# FRACTAL SOLITARY WAVES OF THE (3+1)-DIMENSIONAL FRACTAL MODIFIED KdV-ZAKHAROV-KUZNETSOV

by

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*In this work, the fractal (3+1)-D modified KdV-Zakharov-Kuznetsov (MKdV-ZK) model is studied, which can represent weakly non-linear waves under the unsmooth boundary. With the help of the fractal traveling wave transformation and the semi-inverse method, a fractal variational principle is obtained, which is a strong minimum one according to the He-Weierstrass function. From the variational principle, a fractal solitary wave solution is obtained, and the influence of unsmooth boundary on solitary waves is studied and the behaviors of the solutions are presented via 3-D plots. This paper shows that the fractal dimensions can affect the wave pattern, but cannot influence its crest value.*

Key words: *He's fractal derivatives, fractal variational principle, Semi-inverse method, unsmooth boundary, He-Weierstrass function*

## Introduction

Traveling wave solutions of a non-linear PDE play a significant part in understanding the action of non-linear physical phenomena, and have aroused research interest of physicists and mathematicians. In order to gain precise solution, many methods have been trialed, for instance, the inverse scattering method [1], the modified simple equation method [2], the trigonometric function series [3], Hirota's bilinear method [4], the differential transformation technique [5], the sine-cosine technique [6], the tanh function expansion and its multifarious extension technique [7], F-extension method [8], exp-function method [9], and G-expansion method [10].

The non-linear (3+1)-D MKdV-ZK model reads:

$$w_t + \lambda w^2 w_x + w_{xxx} + w_{xyy} + w_{xzz} = 0 \tag{1}$$

where $\lambda$ is a constant.

The traveling wave solution of eq. (1) was constructed by using the enhanced (G'/G) expansion method [11]. The exact solution of time fractional KdV-ZK equation was determined by using the improved fractional sub-equation method [12]. The improved tan $\phi(\xi)/2$-expansion technique was recommended to build exact specific solutions of (3+1)-D

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MKdV-ZK equation [13]. When MKdV-ZK model has a non-smooth boundary [14], a fractal modification has to be considered, which is:

$$\frac{Dw}{^{H}Dt^{\tau}}+\lambda w^{2}\frac{Dw}{^{H}Dx^{\tau}}+\frac{D^{3}w}{^{H}Dx^{\tau}\,^{H}Dx^{\tau}\,^{H}Dx^{\tau}}+\frac{D^{3}w}{^{H}Dy^{\tau}\,^{H}Dy^{\tau}\,^{H}Dx^{\tau}}+\frac{D^{3}w}{^{H}Dz^{\tau}\,^{H}Dz^{\tau}\,^{H}Dx^{\tau}}=0 \quad (2)$$

where $\tau$ represents the two-scale fractal dimensions [15], $t$ and $x$-, $y$-, and $z$- the time and space coordinates, respectively. He's fractal derivatives [16, 17] are expounded:

$$\frac{Dw}{^{H}Dt^{\tau}}(x^{\tau},y^{\tau},z^{\tau},t_{0}^{\tau})=\Gamma(1+\tau)\lim_{\substack{t-t_{0}=\Delta t\\ \Delta t\neq 0}}\frac{w(x^{\tau},y^{\tau},z^{\tau},t^{\tau})-w(x^{\tau},y^{\tau},z^{\tau},t_{0}^{\tau})}{(t-t_{0})^{\tau}} \quad (3)$$

$$\frac{Dw}{^{H}Dx^{\tau}}(x_{0}^{\tau},y^{\tau},z^{\tau},t^{\tau})=\Gamma(1+\tau)\lim_{\substack{x-x_{0}=\Delta x\\ \Delta x\neq 0}}\frac{w(x^{\tau},y^{\tau},z^{\tau},t^{\tau})-w(x_{0}^{\tau},y^{\tau},z^{\tau},t^{\tau})}{(x-x_{0})^{\tau}} \quad (4)$$

