $\mathcal{A}$ it is called nanointernally (inshort, $\zeta^n\alpha$-internally) undefinable in $(U, \zeta^n\alpha O(U))$.
4. If $\Psi^{-s\alpha}(\mathcal{A}) = \varphi$, and $\Psi_{-s\alpha}(\mathcal{A}) = \mathfrak{A}$. Subsequently, $\mathcal{A}$ it is called nanototally (in short, $\zeta^n\alpha$-totally) undefinable in $(\mathfrak{A}, \zeta^n\alpha O(\mathfrak{A}))$.
5. If $\Psi_{-s\alpha}(\mathcal{A}) = \Psi^{-s\alpha}(\mathcal{A}) = \mathcal{A}$. Subsequently, $\mathcal{A}$ is nano simply alpha exact (in short, $\zeta^n\alpha$-exact) set in $(\mathfrak{A}, \zeta^n\alpha O(\mathfrak{A}))$, where $\zeta^n\alpha$-roughly (resp. $\zeta^n\alpha$-externally, $\zeta^n\alpha$-internally, $\zeta^n\alpha$-totally undefinable, and $\zeta^n\alpha$-exact) denotes to nano simply alpha roughly (resp. nano simply alpha externally, nano simply alpha internally, nano simply alpha totally, and nano simply alpha exact) sets. The next illustration demonstrates the classification of nano simply rough sets briefly ($\zeta^n\alpha$-rough).

Example 1. Let

$$\mathfrak{U} = \left\{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}\right\},\ \mathfrak{U}/R = \left\{\left\{\mathfrak{a}\right\}, \left\{\mathfrak{b}, \mathfrak{c}\right\}, \left\{d\right\}\right\}, \mathfrak{X} = \left\{\mathfrak{a}, b\right\} \text{ and } d$$

$$\overline{R}\left(\mathfrak{X}\right) = \left\{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\right\}, \underline{R}\left(\mathfrak{X}\right) = \left\{\mathfrak{a}\right\}, b\left(A\right) = \left\{\mathfrak{b}, \mathfrak{c}\right\}$$

the nanotopology with respect to $\mathfrak{X}$ on $\mathfrak{U}$ is

$$\tau(\mathfrak{X})_R = \{\mathfrak{U}, \varphi, \{\mathfrak{a}\}, \{\mathfrak{b}, \mathfrak{c}\}, \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}\}$$

Let $\mathcal{A} = \{\mathfrak{a}\}$ and $\mathfrak{B} = \{\mathfrak{a}, \mathfrak{d}\}$ be two sets we show the aforementioned concepts are examples of $\zeta^n\alpha$-roughly definable. The corresponding $\zeta^n\alpha$-approximations, bound aries and the accuracies:

$$\underset{-S\alpha}{\Psi}(\mathcal{A}) = \{\mathfrak{a}\},\ \overset{-S\alpha}{\Psi}(\mathcal{A}) = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\},\ \overset{n}{\zeta}\alpha\, b(\mathcal{A}) = \{\mathfrak{b}, c\},\ \mu(\mathcal{A}) = 1/3 = 0.33,$$

$$\underset{-s\alpha}{\Psi}(\mathfrak{B}) = \{\mathfrak{a}\},\ \overline{\Psi}_{s\alpha}(\mathfrak{B}) = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\},\quad \overset{n}{\zeta}\alpha\, b(\mathfrak{B}) = \{b, \mathfrak{c}\},\ \mu(\mathfrak{B}) = 0.33.$$

The sets $\mathcal{C} = \{\mathfrak{a}, \mathfrak{c}, \mathfrak{d}\}$ and $\mathfrak{D} = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{d}\}$ are examples of $\zeta^n\alpha$-exter nally undefinable. We have the corresponding $\zeta^n\alpha$-approximations, boundaries and the accuracies:

$$\underset{-S\alpha}{\Psi}(\mathfrak{C}) = \{\mathfrak{a}, \mathfrak{d}\},\ \overset{-S\alpha}{\Psi}(\mathfrak{C}) = \mathfrak{U},\ \overset{n}{\zeta}\alpha\, b(\mathfrak{C}) = \{\mathfrak{b}, \mathfrak{c}\},$$

$$\mu(\mathfrak{C}) = 0.50,\ \underset{-S\alpha}{\Psi}(\mathfrak{D}) = \{\mathfrak{a}, \mathfrak{d}\},\ \overset{-S\alpha}{\Psi}(\mathfrak{D}) = \mathfrak{U},\ \overset{n}{\zeta}\alpha\, b(\mathfrak{D}),\ \mu(\mathfrak{D}) = 0.50.$$

Also if we take the partition$\{\{\mathfrak{a}, \mathfrak{d}\}, \{\mathfrak{b}, \mathfrak{c}\}\}$. Then the sets $\mathfrak{U} = \{\mathfrak{c}\}$ and $\mathfrak{b} = \{\mathfrak{d}\}$ are examples of $\zeta^n\alpha$-internally undefinable. The corresponding $\zeta^n\alpha$-approximations, boundaries and the accuracies:

$$\underset{-S\alpha}{\Psi}(\mathfrak{U}) = \varphi,\ \overset{-S\alpha}{\Psi}(\mathfrak{U}) = \{\mathfrak{b}, \mathfrak{c}\}.$$

The sets $\mathfrak{E} = \{\mathfrak{c}, \mathfrak{d}\}$ and $\mathfrak{F} = \{\mathfrak{a}, \mathfrak{b}\}$ are examples of $\zeta^n\alpha$-totally undefinable. The corre sponding $\zeta^n\alpha$-approximations, boundaries and the accuracies are:

$$\underset{-S\alpha}{\Psi}(\mathfrak{E}) = \varphi, \overset{n}{\overline{\Psi}}_{S\alpha}(\mathfrak{E}) = U,\ \zeta\alpha\, b(\mathfrak{E}) = U,\ \mu(\mathfrak{E}) = 0,\ \underset{-S\alpha}{\Psi}(\mathfrak{F}) = \varphi$$

$$\overset{-S\alpha}{\Psi}(\mathfrak{F}) = U,\ \overset{n}{\zeta}\alpha\, b(\mathfrak{F}) = U,\ \mu(\mathfrak{F}) = 0$$

Remark 3. If $(U, \zeta^n\alpha O(U))$ is a $\zeta^n\alpha$-approximation space and for every na nosubset $A \subseteq U$. Then:

$$\underline{L}\mathcal{A} \subseteq \overline{L}(\mathcal{A}) \subseteq \overset{-S\alpha}{\Psi}(\mathcal{A})$$

The following example indicates this remark.

Example 2. Let $\mathfrak{U} = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}\}$ the partition of Pawlak:

$$\frac{\mathfrak{A}}{\mathfrak{N}} = \{\{\mathfrak{a}\}, \{\mathfrak{b}\}, \{\mathfrak{c}, \mathfrak{d}\}\},$$

and $\mathfrak{X} = \{\mathfrak{a}, \mathfrak{c}\}$ the class of subsets of $\mathfrak{A}$. Then the nanotopology with respect to $\mathfrak{X}$ over $\mathfrak{U}$ is

$$\mathcal{T}_{\mathfrak{K}}(\mathfrak{X}) = \{\mathfrak{A}, \varnothing, \{\mathfrak{a}\}, \{\mathfrak{c}, \mathfrak{d}\}, \{\mathfrak{a}, \mathfrak{c}, \mathfrak{d}\}\}.$$

Then let $\mathfrak{D} = \{\mathfrak{b}\}$, then we see that $\{\mathfrak{b}\} \in \underline{\Psi}_{-s\alpha}(\mathfrak{A})$, and $\{\mathfrak{a}, \mathfrak{b}\} \in \Psi^{-s\alpha}(\mathfrak{D})$ but $\{\mathfrak{b}\} \notin \overline{L}(\mathfrak{D})$ and $\varphi \in \underline{L}(\mathfrak{D})$). Where $\Psi_{-s\alpha}(\mathfrak{D})$, $\Psi^{-s\alpha}(\mathfrak{D})$, $\underline{L}(\mathfrak{D})$, and $\overline{L}(\mathfrak{D})$ are the family of all lower and upper approximations for every $\mathfrak{D} \subseteq \mathfrak{A}$.

