ON A FRACTAL RLC-PARALLEL RESONANT CIRCUIT MODELED WITHIN THE LOCAL FRACTIONAL DERIVATIVE

by

Yu TIAN[a], Wen-Bo GENG[b], Shao-Hui WANG[a], and Kang-Jia WANG[c*]

[a] College of Physics and Telecommunication Engineering, Zhoukou Normal University, Zhoukou, China
[b] College of Network Engineering, Zhoukou Normal University, Zhoukou, China
[c] School of Physics and Electronic Information Engineering, Henan Polytechnic University, Jiaozuo, China

Original scientific paper
https://doi.org/10.2298/TSCI2404505T

In recent years, the theory of local fractional calculus has been widely used in the description of the fractional circuits. This paper presents a fractal RLC-parallel resonant circuit (FRLC-PRC) using the local fractional derivative (LFD). The FRLC-PRC is modeled by studying the non-differentiable (ND) lumped elements, then the ND conductance is obtained with the help of the local fractional Laplace transform (LFLT) and the ND parallel-resonant angular frequency (ND PRAF) is analyzed. It is found that the FRLC-PRC becomes the ordinary one when the fractional order $\delta = 1$. The obtained results show that the LFD is a powerful tool in the description of fractal circuit systems.

Key words: *fractal RLC-parallel resonant circuit, local fractional derivative, fractal circuit systems, local fractional Laplace transform*

Introduction

The theory of fractional calculus has been put forward for more than 300 years. At first, due to the lack of application background, its development is slow and there is no substantive research progress. In recent years, many researchers have been devoted to extending the traditional integer order system to the fractional order domain and exploring the new characteristics and laws of fractional order system. Therefore, the theory of fractional calculus has become a research hotspot, and has been widely used in many fields, such as porous media [1-3], oscillator [4], physics [5, 6], diffusion [7, 8], chaotic system [9] and other fields. The existing research results show that the system has more possibilities, flexibility and DoF due to the introduction of new fractional order variables. Recently, a new definition of LFD proposed by Yanghas attracted a lot of attention in various fields such as physics [10-12], diffusion [13], wave [14], circuits [15, 16] and so on. This paper mainly develops a new FRLC-PRC using the LFD motivated by recent work in the fractal circuit systems.

Basic definitions

Definition 1. The local fractional derivative of function $j(v)$ with the fractal dimension, δ, $(< 0 < \delta \leq 0)$ is defined as [17]:

* Corresponding author, e-mail: kangjiaw@hpu.edu.cn

$$j^{(\delta)}(v_0)=\frac{\mathrm{d}^\delta j(v)}{\mathrm{d}v^\delta}\Big|_{v=v_0}=\lim_{v\to v_0}\frac{\Delta^\delta\left[j(v)-j(v_0)\right]}{\left(v-v_0\right)^\delta} \tag{1}$$

where

$$\Delta^\delta\left[j(x)-j(x_0)\right]\cong\Gamma(1+\delta)\left[j(x)-j(x_0)\right]$$

Definition 2. The Yang's sine function, Yang's cosine function and Mittag-Leffler function on Cantor sets with the fractal dimension ℵ are defined [17]:

$$\sin(v^\delta)=\sum_{k=0}^{\infty}(-1)^k\frac{v^{(2k+1)\delta}}{\Gamma[1+(2k+1)\delta]} \tag{2}$$

$$\cos(v^\delta)=\sum_{k=0}^{\infty}(-1)^k\frac{v^{2k\delta}}{\Gamma[1+(2k+1)\delta]} \tag{3}$$

$$\mathrm{E}(v^\delta)=\sum_{k=0}^{\infty}\frac{v^{k\delta}}{\Gamma[1+k\delta]} \tag{4}$$

where $k\in N$.

Definition 3. Suppose the Yang local fractional Laplace type transform of function $j(v)$ expressed:

$$L_\delta[j^{(\delta)}(v)]=\Xi_\delta^j(\lambda)$$

there is the definition [17]:

$$L_\delta[j(v)]=\Xi_\delta^j(\lambda)=\frac{1}{\Gamma(1+\delta)}\int_0^\infty j(v)E_\delta(-v^\delta\lambda^\zeta)(\mathrm{d}v)^\delta \tag{5}$$

where L_N is the LFLT operator.