In terms of fractal derivatives, we have the following chain rules:

$$\frac{D^{2}w}{^{H}Dt^{\tau}\,^{H}Dx^{\tau}}=\frac{Dw}{^{H}Dt^{\tau}}\frac{Dw}{^{H}Dx^{\tau}} \quad (5)$$

$$\frac{D^{3}w}{^{H}Dx^{3\tau}}=\frac{Dw}{^{H}Dx^{\tau}}\frac{Dw}{^{H}Dx^{\tau}}\frac{Dw}{^{H}Dx^{\tau}} \quad (6)$$

The fractal derivative is now widely used to describe non-smooth boundary problems or porous medium problems, for examples, the fractal Harry Dym equation [18], the fractal Klein-Gordon equation [19], the fractal KdV-Burgers-Kuramoto equation [20], the fractal Klein-Gordon equation [21], the fractal Fisher's equation [22], the fractal Boiti-Leon-Manna-Pempinelli equation [23], the fractal stress model [24], the fractal Fisher's equation [25], the fractal variational principles [26-32], the fractal MEMS systems [33-36], the fractal vibration systems [37-40], and the fractal thermodynamics [41, 42].

In this paper, we suggest an effective technique to settle the fractal (3+1)-D MKdV--ZK equation.

## Fractal variational principle

In order to seek the travelling wave solution of eq. (2), we present a fractal complex transformation [14]:

$$w(x^{\tau},y^{\tau},z^{\tau},t^{\tau})=w(\varsigma^{\tau}) \quad (7)$$

$$\varsigma^{\tau}=h_{1}^{\tau}x^{\tau}+h_{2}^{\tau}y^{\tau}+h_{3}^{\tau}z^{\tau}-k^{\tau}t^{\tau} \quad (8)$$

where $h_1$, $h_2$, $h_3$, and $k$ are constants, $\tau$-fractal dimension.

Using the transformations eqs. (7) and (8), eq. (2) becomes:

$$-k^{\tau}\frac{Dw}{^{H}D\varsigma^{\tau}}+\lambda h_{1}^{\tau}w^{2}\frac{Dw}{^{H}D\varsigma^{\tau}}+(h_{1}^{3\tau}+h_{1}^{\tau}h_{2}^{2\tau}+h_{1}^{\tau}h_{3}^{2\tau})\frac{Dw}{^{H}D\varsigma^{\tau}}\frac{Dw}{^{H}D\varsigma^{\tau}}\frac{Dw}{^{H}D\varsigma^{\tau}}=0 \quad (9)$$

where $Dw/{}^{H}D\varsigma^{\tau}$ is He's the fractal derivative with respect to $\varsigma^{\tau}$.

According to the fractal derivative chain rules, eq. (9) can be rewritten:

$$(-k^{\tau}+\lambda h_1^{\tau}w^2)\frac{Dw}{{}^{H}D\varsigma^{\tau}}+(h_1^{3\tau}+h_1^{\tau}h_2^{2\tau}+h_1^{\tau}h_3^{2\tau})\frac{D^3w}{{}^{H}D\varsigma^{3\tau}}=0 \tag{10}$$

Integrating eq. (10) once with respect to $\varsigma^{\tau}$ and setting the integral constant to be zero, we have:

$$-k^{\tau}+\lambda h_1^{\tau}w^2+(h_1^{3\tau}+h_1^{\tau}h_2^{2\tau}+h_1^{\tau}h_3^{2\tau})\frac{D^2w}{{}^{H}D\varsigma^{2\tau}}=0 \tag{11}$$

We can construct the fractal variational principle of eq. (11) [43]:

$$V(w)=\int_0^{\infty}\left[\frac{1}{2}\left(\frac{Dw}{{}^{H}D\varsigma^{\tau}}\right)^2-\frac{\lambda w^3}{3(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})}+\frac{k^{\tau}w}{h_1^{3\tau}+h_1^{\tau}h_2^{2\tau}+h_1^{\tau}h_3^{2\tau}}\right]\mathrm{d}\varsigma^{\tau} \tag{12}$$