Proposition 2. The complement of all nano simply alpha open sets in any nanotopological space ($\mathfrak{A}$, τ_R ($\mathfrak{X}$)) are nano simply alpha open sets. Moreover, finite intersection of nano simply alpha open sets is nano simply alpha open set.

Proof. Obvious.

Proposition 3. If (U, $\zeta^n\alpha O(U)$) is a $\zeta^n\alpha$-approximation space and for every nanosubset $A \subseteq U$. Then:

1. A set A is nano simply alpha definable (resp. nano simply alpha roughly definable, nano simply alpha totally undefinable) if and only if so is A^C.
2. A set A is nano simply alpha externally (resp. nano simply alpha internally undefinable) if and only if A^C is nano simply internally (resp. nano simply alpha externally) undefinable where, A^C denoted to the complement of the set A.

Proof.

i. If A is nano, simply alpha definable. Then:

$$A = \underset{-S\alpha}{\Psi}(A) = \overset{-S\alpha}{\Psi}(A).$$

By taken the complement for the sides we have:

$$A^C = (\underset{-S\alpha}{\Psi}(A))^C = (\overset{-S\alpha}{\Psi}(A))^C$$

if and only if. Then A^C is also nano simply alpha definable.

ii. If A is a nano simply alpha roughly definable, then $\Psi_{-s\alpha}(A) \neq \varphi$ and $\Psi^{-s\alpha}(A) \neq U$ since $\Psi(A) \neq \varphi$. Then there exists $x \in U$ and nano simply alpha open set G such that $x \in G \subseteq A$ if and only if $U - G \supseteq U - A$ if and only if $A^C = U$ equivalently $\Psi^{-s\alpha}(A) = U$, where $U - G$ is nano simply alpha closed set. Similarly $\Psi^{-s\alpha}(A) = U$ equivalently there exists $x \in U$ and F is nano simply alpha closed set such that $x \in F \supseteq A$ if and only if $U - F \subseteq U - A$ if and only if $A^C \neq \varphi$ if and only if $\Psi_{-s\alpha}(A)^C$ and $\Psi^{-s\alpha}(A)^C \neq \varphi$. Subsequently, A^C is nano simply alpha roughly definable.

iii. Since A^C is nano simply alpha totally undefinable. Then $\Psi_{-s\alpha}(A) = \varphi$ and $\Psi^{-s\alpha}(A) = U$. Then by taken the complement for both sides we get:

$$(\underset{-S\alpha}{\Psi}(A))^C = X,\ (\overset{-S\alpha}{\Psi}(A))^C = \varphi$$

hence $\Psi_{-s\alpha}(A) = \phi = \varphi$, $\Psi^{-s\alpha}(A) = U$, if and only if A^C is nano simply alpha totally undefinable.

Similarly, as *i*.

*Proposition 4***.** If ($\mathfrak{A}$, $\zeta^n\alpha O(\mathfrak{A})$) is a $\zeta^n\alpha$-approximation space and for every nanosubset $\mathfrak{M} \subseteq \mathfrak{A}$. Then:

$$1.\ \underset{-S\alpha}{\Psi}(\mathfrak{M}) \subseteq \mathfrak{M} \subseteq \overset{-S\alpha}{\Psi}(\mathfrak{M})$$

$$2.\ \overset{-S\alpha}{\Psi}(\mathfrak{M} \cup \mathfrak{P}) = \overset{-S\alpha}{\Psi}(\mathfrak{M} \cup \overset{-S\alpha}{\Psi}(\mathfrak{P})$$

$$3.\ \underset{-S\alpha}{\Psi}(\mathfrak{M} \cap \mathfrak{P}) = \underset{-S\alpha}{\Psi}(\mathfrak{M}) \cap \underset{-S\alpha}{\Psi}(\mathfrak{P})$$

$$4.\ \mathfrak{M} \subseteq \mathfrak{P} \text{ then } \underset{-S\alpha}{\Psi}(\mathfrak{M}) \subseteq \underset{-S\alpha}{\Psi}(\mathfrak{B}) \text{ and } \overset{-S\alpha}{\Psi}(\mathfrak{M}) \subseteq \overset{-S\alpha}{\Psi}(\mathfrak{P})$$

$$5.\ \overset{-S\alpha}{\Psi}(\mathfrak{M} \cup \mathfrak{P}) \supseteq \overset{-S\alpha}{\Psi}(\mathfrak{M}) \cup \overset{-S\alpha}{\Psi}(\mathfrak{P})$$

$$6.\ \overset{-S\alpha}{\Psi}(\mathfrak{M}) = \left(\underset{-S\alpha}{\Psi}(\mathfrak{M}^C)\right)^C$$

$$7.\ \underset{-S\alpha}{\Psi}(\mathfrak{M}) = \left(\overset{-S\alpha}{\Psi}\left(\mathfrak{M}^C\right)\right)^C$$

$$8.\ \underset{-S\alpha}{\Psi}\left(\underset{-S\alpha}{\Psi}(\mathfrak{M})\right) = \underset{-S\alpha}{\Psi}(\mathfrak{M})$$

$$9.\ \overset{-S\alpha}{\Psi}\left(\overset{-S\alpha}{\Psi}(\mathfrak{M})\right) = \overset{-S\alpha}{\Psi}(\mathfrak{M})$$

$$10.\ \overset{-S\alpha}{\Psi}(\mathfrak{M}\cap\mathfrak{P}) \subseteq \overset{-S\alpha}{\Psi}(\mathfrak{M}) \cap \overset{-S\alpha}{\Psi}(\mathfrak{P})$$

$$11.\ \underset{-S\alpha}{\Psi}(\phi) = \overset{-S\alpha}{\Psi}(\phi) = \phi \text{ and } \underset{-S\alpha}{\Psi}(\mathfrak{A}) = \overset{-S\alpha}{\Psi}(\mathfrak{A}) = \mathfrak{A}$$

Proof. 1 and 2 are obvious.

3. Let $x \in \Psi^{-S\alpha}(\mathfrak{M} \cap \mathfrak{P})$. Then there exists nano simply alpha open set G such that:

$$x \in \cup\left\{G : G \subseteq \mathfrak{M}\cap\mathfrak{B}, G \in \overset{n}{\zeta}\alpha O(\mathfrak{U})\right\}$$

if and only if

$$x \in \cup\left\{G : G \subseteq \mathfrak{M},\ G \in \overset{n}{\zeta}\alpha O(\mathfrak{U})\right\} \text{ and } x \in \cup\left\{G : G \subseteq \mathfrak{B}, G \in \overset{n}{\zeta}\alpha O(\mathfrak{U})\right\}$$

if and only if

$$x \in \overset{-S\alpha}{\Psi}(\mathfrak{M}), x \in \overset{-S\alpha}{\Psi}(\mathfrak{B}).$$

4. and 5. are obvious.

