NEW PERSPECTIVE TO DISEASE AND INSECT INFECTION MODEL

by

Qing ZHU and Chun-Fu WEI *

School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo, China

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The main purpose of this paper is to study a class of forest disease and insect infection models with time delay. The exact solutions are successfully obtained by using the homotopy perturbation method.

Key words: *forest pest and disease, time-delay differential equations, homotopy perturbation method*

Introduction

Non-linear differential equations are one of the most common mathematical models in physical sciences and are the basic content of the development of various fields.The model in practice is very complex, and the approximate analytical solution has strong practical significance when solving the problem. There are some methods of approximating, such as the variational iteration method (VIM) [1-4], Adomain decomposition method (ADM) [5], successive approximation method (SAM), and homotopy analysis method (HAM) [6], *etc*. [7-12]. This paper mainly focuses on the use of homotopy perturbation method (HPM) in time-delay differential equations.

In the early days, He proposed the HPM [13]. For a long time afterward, a large number of scholars studied and revised,as well as in combination with other mathematical methods. For example, the HPM is combined with Laplace [14] and Madani [15] transform method. In addition, the HPM also has many applications in biomedicine. In this paper, the time-delay differential equation is studied by the HPM to calculate the approximate solution of the problem.

The HPM method has been widely used in infectious disease models. For example, the application of the SIR model, the COVID-19 pandemic model was estimated using the HPM method [16]. In this article, through an iterative approach,in the absence of linearization and discretization, an approximate solution of the equation is obtained with proper calculations.

In this paper, on the basis of non-linear differential equations, the relevant methods for finding the approximate solution of non-linear differential equation are described. The method of this paper, the HPM introduced, additionally the research status of the HPM is introduced. Secondly, the model of this paper is described, which is a type of infection model with time lag forest pest and disease. The basic theory of HPM was briefly introduced. Finally, the HPM is used to solve the model. The results are analyzed in this paper, which is summarized and respected.

* Corresponding author, e-mail: mathwcf@163.com

Problem description

In [17], the author constructs a time-delay differential equation model:

$$\frac{\mathrm{d}x}{\mathrm{d}t}=rx\left(1-\frac{x}{N}\right)-\frac{\beta xy}{1+\alpha x}+my\left(t-\tau\right) \tag{1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\beta xy}{1+\alpha x}-my\left(t-\tau\right)-by \tag{2}$$

where $x(t)$ is the area of healthy trees in that forest area at time, t, $y(t)$ – the area of diseased trees in the forest area at time, t, and t – the time variable.

Taking into account the natural growth and death of trees, the aforementioned model is established. The r and N represent the growth rate of the area of healthy forest area and the environmental capacity of healthy forest area. The $\beta xy/1+\alpha x$ is the saturation exposure rate for the spread of forest pest and disease, where α, β represent the saturation parameter and the effective contact rate. The m is the control rate of forest disease and pest in forest areas.The b indicates the mortality rate of forest pest to the number of forest areas, τ indicates the recovery time of the diseased tree. In the previous model, all coefficients are positive.

Basic idea of homotopy perturbation method

Consider the non-linear differential equations:

$$L\left(u\right)+N\left(u\right)=f\left(r\right),\ \ r\in\Omega \tag{3}$$

where L is the linear operator, N – the non-linear operator, and $f(r)$ – the known analytical function:

$$B\left(u,\partial u/\partial n\right)=0,\ \ r\in\Gamma \tag{4}$$

The eq. (4) is the boundary condition of the aforementioned equation, B – the boundary operator, and Γ – the boundary of the neighborhood, Ω.

Below constructs a homotopy:

$$V\left(r,p\right):\Omega\times\left[0,1\right]\rightarrow R$$

it satisfies:

$$H\left(v,p\right)=\left(1-p\right)\left[L\left(v\right)-L\left(u_0\right)\right]+p\left[L\left(v\right)+N\left(v\right)-f\left(r\right)\right]=0,\ \ p\in\left[0,1\right],\ \ r\in\Omega \tag{5a}$$

or

$$H\left(v,p\right)=L\left(v\right)-L\left(u_0\right)+pL\left(u_0\right)+p\left[N\left(v\right)-f\left(r\right)\right]=0 \tag{5b}$$

where $p\in[0, 1]$ is the small parameter, u_0 – the initial variable in eq. (1). It satisfies the boundary condition:

$$H\left(v,0\right)=L\left(v\right)-L\left(u_0\right)=0 \tag{6}$$

$$H\left(v,1\right)=L\left(v\right)+N\left(v\right)-f\left(r\right)=0 \tag{7}$$

where p from $0\rightarrow 1$ in eq. (5a) refers to the process of $v(r, p)$ from $u_0(r)\rightarrow u(r)$. In topology, this is called deformation:

$$L\left(v\right)-L\left(u_0\right),\ \ L\left(v\right)+N\left(v\right)-f\left(r\right)$$

call it perturbation.