The Euler-Lagrange equation of eq. (12) can be described:

$$\frac{k^2}{h_1^{3\tau}+h_1^{\tau}h_2^{2\tau}+h_1^{\tau}h_3^{2\tau}}-\frac{\lambda}{h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau}}w^2-\frac{D^2w}{{}^{H}D\varsigma^{2\tau}}=0 \tag{13}$$

From eq. (12) , we can obtain He's modified Weierstrass function [30] is:

$$\Theta[\varsigma,w,w^{(\tau)},p]=\frac{1}{2}p^2-\left[\frac{\lambda w^3}{3(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})}-\frac{k^{\tau}w}{h_1^{3\tau}+h_1^{\tau}h_2^{2\tau}+h_1^{\tau}h_3^{2\tau}}\right]-$$

$$-\left[\frac{1}{2}\left(\frac{Dw}{{}^{H}D\varsigma^{\tau}}\right)^2-\frac{\lambda w^3}{3(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})}+\frac{k^{\tau}w}{h_1^{3\tau}+h_1^{\tau}h_2^{2\tau}+h_1^{\tau}h_3^{2\tau}}\right]-\left(p-\frac{Dw}{{}^{H}D\varsigma^{\tau}}\right)\frac{Dw}{{}^{H}D\varsigma^{\tau}}=$$

$$=\frac{1}{2}\ {}^2-\frac{1}{2}\left(\frac{Dw}{{}^{H}D\varsigma^{\tau}}\right)^2-\left(p-\frac{Dw}{{}^{H}D\varsigma^{\tau}}\right)\frac{Dw}{{}^{H}D\varsigma^{\tau}} \tag{14}$$

where the variable $p$ is defined:

$$p=\frac{Dw}{{}^{H}D\varsigma^{\tau}} \tag{15}$$

From eq. (14), it is obvious that:

$$\Theta(\varsigma,w,w^{\tau},p)=0 \text{ and } \frac{\partial^2\Theta}{\partial p^2}>0 \tag{16}$$

Equation (16) indicates that eq. (12) is a minimal variational principle.

## Solitary wave solution

Now we postulate the solution of eq. (12) [43]:

$$w(\varsigma^{\tau})=\gamma\,\mathrm{sech}(\varsigma^{\tau}) \tag{17}$$

where $\gamma$ can be dictated soon afterwards. Substituting eq. (17) into eq. (12), we obtain:

$$V(\gamma)=\int_0^\infty\left\{\frac{1}{2}\left[-\gamma\tanh(\varsigma^\tau)\operatorname{sech}(\varsigma^\tau)\right]^2-\frac{\lambda\gamma^3\operatorname{sech}^3(\varsigma^\tau)}{3(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})}+\frac{\gamma k^\tau\operatorname{sech}(\varsigma^\tau)}{h_1^\tau(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})}\right\}\mathrm{d}\varsigma^\tau=$$

$$=\frac{2h_1^\tau(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})\gamma^2-\lambda h_1^\tau\pi\gamma^3+6k^2\pi\gamma}{12h_1^\tau(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})} \tag{18}$$

Taking stability condition of the above equation provides:

$$\frac{DV}{D\gamma}=0 \tag{19}$$

which gives rise to:

$$\frac{4h_1^\tau(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})\gamma-3\lambda h_1^\tau\pi\gamma^2+6k^2\pi}{12h_1^\tau(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})}=0 \tag{20}$$

Consequently, from the previous equations, we achieve:

$$\gamma=\frac{2h_1^\tau(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})\pm\sqrt{4h_1^{2\tau}(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})^2+18\lambda\pi^2k^2h_1^\tau}}{3\lambda\pi h_1^\tau} \tag{21}$$