6. Since

$$\left(\underset{-S\alpha}{\Psi}(\mathfrak{M})^C\right)^C = \mathfrak{U} - \cup\left\{G : G \subseteq U - \mathfrak{M}, G \in \overset{n}{\zeta}\alpha O(\mathfrak{U})\right\} =$$
$$= \cap\left\{\mathfrak{U} - G : \mathfrak{U} - G \subseteq U - (\mathfrak{U} - \mathfrak{M}), \mathfrak{U} - G \in \overset{n}{\zeta}\alpha C(\mathfrak{U})\right\}$$

Then:

$$\overset{-S\alpha}{\Psi}(\mathfrak{M}) = \left(\underset{-S\alpha}{\Psi}\left(\mathfrak{M}^C\right)\right)^C$$

7. Let:

$$\left(\overset{-S\alpha}{\Psi}\left(\mathfrak{M}^C\right)\right)^C = \left(\overset{-S\alpha}{\Psi}\left(\mathfrak{M}^C\right)\right)^C = \mathfrak{U} - \cap\left\{\mathfrak{F} : \mathfrak{F} \supseteq \mathfrak{M}^C, \mathfrak{F} \in \overset{n}{\zeta}\alpha C(\mathfrak{U})\right\}^C =$$
$$= \cup\left\{\mathfrak{U} - \mathfrak{F} : \mathfrak{U} - \mathfrak{F} \subseteq \mathfrak{U} - (\mathfrak{U} - \mathfrak{M}), \mathfrak{U} - \mathfrak{F} \in \overset{n}{\zeta}\alpha O(\mathfrak{U})\right\} = \underset{-S\alpha}{\Psi}(\mathfrak{M}).$$

Then:

$$\left(\overset{-S\alpha}{\Psi}(\mathfrak{M}^C)\right)^C = \underset{-S\alpha}{\Psi}(\mathfrak{M})$$

8. Let:

$$\underset{-S\alpha}{\Psi}\left(\underset{-S\alpha}{\Psi}(\mathfrak{M})\right)=\cup\left\{G:G\subseteq\underset{-S\alpha}{\Psi}(\mathfrak{M})\subseteq\mathfrak{M},G\in\overset{n}{\zeta}\alpha\,O(\mathfrak{U})\right\}=$$

$$=\cup\left\{G:G\subseteq\underset{-S\alpha}{\Psi}(\mathfrak{M})\subseteq\mathfrak{M},G\in\overset{n}{\zeta}\alpha\,O(\mathfrak{U})\right\}=$$

$$=\cup\left\{G:G\subseteq\underset{-S\alpha}{\Psi}(\mathfrak{M})\subseteq\mathfrak{M},G\in\overset{n}{\zeta}\alpha\,O(\mathfrak{U})\right\}=\underset{-S\alpha}{\Psi}(\mathfrak{M})$$

Hence:

$$\underset{-S\alpha}{\Psi}\left(\underset{-S\alpha}{\Psi}(\mathfrak{M})\right)=\underset{-S\alpha}{\Psi}(\mathfrak{M})$$

9. Let:

$$\overset{-S\alpha}{\Psi}\left(\overset{-S\alpha}{\Psi}(\mathfrak{M})\right)=\cup\left\{\mathfrak{F}:\mathfrak{F}\supseteq\overset{-S\alpha}{\Psi}(\mathfrak{M})\supseteq\mathfrak{M},\mathfrak{F}\in\overset{n}{\zeta}\alpha\,C(\mathfrak{U})\right\}=\overset{-S\alpha}{\Psi}(\mathfrak{M})$$

Thus:

$$\overset{-S\alpha}{\Psi}\left(\overset{-S\alpha}{\Psi}(\mathfrak{M})\right)=\overset{-S\alpha}{\Psi}(\mathfrak{M})$$

10. Obvious.
11. Since $\mathfrak{X}$ and φ are nano simply exact.
Then:

$$\underset{-S\alpha}{\Psi}(\mathfrak{A})=\overset{-S\alpha}{\Psi}(\mathfrak{A})=U,\quad \underset{-S\alpha}{\Psi}(\varphi)=\overset{-S\alpha}{\Psi}(\varphi)=\varphi$$

Proposition 5. If $(\mathfrak{U}, \zeta^n\alpha O(\mathfrak{U}))$ is a $\zeta^n\alpha$-approximation space and for every nanosubset $A, B \subseteq U$. Then the following statement are not hold:

1. $\overset{-S\alpha}{\Psi}(\mathfrak{M}-\mathfrak{B})\subseteq\overset{-S\alpha}{\Psi}(\mathfrak{M})-\overset{-S\alpha}{\Psi}(\mathfrak{B})$
2. $\underset{-S\alpha}{\Psi}(\mathfrak{M}-\mathfrak{B})\subseteq\underset{-S\alpha}{\Psi}(\mathfrak{M})-\underset{-S\alpha}{\Psi}(\mathfrak{B})$

Proof. Since:

$$\overset{-S\alpha}{\Psi}(\mathfrak{M}-\mathfrak{B})=\overset{-S\alpha}{\Psi}\left(\mathfrak{M}\cap\mathfrak{B}^C\right)\subseteq\overset{-S\alpha}{\Psi}(\mathfrak{M})\cap\overset{-S\alpha}{\Psi}(\mathfrak{B}^C)=$$

$$=\overset{-S\alpha}{\Psi}(\mathfrak{M})\cap\left(\overset{-S\alpha}{\Psi}(\mathfrak{B})\right)^C=\overset{-S\alpha}{\Psi}(\mathfrak{M})-\underset{-S\alpha}{\Psi}(\mathfrak{B})$$

Then

$$\overset{-S\alpha}{\Psi}(\mathfrak{M}-\mathfrak{B})\subseteq\overset{-S\alpha}{\Psi}(\mathfrak{M})-\overset{-S\alpha}{\Psi}(\mathfrak{B})$$

1. Since

$$\underset{-S\alpha}{\Psi}(\mathfrak{M}-\mathfrak{B})=\underset{-S\alpha}{\Psi}\left(\mathfrak{M}\cap\mathfrak{B}^C\right)\subseteq\underset{-S\alpha}{\Psi}(\mathfrak{M})\cap\underset{-S\alpha}{\Psi}\left(\mathfrak{B}^C\right)=$$

$$=\underset{-S\alpha}{\Psi}(\mathfrak{M})\cap(\underset{-S\alpha}{\Psi}(\mathfrak{B}))^C=\underset{-S\alpha}{\Psi}(\mathfrak{M})-\underset{-S\alpha}{\Psi}(\mathfrak{B})$$

Therefore:

$$\underset{-S\alpha}{\Psi}(\mathfrak{M}-\mathfrak{V}) \subseteq \underset{-S\alpha}{\Psi}(\mathfrak{M}) - \underset{-S\alpha}{\Psi}(\mathfrak{V}).$$

Proposition 6. If $(U, \zeta^n\alpha O(U))$ is a ζ^n-approximation space and for every nano subset $A, B \subseteq U$. Then:

$$1.\ [\overset{-S\alpha}{\Psi}(\mathfrak{M}) \cup \overset{-S\alpha}{\Psi}(\mathfrak{V})]^C = \underset{-S\alpha}{\Psi}(\mathfrak{M})^C \cap \underset{-S\alpha}{\Psi}(\mathfrak{V})^C$$

$$2. \Big[\underset{-S\alpha}{\Psi}(\mathfrak{M}) \cup \Big[\overset{-S\alpha}{\Psi}(\mathfrak{V})]^C = \overset{-S\alpha}{\Psi}(\mathfrak{M})^C \cap \underset{-S\alpha}{\Psi}(\mathfrak{V})^C$$

$$3.\ [\overset{-S\alpha}{\Psi}(\mathfrak{M}) \cup \underset{-S\alpha}{\Psi}(\mathfrak{V})]^C = \underset{-S\alpha}{\Psi}(\mathfrak{M})^C \cap \overset{-S\alpha}{\Psi}(\mathfrak{V})^C$$

$$4.\ [\underset{-S\alpha}{\Psi}(\mathfrak{M}) \cup \underset{-S\alpha}{\Psi}(\mathfrak{V})]^C = \overset{-S\alpha}{\Psi}(\mathfrak{M})^C \cap \overset{-S\alpha}{\Psi}(\mathfrak{V})^C$$