Theorem 1. There is the theorem for the LFLT of function $j(v)$ with high order [17]:

$$L_N[j^{(i\delta)}(v)]=\lambda^{i\delta}L_N[j(v)]-\sum_{j=0}^{i-1}\lambda^{(i-1-j)}j^{(i\delta)}(0) \tag{6}$$

The ND lumped elements and KCL

The ND capacitor

The relation between the ND charge $\Phi_\delta(v)$ and ND $i_\delta(v)$ within the LFD can be expressed [18]:

$$i_\delta(v)=\frac{\mathrm{d}^\delta\Phi_\delta(v)}{\mathrm{d}v^\delta} \tag{7}$$

Definition 4. The ND capacitance of the ND capacitor (NDC) can be defined [18]:

$$C_\delta=\frac{\Phi_{\delta,C}(v)}{u_{\delta,C}(v)} \tag{8}$$

where $u_{\delta,C}(v)$ indicates the ND voltage of the ND capacitor (NDC). By using eqs. (7) and (8), gives:

$$i_{\delta,C}(v)=C_\delta\frac{\mathrm{d}^\delta u_{\delta,C}(v)}{\mathrm{d}v^\delta} \tag{9}$$

The ND inductor

According to the Faraday law of electromagnetic induction, there is the relation between ND magnetic flux $\Phi_\delta(v)$ and ND voltage:

$$u_{\delta,L}(v) = \frac{\mathrm{d}^\delta \Theta_{\delta,L}(v)}{\mathrm{d}v^\delta} \tag{10}$$

Definition 5. We define the ND inductance of the ND inductor (NDI) within the LFD as [18]:

$$L_\delta = \frac{\Theta_{\delta,L}(v)}{i_{\delta,L}(v)} \tag{11}$$

By combining eqs. (10) and (11), we have:

$$u_{N,L}(v) = L_N \frac{\mathrm{d}^N i_{N,L}(v)}{\mathrm{d}v^N} \tag{12}$$

The ND resistor

Definition 6. We get the Ohm's Law of the ND resistor (NDR) as:

$$i_{\delta,R}(v) = \frac{u_{\delta,R}(\tau)}{R_\delta} \tag{13}$$

where $u_{\delta,R}(\tau)$, $i_{\delta,R}(v)$, and R_δ represent resistance value, the ND voltage, and ND current of the ND resistor (NDR), respectively.

Kirchhoff 's Current Law

Theorem 2. The Kirchhoff Current law (KCL) is expressed in detail that at any node in the circuit,at any moment, the sum of the current flowing into the node is equal to the sum of the current flowing out of the node, which can be expressed:

$$\sum i_N = 0 \tag{14}$$

The FRLC-PRC modeled by LFD

The FRLC-PRC modeled by the LFD is shown in fig. 1, there is the following expression according to the parallel theory:

$$u_{\delta,i}(v) = u_{\delta,R}(v) = u_{\delta,L}(v) = u_{\delta,C}(v) \tag{15}$$

According to eq. (12), there is:

$$u_{\delta,L}(v) = L_\delta \frac{\mathrm{d}^\delta i_{\delta,L}(v)}{\mathrm{d}v^\delta} \tag{16}$$

By using eq. (15), we obtain:

$$u_{\delta,R}(v) = L_\delta \frac{\mathrm{d}^\delta i_{\delta,L}(v)}{\mathrm{d}v^\delta} \tag{17}$$

We now plug eq. (17) into the eq. (13), giving:

$$i_{\delta,R}(v) = \frac{L_\delta}{R_\delta} \frac{\mathrm{d}^\delta i_{\delta,L}(v)}{\mathrm{d}v^\delta} \tag{18}$$

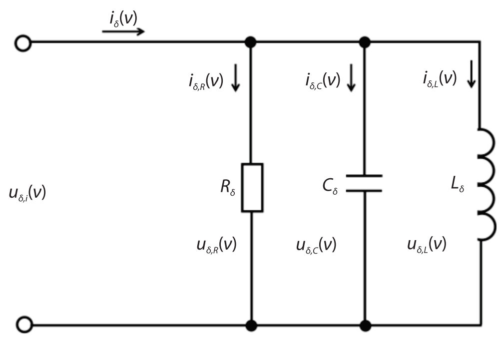

Figure 1. The FRLC-PRC model within LFD

Similarly, there is:

$$u_{\delta,C}(v)=L_\delta\frac{\mathrm{d}^\delta i_{\delta,L}(v)}{\mathrm{d}v^\delta}\tag{19}$$

By using eq. (19) and eq. (9) produces the expression:

$$i_{\delta,C}(v)=L_\delta C_\delta\frac{\mathrm{d}^{2\delta} i_{\delta,L}(v)}{\mathrm{d}v^{2\delta}}\tag{20}$$

Applying the KCL, yields:

$$i_{\delta,i}(v)=i_{\delta,R}(v)+i_{\delta,L}(v)+i_{\delta,C}(v)\tag{21}$$

Take substitution of eqs. (18) and (20) into eq. (21), yields:

$$i_{\delta,i}(v)=\frac{L_\delta}{R_\delta}\frac{\mathrm{d}^\delta i_{\delta,L}(v)}{\mathrm{d}v^\delta}+L_\delta C_\delta\frac{\mathrm{d}^{2\delta} i_{\delta,L}(v)}{\mathrm{d}v^{2\delta}}+i_{\delta,L}(v)\tag{22}$$