This paper assumes that the solutions of eqs. (5a) and (5b) can be expressed by the power series of p:

$$v = v_0 + pv_1 + p^2 v_2 + \cdots \tag{8}$$

When p is equal to 1, the solution is anapproximate solution of eq. (3). It can be expressed: as:

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \cdots \tag{9}$$

Application of homotopy perturbation method

In this section, the HPM method is used to calculate the approximate solution of the system of non-linear differential equations for the aforementioned model. According to HPM, the homotopy of the model can be constructed [1]:

$$(1-p)(\dot{v}_1 - \dot{x}_0) + p\left[\dot{v}_1 - rv_1\left(1-\frac{v_1}{N}\right) + \frac{\beta v_1 v_2}{1+\alpha v_1} - mv_2(t-\tau)\right] = 0 \tag{10}$$

$$(1-p)(\dot{v}_2 - \dot{y}_0) + p\left[\dot{v}_2 - \frac{\beta v_1 v_2}{1+\alpha v_1} + mv_2(t-\tau) + bv_2\right] = 0 \tag{11}$$

The previous equation can be deformed as:

$$\begin{gathered}(1-p)(\dot{v}_1 - \dot{x}_0)(1+\alpha v_1) + \\ +p\left[\dot{v}_1(1+\alpha v_1) - rv_1\left(1-\frac{v_1}{N}\right)(1+\alpha v_1) + \beta v_1 v_2 - mv_2(t-\tau)(1+\alpha v_1)\right] = 0\end{gathered} \tag{12}$$

$$(1-p)(\dot{v}_2 - \dot{y}_0)(1+\alpha v_1) + p\left[\dot{v}_2(1+\alpha v_1) - \beta v_1 v_2 + mv_2(t-\tau)(1+\alpha v_1) + bv_2(1+v_1)\right] = 0 \tag{13}$$

The initial approximation:

$$\begin{gathered} v_{1,0}(t) = x_0(t) = x(0) \\ v_{2,0}(t) = y_0(t) = y(0) \\ v_{1,i}(0) = v_{2,i}(0) = 0 \quad i = 1,2,3,\ldots \end{gathered} \tag{14}$$

and

$$\begin{gathered} v_1 = v_{1,0} + pv_{1,1} + p^2 v_{1,2} + p^3 v_{1,3} + \cdots \\ v_2 = v_{2,0} + pv_{2,1} + p^2 v_{2,2} + p^3 v_{2,3} + \cdots \end{gathered} \tag{15}$$

Among v_{ij} $(i, j = 1, 2, 3,...)$ are the unidentified parts. Substitute eqs. (12) and (13) into eqs. (10) and eq.(11). We have the relations:

$$\begin{gathered}
p^0 : (1+\alpha v_{1,0})\dot{v}_{1,0} = 0,\; p^1 : (1+\alpha v_{1,0})\cdot\left[\dot{v}_{1,1} + \frac{r}{N}v_{1,0}{}^2 - rv_{1,0} - m(t-\tau)v_{2,0}\right] + \beta v_{1,0}v_{2,0} + \alpha\dot{v}_{1,0}v_{1,1} = 0 \\
p^2 : (1+\alpha v_{1,0})\left[\dot{v}_{1,2} + \frac{2r}{N}v_{1,0}v_{1,1} - rv_{1,1} - m(t-\tau)v_{2,1}\right] + \alpha v_{1,1}\left[\dot{v}_{1,1} + \frac{r}{N}v_{1,0}{}^2 - rv_{1,0} - m(t-\tau)v_{2,0}\right] + \\
+\alpha\dot{v}_{1,0}v_{1,2} + \beta(v_{1,0}v_{2,1} + v_{1,1}v_{2,0}) = 0 \\
p^3 : (1+\alpha v_{1,0})\left[\dot{v}_{1,3} + \frac{r}{N}(2v_{1,0}v_{1,2} + v_{1,1}{}^2\right] - rv_{1,2} - m(t-\tau)v_{2,2}) + \alpha v_{1,1}\left[\dot{v}_{1,2} + \frac{2r}{N}v_{1,0}v_{1,1} - rv_{1,1} - m(t-\tau)v_{2,1}\right] + \\
+\alpha v_{1,2}\left[\dot{v}_{1,1} + \frac{r}{N}v_{1,0}{}^2 - rv_{1,0} - m(t-\tau)v_{2,0}\right] + \alpha\dot{v}_{1,0}v_{1,3} + \beta\left(v_{1,0}v_{2,2} + v_{1,1}v_{2,1} + v_{1,2}v_{2,0}\right) = 0
\end{gathered} \tag{16}$$