$$5.\ [\overset{-S\alpha}{\Psi}(\mathfrak{M}) \cup \overset{-S\alpha}{\Psi}(\mathfrak{V})]^C = \underset{-S\alpha}{\Psi}(\mathfrak{M})^C \cap \underset{-S\alpha}{\Psi}(\mathfrak{V})^C$$

$$6.\ [\overset{-S\alpha}{\Psi}(\mathfrak{M}) \cap \underset{-S\alpha}{\Psi}(\mathfrak{V})]^C = \underset{-S\alpha}{\Psi}(\mathfrak{M})^C \cup \overset{-S\alpha}{\Psi}(\mathfrak{V})^C$$

$$7.\ [\underset{-S\alpha}{\Psi}(\mathfrak{M}) \cup \underset{-S\alpha}{\Psi}(\mathfrak{V})]^C = \overset{-S\alpha}{\Psi}(\mathfrak{M})^C \cap \underset{-S\alpha}{\Psi}(\mathfrak{V})^C$$

$$8.\ [\underset{-S\alpha}{\Psi}(\mathcal{A}\mathfrak{M}) \cap \underset{-S\alpha}{\Psi}(B)]^C = \overset{-S\alpha}{\Psi}(\mathfrak{M})^C \cup \overset{-S\alpha}{\Psi}(\mathfrak{V})^C$$

Proof. Obvious.

Proposition 7. If $(\mathfrak{A}, \zeta^n\alpha O(\mathfrak{A}))$ is a ζ^n-approximation space and for every nanosubset $\mathfrak{M} \subseteq \mathfrak{A}$ then:

$$1.\ \underset{-S\alpha}{\Psi}(\mathfrak{M}) \subseteq \overset{-S\alpha}{\Psi}(\underset{-S\alpha}{\Psi}(\mathfrak{M})$$

$$2.\ \underset{-S\alpha}{\Psi}\left(\overset{-S\alpha}{\Psi}\mathfrak{M}\right) \subseteq \overset{-S\alpha}{\Psi}(\mathfrak{M})$$

Proof. Since:

$$\underset{-S\alpha}{\Psi}(\mathfrak{M}) \subseteq \mathcal{A} \subseteq \overset{-S\alpha}{\Psi}(\mathfrak{M})$$

Then:

$$\underset{-S\alpha}{\Psi}\left(\underset{-S\alpha}{\Psi}(\mathfrak{M})\right) \subseteq \underset{-S\alpha}{\Psi}(\mathcal{A}) \subseteq \overset{-S\alpha}{\Psi}\left(\underset{-S\alpha}{\Psi}(\mathfrak{M})\right),\ \text{and hence}\ \underset{-S\alpha}{\Psi}(\mathfrak{M}) \subseteq \overset{-S\alpha}{\Psi}\left(\underset{-S\alpha}{\Psi}(\mathcal{A})\right)$$

1. Since:

$$\underset{-S\alpha}{\Psi}\left(\overset{-S\alpha}{\Psi}(\mathfrak{M})\right) \subseteq \overset{-S\alpha}{\Psi}(\mathfrak{M}) \subseteq \overset{-S\alpha}{\Psi}\left(\overset{-S\alpha}{\Psi}(\mathfrak{M})\right)$$

Then we get:

$$\underset{-S\alpha}{\Psi}\left(\overset{-S\alpha}{\Psi}(\mathfrak{M})\right) \subseteq \overset{-S\alpha}{\Psi}(\mathfrak{M})$$

Proposition 8. If $(\mathfrak{A}, \zeta^n\alpha O(\mathfrak{A}))$ is a $\zeta^n\alpha$-approximation space and for every nanosubset $\mathfrak{M} \subseteq \mathfrak{A}$. Then:

1. $\underset{-S\alpha}{\Psi}(\mathfrak{M}) \cap \underset{-S\alpha}{\Psi}(\mathfrak{M}^C) = \varphi$
2. $\underset{-S\alpha}{\Psi}(\mathfrak{M}) \cap \overset{-S\alpha}{\Psi}(\mathfrak{M}^C) = \varphi$
3. $\underset{-S\alpha}{\Psi}(\mathfrak{M}^C) \cap \overset{-S\alpha}{\Psi}(\mathfrak{M}) = \varphi$
4. $\overset{-S\alpha}{\Psi}(\mathfrak{M}) \cup \overset{-S\alpha}{\Psi}(\mathfrak{M}^C) = \mathfrak{A}$
5. $\overset{-S\alpha}{\Psi}(\mathfrak{M}^C) \cup \underset{-S\alpha}{\Psi}(\mathfrak{M}) = \mathfrak{A}$
6. $\underset{-S\alpha}{\Psi}(\mathfrak{M}^C) \cup \overset{-S\alpha}{\Psi}(\mathfrak{M}) = \mathfrak{A}$

Proof. Since:

$$\underline{\Psi}_{s\alpha}(\mathfrak{M}) \cap \underline{\Psi}_{s}(\mathfrak{M}^C) \subseteq \mathfrak{M} \cap \mathfrak{M}^C = \varphi$$

But:

$$\varphi \subseteq \underline{\Psi}_{s\alpha}(\mathfrak{M}) \cap \underline{\psi}_{s\alpha}(\mathfrak{M}^C)$$

Therefore:

$$\Psi_{s\alpha}(\mathfrak{M}) \cap \underline{\Psi}_{s\alpha}(\mathfrak{M}^C) = \varphi$$

1. Since:

$$\bar{\Psi}^{s\alpha}(\mathfrak{M}^C) = (\underline{\Psi}_{s\alpha}(\mathfrak{M}))^C, \text{ then } \underline{\Psi}_{s\alpha}(\mathfrak{M}) \cap (\underline{\Psi}_{s\alpha}(\mathfrak{M}))^C = \varphi$$
$$\text{and hence } \underline{\psi}_{s\alpha}(\mathfrak{M}) \cap \bar{\psi}^{s\alpha}(\mathfrak{M}^C) = \varphi$$

2. Since:

$$(\bar{\Psi}^{s\alpha}(\mathfrak{M}))^C = \underline{\Psi}_{s\alpha}(\mathfrak{M})^C, \text{ then } \underline{\Psi}_{s\alpha}(\mathfrak{M}^C) \cap \bar{\Psi}^{s\alpha}(\mathfrak{M}) = (\bar{\Psi}^{s\alpha}(\mathfrak{M}))^C \cap \bar{\Psi}^{s\alpha}(\mathfrak{M}) = \varphi$$
$$\text{and then } \underline{\Psi}_{s\alpha}(\mathfrak{M}^C) \cap \bar{\Psi}^{s\alpha}(\mathfrak{M}) = \varphi$$

3. Since:

$$\bar{\Psi}^{s\alpha}(\mathfrak{M})\bar{\Psi}^{s\alpha}(\mathfrak{M}) \cup \bar{\Psi}^{s\alpha}(\mathfrak{M}^C) \supseteq \mathfrak{M} \cup \mathfrak{M}^C \supseteq \mathfrak{A}., \text{ then}$$
$$\mathfrak{A} \subseteq \bar{\Psi}^{s\alpha}(\mathfrak{M}) \cup \bar{\Psi}^{s\alpha}(\mathfrak{M}^C), \text{ but } \bar{\Psi}^{s\alpha}(\mathfrak{M}) \cup \bar{\Psi}^{s\alpha}(\mathfrak{M}^C) \subseteq \mathfrak{A}$$
$$\text{and therefore } [\bar{\Psi}^{s\alpha}(\mathfrak{M}^C)] \cup [\underline{\Psi}_{s\alpha}(\mathfrak{M})] = \mathfrak{A}$$