With the help of eqs. (15) and (16), we have:

$$u_{\delta,i}(v)=L_\delta\frac{\mathrm{d}^\delta i_{\delta,L}(v)}{\mathrm{d}v^\delta}\tag{23}$$

Having LFLT for eqs. (22) and (23), respectively, gives:

$$\Xi_\delta^{i_{\delta,i}}(\lambda)=\frac{L_\delta}{R_\delta}\left[\lambda^\delta\Xi_\delta^{i_{\delta,L}}(\lambda)-i_{\delta,L}(0)\right]+L_\delta C_\delta\left[\lambda^{2\delta}\Xi_\delta^{i_{\delta,L}}(\lambda)-\lambda^\delta i_{\delta,L}^{(\delta)}(0)-i_{\delta,L}(0)\right]+\Xi_\delta^{i_{\delta,L}}(\lambda)\tag{24}$$

and

$$\Xi_\delta^{u_{\delta,i}}(\lambda)=L_\delta\left[\lambda^\delta\Xi_\delta^{i_{\delta,L}}(\lambda)-i_{\delta,L}(0)\right]\tag{25}$$

Forzero-state of the circuit that $i_{\delta,L}(0)=0$, eqs. (24) and (25) can be reduced:

$$\Xi_\delta^{i_{\delta,i}}(\lambda)=\frac{L_\delta}{R_\delta}\lambda^\delta\Xi_\delta^{i_{\delta,L}}(\lambda)+L_\delta C_\delta\lambda^{2\delta}\Xi_\delta^{i_{\delta,L}}(\lambda)+\Xi_\delta^{i_{\delta,L}}(\lambda)\tag{26}$$

and

$$\Xi_\delta^{u_{\delta,i}}(\lambda)=L_\delta\lambda^\delta\Xi_\delta^{i_{\delta,L}}(\lambda)\tag{27}$$

Dividing eq. (26) by eq. (27) to get:

$$Y_\delta(\lambda)=G_\delta+G_\delta\lambda^\delta+\frac{1}{L_\delta\lambda^\delta}\tag{28}$$

where G_δ is the ND conductance, and $G_\delta = 1/R_\delta$. Based on the aforementioned expression, we get the ND input admittance by letting $\lambda = i\varpi$ as:

$$Y_\delta(i\varpi)=G_\delta+G_\delta(i\varpi)^\delta+\frac{1}{L_\delta(i\varpi)^\delta}\tag{29}$$

That is:

$$Y_\delta(i\varpi)=G_\delta+i^\delta\left(C_\delta\varpi^\delta-\frac{1}{L_\delta\varpi^\delta}\right)\tag{30}$$

Then we can get the modulus value of input admittance:

$$|Y_\delta(i\varpi)|=\sqrt{G_\delta^2+\left(C_\delta\varpi^\delta-\frac{1}{L_\delta\varpi^\delta}\right)^2}\tag{31}$$

when the ND circuit resonates, the ND input voltage and ND current are in the same phase, and input admittance imaginary part is zero:

$$C_\delta \varpi^\delta - \frac{1}{L_\delta \varpi^\delta} = 0 \tag{32}$$

which gives

$$\varpi_{\delta,0} = \frac{1}{\sqrt[2N]{L_N C_N}} \tag{33}$$

where $\varpi_{\delta,0}$ is the ND resonant angular frequency (ND RAF). When resonance occurs, the input admittance has the minimum modulus, the ND input admittance is a pureconductance, and the shunt part of NDC and NDI is equivalent to an open circuit. It is easily seen that different $\varpi_{\delta,0}$ can be obtained by using different L_N, C_N and fractional orders δ. It's interesting that when $\delta = 1$, the fractal RLC-parallel resonant circuit becomes the ordinary one, the ND input admittance:

$$Y(i\varpi) = G + i\left(C\varpi - \frac{1}{L\varpi}\right) \tag{34}$$

and the ND resonant angular frequency changes:

$$\varpi_0 = \frac{1}{\sqrt{LC}} \tag{35}$$

In accordance with eq. (26), the ratio of the modulus of the ND input voltage to the modulus of the ND input current:

$$\left|i_N(i\varpi_\zeta)\right| = \left|u_{N,i}(i\varpi)\right| \sqrt{G_N^2 + \left(C_N\varpi^N - \frac{1}{L_N\varpi^N}\right)^2} \tag{36}$$

Conclusion

By studying the ND lumped elements within the LFD, we successfully propose the FRLC-PRC for the first time, then the ND conductance and ND PRAF are obtained with help of the LFLT. The ND PRAF and the characteristics of FRLC-PRC are analyzed by using different input signals and parameters in detail. Interestingly, it is found that the FRLC-PRC becomes the ordinary one in the special situation $\delta = 1$. The obtained results are expected to open some new perspectives towards the characterization of ND electric circuits via LFD.

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Paper submitted: February 3, 2024
Paper revised: March 10, 2024
Paper accepted: May 9, 2024