So the solution of eq. (11) can be obtained:

$$w(\varsigma^\tau)=\frac{2h_1^\tau(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})\pm\sqrt{4h_1^{2\tau}(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})^2+18\lambda\pi^2k^2h_1^\tau}}{3\lambda\pi h_1^\tau}\operatorname{sech}(\;^\tau) \tag{22}$$

In view of the aforementioned, we achieve solitary wave solution of MKdV-ZK equation according to eq. (22):

$$w(x^\tau,y^\tau,z^\tau,t^\tau)=\frac{2h_1^\tau(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})\pm\sqrt{4h_1^{2\tau}(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})^2+18\lambda\pi^2k^2h_1^\tau}}{3\lambda\pi h_1^\tau}\cdot$$

$$\cdot\operatorname{sech}(h_1^\tau x^\tau+h_2^\tau y^\tau+h_3^\tau z^\tau-k^\tau t^\tau) \tag{23}$$

Without loss of generality, we only consider the following situation:

$$w(x^\tau,y^\tau,z^\tau,t^\tau)=\frac{2h_1^\tau(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})+\sqrt{4h_1^{2\tau}(h_1^{2\tau}+h_2^{2\tau}+h_3^{2\tau})^2+18\lambda\pi^2k^2h_1^\tau}}{3\lambda\pi h_1^\tau}\cdot$$

$$\cdot\operatorname{sech}(h_1^\tau x^\tau+h_2^\tau y^\tau+h_3^\tau z^\tau-k^\tau t^\tau) \tag{24}$$

We draw the demeanours of eq. (24) with different fractal dimensions as shown in fig. 1. Figure 2 manifests the comparisons of eq. (24) with dissimilar fractal dimensions when $t^\tau = 1$ by the aid of the 2-D curved line. On the basis of figs. 1 and 2, we clearly see that for unequal fractal dimension values, the shape and peak position of waves will change, but without correcting vertex values. The final result is that the non-smooth boundary will not involve the peak of solitary waves, and it is expected that solitary waves will be used for coastal protection.