4. Since:

$$\bar{\Psi}^{s\alpha}(\mathfrak{M}^C) = (\underline{\Psi}_{s\alpha}(\mathfrak{M}))^C, \text{ then } \underline{\Psi}_{s\alpha}(\mathfrak{M}) \cap \bar{\Psi}^{s\alpha}(\mathfrak{M}^C) = \underline{\Psi}_{s\alpha}(\mathfrak{M}) \cup (\underline{\Psi}_{s\alpha}(\mathfrak{M}))^C = \mathfrak{A}$$

5. Since

$$\underline{\Psi}_{s\alpha}\left(\mathfrak{M}^C\right)=\left(\overline{\Psi}^{s\alpha}\left(\mathfrak{M}\right)\right)^C,\ \left(\overline{\Psi}^{s\alpha}\left(\mathfrak{M}\right)\right)^C$$

$$\text{then }\ \underline{\Psi}_{s\alpha}\left(\mathfrak{M}^C\right)\cap\overline{\Psi}^{s\alpha}\left(\mathfrak{M}\right)=\left(\overline{\Psi}^{s\alpha}\left(\mathfrak{M}\right)\right)^C\cup\overline{\Psi}^{s\alpha}\left(\mathfrak{M}\right)=\mathfrak{A}$$

Proposition 9. If $(U, \zeta^n\alpha O(U))$ is a $\zeta^n\alpha$-approximation space and for every nanosubset $A \subseteq U$. Then:

1. The A is nano $\zeta^n\alpha$-exact $\Leftrightarrow \zeta^n\alpha b(A) = \varphi$.
2. The A is nano $\zeta^n\alpha$-rough $\Leftrightarrow \zeta^n\alpha b(A) \neq \varphi$.

Proof. Let A is nano $\zeta^n\alpha$-exact. Then $\Psi_{-s\alpha}(A) = \Psi^{-s\alpha}(A) = A$. Hence $\zeta^n\alpha b(A) = \varphi$.

Conversely, let $\zeta^n\alpha b(A) = \varphi$. There is two cases:

– $\Psi_{-s\alpha}(A) = \Psi^{-s\alpha}(A)$.
– $\Psi_{-s\alpha}(A) \supseteq \Psi^{-s\alpha}(A)$ in case give $\zeta^n\alpha b(A) = \varphi$, and hence A is nano $\zeta^n\alpha$-exact and Case 2 it is impossible.

1. Obvious.

Proposition 10. If $(\mathfrak{A}, \zeta^n\alpha O(\mathfrak{A}))$ is a $\zeta^n\alpha$-approximation space and for every nanosubset $\mathfrak{M} \subseteq \mathfrak{A}$. Then:

$$1.\ \underset{-S\alpha}{\Psi}\left(\mathfrak{M}\right)\cap\overset{-S\alpha}{\Psi}\left(\mathfrak{M}\right)^C=[\overset{n}{\zeta}\alpha\, b\left(\mathfrak{M}\right)]^c$$

$$2.\ \overset{-S\alpha}{\Psi}\left(\mathfrak{M}\right)\cap\overset{-S\alpha}{\Psi}\left(\mathcal{A}\right)^C=\overset{n}{\zeta}\alpha\, b\left(\mathfrak{M}\right)$$

Proof. Obvious.

Proposition 11. If $(U, \zeta^n\alpha O(U))$ is a ζ^n-approximation space and for every nanosubset $A \subseteq U$. Then:

$$1.\ [\underset{-S\alpha}{\Psi}\left(\mathcal{A}\right)]^C=\overset{-S\alpha}{\Psi}\left(\mathcal{A}\right)^C$$

$$2.\ [\overset{-S\alpha}{\Psi}\left(\mathcal{A}\right)]^C=\underset{-S\alpha}{\Psi}\left(\mathcal{A}\right)^C$$

Proof. Since:

$$\left[\underset{-S\alpha}{\Psi}\left(\mathcal{A}\right)\right]^C=\mathfrak{A}-\cup\left\{G:G\subseteq\mathcal{A},G\in\overset{M}{S}\alpha\left(\mathfrak{A}\right)\right\}=$$

$$=\cap\left\{U-G:\mathfrak{A}-G\supseteq\mathfrak{A}-\mathcal{A},\mathfrak{A}-G\in\overset{M}{S}\alpha C\left(\mathfrak{A}\right)\right\}=\overset{-S\alpha}{\Psi}\left(\mathcal{A}\right)^C,\ \text{then}\ \left[\underset{-S\alpha}{\Psi}\left(\mathcal{A}\right)\right]^C=\overset{-S\alpha}{\Psi}\left(\mathcal{A}\right)^C$$

1. Since:

$$\left[\overset{-S\alpha}{\Psi}\left(\mathcal{A}\right)\right]^C=\mathfrak{A}-\cap\left\{\mathfrak{F}:\mathfrak{F}\supseteq\mathcal{A},\mathfrak{F}\in\overset{n}{\zeta}\alpha\, C\left(U\right)\right\}=$$

$$=\cup\left\{U-\mathfrak{F}:U-\mathfrak{F}\subseteq\mathfrak{A}-\mathcal{A},\mathfrak{A}-\mathfrak{F}\in\overset{n}{\zeta}\alpha\, O\left(U\right)\right\}=\underset{-S\alpha}{\Psi}\left(\mathcal{A}\right)^C$$

$$\text{then}\ \left[\overset{-S\alpha}{\Psi}\left(\mathcal{A}\right)\right]^C=\underset{-S\alpha}{\Psi}\left(\mathcal{A}\right)^C$$

Generalized rough sets based on general neighborhoods

The concept of a new neighborhood called *general neighborhood*, derived from a general dyadic relationship is presented and the relevance of this neighborhood in the medical application of chronic kidney disease (CKD) is examined. In accordance with the concept of a generalized neighborhood (general-neighborhood), novel generalized rough set theory (called general-approximations) is created.

Definition 10. Take into account $\mathfrak{U} = \{\mathfrak{Y}_1, \mathfrak{Y}_2, ..., \mathfrak{Y}_n\}$ is a finite set and let $\mathfrak{Q} = \{\mathfrak{S}_1, \mathfrak{S}_2, ..., \mathfrak{S}_n\}$ be a set of attributes. We define a function $\mathfrak{F} : \mathfrak{A} \rightarrow P(\mathfrak{Q})$ such that for every $\mathfrak{Y} \in \mathfrak{A}$ then $\mathfrak{F}(\mathfrak{Y}) \in P(\mathfrak{Q})$ where $P(\mathfrak{Q})$ is the power set of the set of attributes.

Definition 11. Take into account $\mathfrak{R}$ is a binary relation on the universe $\mathfrak{U}$. For each $\alpha \in \mathfrak{U}$, kknown as (in short, *general neighborhood*) in the manner described:

$$N(\mathfrak{Y}) = \{\mathfrak{W} \in \mathfrak{A} : \mathfrak{F}(\mathfrak{W}) \subseteq \mathfrak{F}(\mathfrak{Y})$$

Definition 12. Take into account $\mathfrak{R}$ is a binary relation on $\mathfrak{U}$. The general-lower as well as general-upper approximations of $\mathfrak{H} \subseteq \mathfrak{U}$ are proposed, respectively:

$$\underline{\mathfrak{L}}(\mathfrak{H}) = \{\mathfrak{W} \in \mathfrak{U} : N(\mathfrak{W}) \subseteq \mathfrak{H}\} \text{ and } \overline{\mathfrak{L}}(\mathfrak{H}) = \{\mathfrak{W} \in \mathfrak{U} : N(\mathfrak{W}) \cap \mathfrak{H} \neq \varphi\}$$

Definition 13. Assume that $\mathfrak{R}$ is a binary relation on $\mathfrak{U}$. The general-positive, general negative as well as general border areas and the accuracy of general approximations of a subset $\mathbb{A} \subseteq \mathfrak{U}$ are provided, respectively,:

$$\mathfrak{p}os(\mathfrak{H}) = \underline{\mathfrak{L}}(\mathfrak{H}),\ \mathcal{N}eg_i(\mathfrak{H}) = \mathfrak{U} - \overline{\mathfrak{L}}(\mathfrak{H}),\ \mathcal{B}nd(\mathfrak{H}) = \overline{\mathcal{U}}_i(\mathfrak{H}) - \underline{\mathcal{L}}_i(\mathfrak{H}) \text{ and}$$

$$\kappa_i(\mathfrak{H}) = \frac{|\underline{\mathfrak{L}}(\mathfrak{H})|}{|\overline{\mathfrak{L}}(\mathfrak{H})|}, \text{ where } \overline{\mathcal{U}}_i(\mathfrak{H}) \neq \varphi$$

Generalized nanotopology and its applications in medical

The purpose of this portion is to expand the concept of nanotopology [7] to encompass all types of generalized rough sets. We create a nanotopology that is caused through rough set approximations in a general manner. The state, situation needed to create this topology is suggested.