$$\begin{gathered}
p^0 : (1+\alpha v_{1,0})\dot{v}_{2,0} = 0 \\
p^1 : \left[(1+\alpha v_{1,0})\dot{v}_{2,1} + \alpha v_{1,1}\dot{v}_{2,0}\right] - \beta v_{1,0}v_{2,0} + \left[m(t-\tau)+b\right](1+\alpha v_{1,0})v_{2,0} = 0 \\
p^2 : \left[(1+\alpha v_{1,0})\dot{v}_{2,2} + \alpha v_{1,1}\dot{v}_{2,1} + \alpha v_{1,2}\dot{v}_{2,0}\right] - \beta(v_{1,0}v_{2,1} + v_{1,1}v_{2,0}) + \\
+\left[m(t-\tau)+b\right]\left[(1+\alpha v_{1,0})v_{2,1} + \alpha v_{1,1}v_{2,0}\right] = 0 \\
p^3 : \left[(1+\alpha v_{1,0})\dot{v}_{2,3} + \alpha v_{1,1}\dot{v}_{2,2} + \alpha v_{1,2}\dot{v}_{2,1} + \alpha v_{1,3}\dot{v}_{2,0}\right] - \beta(v_{1,0}v_{2,2} + v_{1,1}v_{2,1} + v_{1,2}v_{2,0}) + \\
+\left[m(t-\tau)+b\right]\left[(1+\alpha v_{1,0})v_{2,2} + \alpha v_{1,1}v_{2,1} + \alpha v_{1,2}v_{2,0}\right] = 0
\end{gathered} \tag{17}$$

The system of eqs. (16) and (17) is established to obtain the result v_{ij} $(i, j = 1, 2, 3,...)$.

Consider the initial conditions $v_{1,i}(0) = v_{2,i}(0)$ $i = 1, 2, 3, k$ and approximations are valid in eq. (14). According to eq. (9), the equation can be obtained:

$$\begin{gathered}
x(t) = \lim_{p\to 1} v_1(t) = \sum_{k=0}^{k=3} v_{1,k}(t) \\
y(t) = \lim_{p\to 1} v_2(t) = \sum_{k=0}^{k=3} v_{2,k}(t)
\end{gathered} \tag{18}$$

Numerical results and discussion

In this section, it is described that with the appropriate parameters. Determine the area of healthy and diseased trees at time, *t*. Equations (1) and (2) are calculated using HPM. The parameters in this section are provided by Fei *et al.* [18]: A class of time-lag forest pest and disease is provided in the infectious disease model:

$$r = 0.01,\ N = 33,\ \beta = 0.83,\ \alpha = 0.79,\ m = 0.84,\ b = 0.16,\ \tau = 1.15 \tag{19}$$

$$x_0(t) = 0,\ y_0(t) = 1 \tag{20}$$

Bring the aforementioned parameters and the initial conditions eq. (14) into eqs. (16) and (17), and acquire:

$$\begin{gathered}
v_{1,0} = 0 \\
v_{1,1} = -0.966t \\
v_{1,2} = 0.006762t^2 \\
v_{1,3} = 0.4419210023t^3
\end{gathered} \tag{21}$$

$$\begin{gathered}
v_{2,0} = 1 \\
v_{2,1} = 0.806t \\
v_{2,2} = -0.076072t^2 \\
v_{2,3} = -0.437935547t^3
\end{gathered} \tag{22}$$

where $x(t)$ and $y(t)$ can, respectively be obtained using eq. (18).

Conclusion

This paper mainly studies a differential equation model of forest pest and disease transmission with a time delay and saturation incidence. Select the appropriate parameters, the approximate solution of the equation is obtained by using the HPM. During the calculation, it was only calculated to the third order, and no deeper and more complex calculations were performed.

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