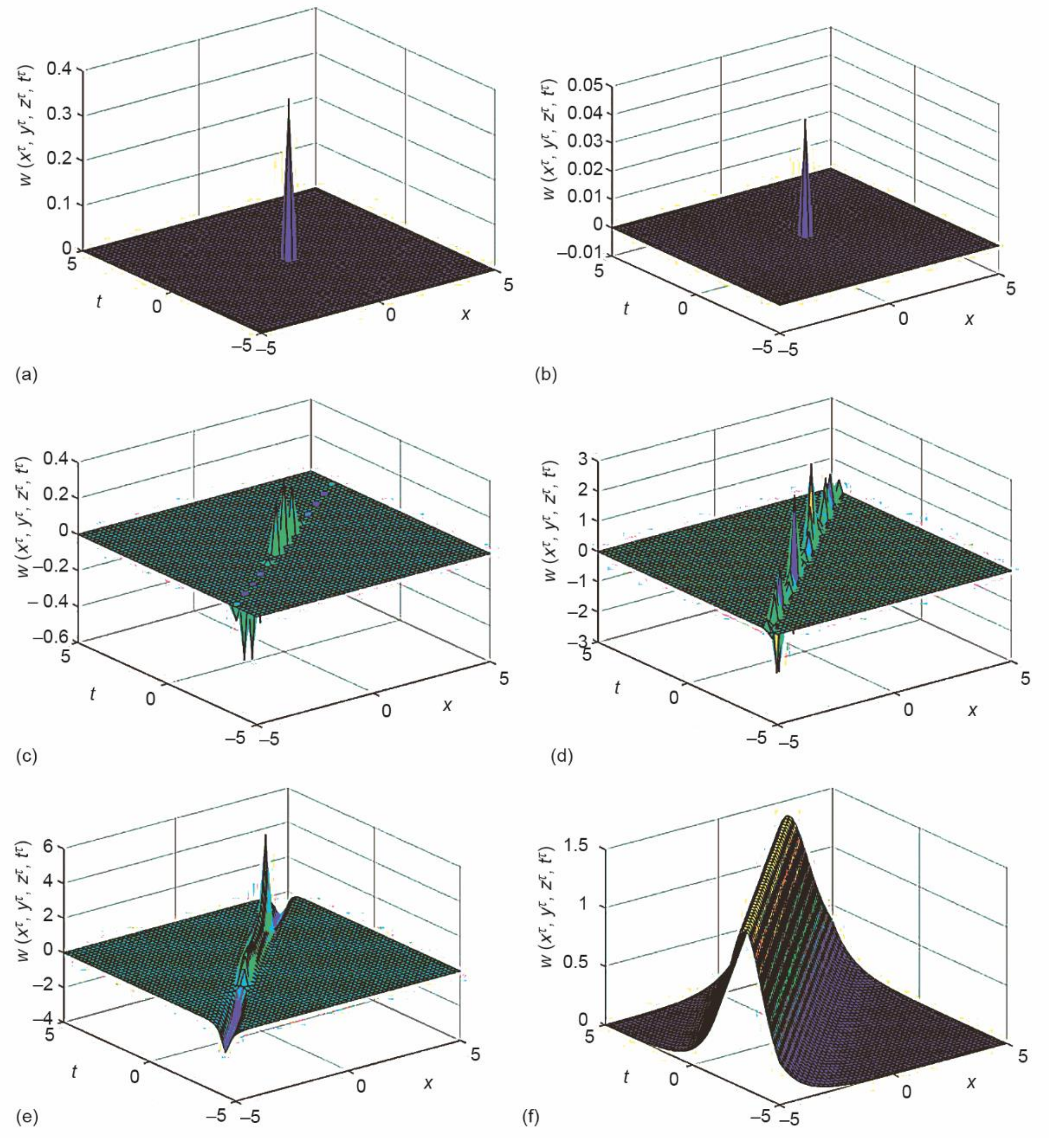

**Figure 1. The demeanours of eq. (24) with different fractal dimensions when $\lambda = 2$, $k = 1$, $h_1^\tau = 1$, $h_2^\tau = 1$, $h_3^\tau = 1$, $y^\tau = 1$, $z^\tau = 1$, (a) $\tau = 0.1$, (b) $\tau = 0.3$, (c) $\tau = 0.6$, (d) $\tau = 0.8$, (e) $\tau = 0.8$, (f) $\tau = 1$**

## Conclusion

In this exploration, we investigate the MKdV-ZK equation in a fractal space in the frame of the variational principle. The basic properties of fractal solitary waves are revealed graphically. The precedent is furnished to manifest that the proposed methodology is effica-

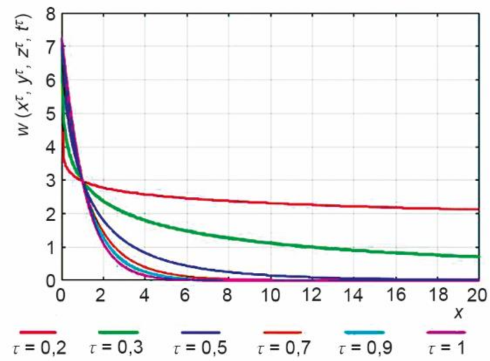


**Figure 2. Comparisons of eq. (24) with dissimilar fractal dimensions when $\lambda = 2$, $k = 1$, $h_1^\tau = 1$, $h_2^\tau = 1$, $h_3^\tau = 1$, $y^\tau = 1$, $z^\tau = 1$, $\tau^\tau = 1$**

cious, plain and very appealing, and it can be augmented to tackle with other various genres of fractional wave equations [44] and fractional-order convolutional neural networks [45-47]. The fractal thermodynamics [48] offers a novel window for investigating fractal solitary waves including the desert wave. The mountain-river-desert conjecture proposed by Mei, *et al*. [49, 50] might be solved by the fractal solitary theory.

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