Definition 14. Consider $\mathfrak{U}$ is a finite set and let $\underline{\mathfrak{L}}(\mathfrak{H})$ and $\overline{\mathfrak{L}}(\mathfrak{H})$ be the lower and upper approximations of $\mathfrak{H} \subseteq \mathfrak{U}$. The class:

$$\mathfrak{T} = \{\mathfrak{U},\ \varphi,\ \underline{\mathfrak{L}}(\mathfrak{H}),\ \overline{\mathfrak{L}}(\mathfrak{H}),\ \mathcal{B}nd(\mathfrak{H})\}$$

where $\mathcal{B}nd(\mathfrak{H})$ is the boundary region of $\mathfrak{H} \subseteq \mathfrak{U}$, is a topology on $\mathfrak{U}$ with respect to $\mathfrak{H}$ if $\underline{\mathfrak{L}}(\mathfrak{H})$ and $\overline{\mathfrak{L}}(\mathfrak{H})$.

Data gathering

The CKD is a progressive condition that hinders the kidneys' ability to remove waste and excess fluids, leading to significant health issues. This study's recommended approaches are critical for advancing our understanding of CKD and refining diagnostic and treatment methods. By expanding nanotopology through advanced computational techniques and binary models, we aim to better identify key factors driving CKD progression. This approach enables the customization of treatment plans based on individual patient characteristics, leading to more effective, personalized therapies. Greater precision in identifying these factors is essential for slowing disease progression and enhancing patient outcomes, creating a pathway for reliable, targeted tools in CKD management. The set of objects as $\mathfrak{U} = \{\mathfrak{Y}_1, \mathfrak{Y}_2, ..., \mathfrak{Y}_n\}$ denotes ten listed patients, the attributes as $\{a_1, a_2, ..., a_5\}$ = {diabetes, chest pain, smoking and alcohol, hypertension, obesity, family history} and decision of chronic kidney disease, as follows in information tab. 1. was collected by [17] chronic kidney disease. Take into account the information system presented in the tab. 1.

Table 1. The set of data that contains decisions made based on information

Patients	Diabetes, a_1	Smoking and alcohol, a_2	Hypertension, a_3	Obesity, a_4	Family history, a_5	Decision of chronic kidney disease
$\mathfrak{Y}_1$	1	1	0	1	0	1
$\mathfrak{Y}_2$	1	0	1	0	1	1
$\mathfrak{Y}_3$	1	1	0	1	0	0
$\mathfrak{Y}_4$	1	1	0	1	0	0
$\mathfrak{Y}_5$	0	1	1	1	0	1
$\mathfrak{Y}_6$	1	1	1	1	0	1
$\mathfrak{Y}_7$	0	1	1	1	0	0
$\mathfrak{Y}_8$	1	0	0	0	1	0

Appyling *Definition 10*. we get a set of diseases for each patient the result:

$$f(\mathfrak{Y}_1) = \{a_1, a_2, a_4\},\ f(\mathfrak{Y}_2) = \{a_1, a_3, a_5\},\ f(\mathfrak{Y}_3) = \{a_2, a_4\},\ f(\mathfrak{Y}_4) = \{a_1, a_2, a_4\}$$

$$f(\mathfrak{Y}_5) = \{a_2, a_3, a_4\}, f(\mathfrak{Y}_6) = \{a_1, a_2, a_3, a_4\},\ f(\mathfrak{Y}_7) = \{a_2, a_3, a_4\},\ \text{and}\ f(\mathfrak{Y}_8) = \{a_1, a_5\}$$

Hence the general neighborhood for every patient:

$$N(\mathfrak{Y}) = \{\mathfrak{W} \in \mathfrak{A}:\ \mathfrak{F}(\mathfrak{W}) \subseteq \mathfrak{F}(\mathfrak{Y}),\ N(\mathfrak{Y}_1) = \{\mathfrak{Y}_1, \mathfrak{Y}_3, \mathfrak{Y}_4\},\ N(\mathfrak{Y}_2) = \{\mathfrak{Y}_2, \mathfrak{Y}_8\}$$

$$N(\mathfrak{Y}_3) = \{\mathfrak{Y}_3\},\ N(\mathfrak{Y}_4) = \{\mathfrak{Y}_1, \mathfrak{Y}_3, \mathfrak{Y}_4\}, N(\mathfrak{Y}_5) = \{\mathfrak{Y}_3, \mathfrak{Y}_5, \mathfrak{Y}_7\}$$

$$N(\mathfrak{Y}_6) = \{\mathfrak{Y}_1, \mathfrak{Y}_3, \mathfrak{Y}_4, \mathfrak{Y}_5, \mathfrak{Y}_6, \mathfrak{Y}_7\},\ N(\mathfrak{Y}_7) = \{\mathfrak{Y}_3, \mathfrak{Y}_5, \mathfrak{Y}_7\},\ N(\mathfrak{Y}_8) = \{\mathfrak{Y}_8\}$$

Case 1: (Patients with chronic kidney disease)

Assuming that $\mathfrak{X} = \{\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_5, \mathfrak{Y}_6\}$ be the patients with chronic kidney disease thus:

$$\overline{\mathfrak{L}\mathfrak{X}} = \{\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_4, \mathfrak{Y}_5, \mathfrak{Y}_6, \mathfrak{Y}_7\}, \underline{\mathfrak{L}}(\mathfrak{X}) = \varnothing,\ \text{and}\ Bnd(\mathfrak{X}) = \{\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_4, \mathfrak{Y}_5, \mathfrak{Y}_6, \mathfrak{Y}_7\}$$

hence the nanogeneralized topology is given:

$$T_A = \{\mathfrak{A},\ \varnothing,\ \{\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_4, \mathfrak{Y}_5, \mathfrak{Y}_6, \mathfrak{Y}_7\}\}$$

and the base

$$\mathfrak{V}_A = \{U,\ \varnothing, \{\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_4, \mathfrak{Y}_5, \mathfrak{Y}_6, \mathfrak{Y}_7\}\}$$

Step 1: When the attribute a_1 *diabetes*. As a result of its removal, the symptoms of each patient are:

$$f(\mathfrak{Y}_1) = \{a_2, a_4\},\ f(\mathfrak{Y}_2) = \{a_3, a_5\},\ f(\mathfrak{Y}_3) = \{a_2, a_4\},\ f(\mathfrak{Y}_4) = \{a_2, a_4\}$$

$$f(\mathfrak{Y}_5) = \{a_2, a_3, a_4\},\ f(\mathfrak{Y}_6) = \{a_2, a_3, a_4\},\ f(\mathfrak{Y}_7) = \{a_2, a_3, a_4\},\ \text{and}\ f(\mathfrak{Y}_8) = \{a_5\}$$

Hence the general neighborhood for every patient:

$$N(\mathfrak{Y}_1)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4\},\ N(\mathfrak{Y}_2)=\{\mathfrak{Y}_2,\mathfrak{Y}_8\},\ N(\mathfrak{Y}_3)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4\},\ N(\mathfrak{Y}_4)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4\}$$

$$N(\mathfrak{Y}_5)=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\},\ N(\mathfrak{Y}_6)=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}$$

$$N(\mathfrak{Y}_7)=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\},\ \text{and}\ N(\mathfrak{Y}_8)=\{\mathfrak{Y}_8\}$$

assume that

$$\mathfrak{X}=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_5,\mathfrak{Y}_6\}$$

then

$$\overline{\mathfrak{L}\mathfrak{X}}=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\},\ \underline{\mathfrak{L}}(\mathfrak{X})=\varnothing,\quad \mathcal{B}nd(\mathfrak{X})=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}$$

Then the nano generalized topology is given:

$$T_{A-\{a_1\}}=\{\mathfrak{A},\ \varnothing,\ \{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}\}$$

The base:

$$\mathfrak{V}_A=\{U,\ \varnothing,\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}\},\ \text{since}\ T_{A-\{a_1\}}\neq T_A\ \text{and}\ \mathfrak{V}_{A-\{a_1\}}\neq\mathfrak{V}_A$$

Step 2: When the attribute a_2 *Smoking and alcohol* is left out: In the same way as in *Step 1*. We get a set of diseases for each patient:

$$f(\mathfrak{Y}_1)=\{a_1,a_4\},\ f(\mathfrak{Y}_2)=\{a_3,a_5\},\ f(\mathfrak{Y}_3)=\{a_4\},\ f(\mathfrak{Y}_4)=\{a_1,a_4\}$$

$$f(\mathfrak{Y}_5)=\{a_3,a_4\},\ f(\mathfrak{Y}_6)=\{a_1,a_3,a_4\},\ f(\mathfrak{Y}_7)=\{a_3,a_4\},\ \text{and}\ f(\mathfrak{Y}_8)=\{a_5\}$$

Hence the general neighborhood for every patients:

$$N(\mathfrak{Y}_1)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4\},\ N(\mathfrak{Y}_2)=\{\mathfrak{Y}_2,\mathfrak{Y}_8\},\ N(\mathfrak{Y}_3)=\{\mathfrak{Y}_3\},\ N(\mathfrak{Y}_4)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4\}$$

$$N(\mathfrak{Y}_5)=\{\mathfrak{Y}_3,\mathfrak{Y}_5,\mathfrak{Y}_7\},\ N(\mathfrak{Y}_6)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6 a_1,a_3,a_4\mathfrak{Y}_7\},\ N(\mathfrak{Y}_7)=\{\mathfrak{Y}_3,\mathfrak{Y}_5,\mathfrak{Y}_7\}$$

$$\text{and}\ N(\mathfrak{Y}_8)=\{\mathfrak{Y}_8\}$$

$$\mathfrak{X}=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_5,\mathfrak{Y}_6\},\ \overline{\mathfrak{L}\mathfrak{X}}=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\},\ \underline{\mathfrak{L}}\mathfrak{X}=\varnothing$$

$$\text{and}\ \mathcal{B}nd(\mathfrak{X})=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}$$

Then the nanogeneralized topology is given:

$$T_{A-\{a_2\}}=\{\mathfrak{A},\ \varnothing,\ \{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}\}$$

The base:

$$\mathfrak{V}_{A-\{a_2\}}=\{U,\ \varnothing,\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}\},\ \text{since}\ T_{A-\{a_2\}}=T_A,\ \text{and}\ \mathfrak{V}_{A-\{a_2\}}=\mathfrak{V}_A$$

Step 3: When the attribute a_3 *Hypertension* is left out: In the same way as in *Step 1*, We get a set of diseases for each patient:

$$f(\mathfrak{Y}_1)=\{a_1,a_2,a_4\},\ f(\mathfrak{Y}_2)=\{a_1,a_5\},\ f(\mathfrak{Y}_3)=\{a_2,a_4\},\ f(\mathfrak{Y}_4)=\{a_1,a_2,a_4\}$$

$$f(\mathfrak{Y}_5)=\{a_2,a_4\},\ f(\mathfrak{Y}_6)=\{a_1,a_2,a_4\},\ f(\mathfrak{Y}_7)=\{a_2,a_4\},\ \text{and}\ f(\mathfrak{Y}_8)=\{a_1,a_5\}$$

Hence the general neighborhood for every patient is:

$$N(\mathfrak{Y}_1)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\},\ N(\mathfrak{Y}_2)=\{\mathfrak{Y}_2,\mathfrak{Y}_8\},\ N(\mathfrak{Y}_3)=\{\mathfrak{Y}_3,\mathfrak{Y}_5,\mathfrak{Y}_7\}$$
$$N(\mathfrak{Y}_4)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\},\ N(\mathfrak{Y}_5)=\{\mathfrak{Y}_3,\mathfrak{Y}_5,\mathfrak{Y}_7\},N(\mathfrak{Y}_6)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}$$
$$N(\mathfrak{Y}_7)=\{\mathfrak{Y}_3,\mathfrak{Y}_5,\mathfrak{Y}_7\},\ \text{and}\ N(\mathfrak{Y}_8)=\{\mathfrak{Y}_2,\mathfrak{Y}_8\}$$

since

$$\mathfrak{X}=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_5,\mathfrak{Y}_6\},\ \text{thus}\ \overline{\mathfrak{L}\mathfrak{X}}=\mathfrak{A},\ \underline{\mathfrak{L}}(\mathfrak{X})=\varnothing,\ \text{and}\ Bnd(\mathfrak{X})=\mathfrak{A}$$

and hence the nanotopology is given

$$T_{A-\{a_3\}}=\{\mathfrak{A},\ \varnothing\}$$

and the base is $\mathfrak{B}=\{U,\varnothing\}$ therefore

$$T_{A-\{a3\}}\neq T_A\ \text{and}\ \mathfrak{V}_{A-\{a3\}}\neq\mathfrak{V}_A$$

Step 4: When the attribute a_4 *Obesity* is omitted: by the same manner as in *Step 1*, we get a set of diseases for each patient:

$$f(\mathfrak{Y}_1)=\{a_1,a_2\},\ f(\mathfrak{Y}_2)=\{a_1,a_3,a_5\},\ f(\mathfrak{Y}_3)=\{a_2\},\ f(\mathfrak{Y}_4)=\{a_1,a_2\},\ f(\mathfrak{Y}_5)=\{a_2,a_3\}$$
$$f(\mathfrak{Y}_6)=\{a_1,a_2,a_3\},\ f(\mathfrak{Y}_7)=\{a_2,a_3\},\ \text{and}\ f(\mathfrak{Y}_8)=\{a_1,a_5\}$$

and hence, the general neighborhood for every patient:

$$N(\mathfrak{Y}_1)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4\},\ N(\mathfrak{Y}_2)=\{\mathfrak{Y}_2,\mathfrak{Y}_8\},\ N(\mathfrak{Y}_3)=\{\mathfrak{Y}_3\},\ N(\mathfrak{Y}_4)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4\}$$
$$N(\mathfrak{Y}_5)=\{\mathfrak{Y}_3,\mathfrak{Y}_5,\mathfrak{Y}_7\},\ N(\mathfrak{Y}_6)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6\},\ N(\mathfrak{Y}_7)=\{\mathfrak{Y}_3,\mathfrak{Y}_5,\mathfrak{Y}_7\}$$
$$\text{and}\ N(\mathfrak{Y}_8)=\{\mathfrak{Y}_8\}$$

since

$$\mathfrak{X}=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_5,\mathfrak{Y}_6\},\ \text{hence}\ \overline{\mathfrak{L}\mathfrak{X}}=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\},\ \underline{\mathfrak{L}}(\mathfrak{X})=\varnothing,\ \text{and}$$
$$Bnd\,\mathfrak{X}=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}$$

and hence the nanogeneralized topology is given:

$$T_{A-\{a_4\}}=\{\mathfrak{A},\ \varnothing,\ \{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}\}$$

the base

$$\mathfrak{V}_{A-\{a_4\}}=\{U,\varnothing,\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}\},\ \text{since}\ T_{A-\{a_4\}}=T_A$$

and the base

$$\mathfrak{V}_{A-\{a_4\}}=\{U,,\ \varnothing,\ \{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}\}=\mathfrak{V}_A$$

Step 5: When the attribute a_5 *Family history* is omitted: By the same manner as in Step *1*, We get a set of diseases for each patient:

$$f(\mathfrak{Y}_1)=\{a_1,a_2,a_4\},\ f(\mathfrak{Y}_2)=\{a_1,a_3\},\ f(\mathfrak{Y}_3)=\{a_2,a_4\},\ f(\mathfrak{Y}_4)=\{a_1,a_2,a_4\}$$
$$f(\mathfrak{Y}_5)=\{a_2,a_3,a_4\},\ f(\mathfrak{Y}_6)=\{a_1,a_2,a_3,a_4\},\ f(\mathfrak{Y}_7)=\{a_2,a_3,a_4\},\ \text{and}\ f(\mathfrak{Y}_8)=\{a_1\}$$

and hence the general neighborhood for patients:

$$N(\mathfrak{Y}_1)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_8\},\ N(\mathfrak{Y}_2)=\{\mathfrak{Y}_2,\mathfrak{Y}_8\},\ N(\mathfrak{Y}_3)=\{\mathfrak{Y}_3\},\ N(\mathfrak{Y}_4)=\{\mathfrak{Y}_1,\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_8\}$$
$$N(\mathfrak{Y}_5)=\{\mathfrak{Y}_3,\mathfrak{Y}_5,\mathfrak{Y}_7\},\ N(\mathfrak{Y}_6)=\mathfrak{A},\ N(\mathfrak{Y}_7)=\{\mathfrak{Y}_3,\mathfrak{Y}_5,\mathfrak{Y}_7\},\ \text{and}\ N(\mathfrak{Y}_8)=\{\mathfrak{Y}_8\}$$

since,

$$\mathfrak{X}=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_5,\mathfrak{Y}_6\},\ \overline{\mathfrak{L}\mathfrak{X}}=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\},\ \underline{\mathfrak{L}}(\mathfrak{X})=\varnothing$$
$$\text{and}\ \mathcal{B}nd\,\mathfrak{X}=\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}$$

The nanogeneralized topology is given:

$$T_{A-\{a_5\}}=\{U,\varnothing,\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}\}\ \text{since}\ T_{A-\{a_5\}}=T_A$$

and the base

$$\mathfrak{V}_{A-\{a_4\}}=\{U,\varnothing,\{\mathfrak{Y}_1,\mathfrak{Y}_2,\mathfrak{Y}_4,\mathfrak{Y}_5,\mathfrak{Y}_6,\mathfrak{Y}_7\}\}=\mathfrak{V}_A$$

Case 2: (Patients are not chronic kidney disease)

The set of infected patients with are not chronic kidney is

$$\mathfrak{H}=\{\mathfrak{Y}_3,\mathfrak{Y}_4,\mathfrak{Y}_7,\mathfrak{Y}_8\}$$

By taking the identical procedures as *Case 1*, we achieve identical outcomes.

Noting: From the CORE, we observed that *Diabetes and Hypertension* are the main elements that contribute to *chronic kidney* disease. Therefore, these characteristics are essential indicators that reflect the influential factors for chronic kidney disease. At the conclusion of the document, we present a decision-making algorithm based on our methods. The abstract graph is show in fig. 1.

Conclusion

This paper introduces a new concept of open simply alpha nanogroups, based on a generalization of the Pawlak approximation space through the lower and upper simply alpha approximations. We explore their properties and relationships with existing approximations, using open nano simple/alpha sets. These methods have potential applications in various decision-making fields, especially in medical diagnosis, offering a deeper understanding of data to support healthcare decisions. A new neighborhood concept, called the general neighborhood, is used to create the general approximation space with general upper and lower approximations. A case study for chronic kidney disease patients highlights how this approach can identify key symptoms, with an algorithm developed for broader applications.

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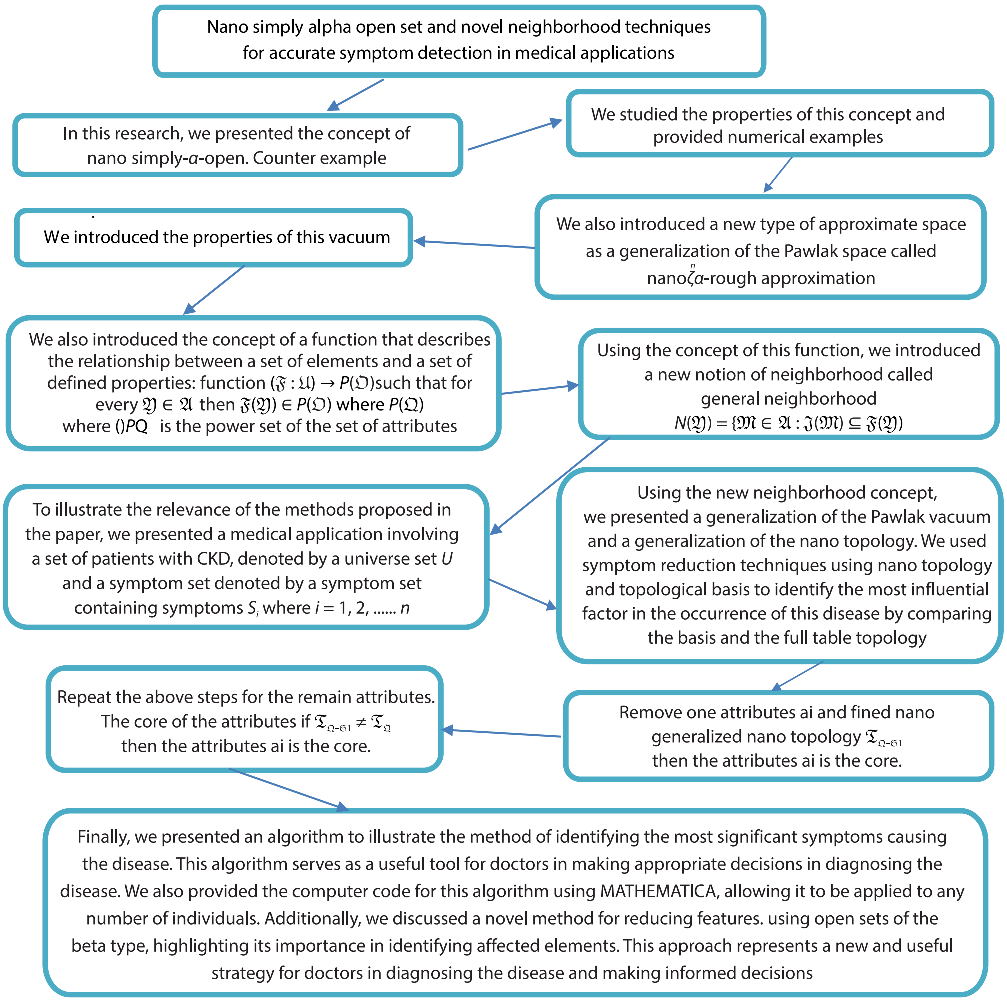

Figure 1. Framework of the proposed method

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