PREDICTING CHINA'S ELDERLY POPULATION USING A FRACTIONAL GRAY PREDICTION MODEL

by

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Original scientific paper
https://doi.org/10.2298/TSCI2403305H

China's aging population is becoming more and more serious, which has a far-reaching influence on the state and society. As the more elderly population grows, it is necessary to strengthen a sound policy system to alleviate the burden on families and society. The importance of accurately predicting the elderly population is therefore highlighted. With the aim of exploring the future development trend of China's older population, in this paper, we establish a new fractional gray prediction model based on time power term to study China's elderly population. We used data from 2010 to 2019 to assess modeling accuracy, demonstrating that the model outperforms the other models. The final step is to use the model to forecast China's elderly population from 2020 to 2029.

Key words: *grey system theory, China's elderly population, FGM(1,1, t^α) model; Simpson formula, fractional order accumulation*

Introduction

The aging process of the population is developing rapidly in China, and the total number of babies born per woman in China has dropped from 6.11 in 1950 to 1.66 in 2015. At the same time, the population mortality rate has also dropped from 22.2 per 10000 population to 7.2 per 10000 population. From 1950 to 2015, the average life span of Chinese people has increased from 44.6 to 75.3 years. It is worth noting that the aging process of China's population is much higher than that of middle-income and high-income countries in the world. It is precisely because that the decline of the birth rate has accelerated the pace of China's entry into the aging country. The burden and responsibility of the society increases rapidly with the increase of the elderly population. In order to take the necessary measures and corresponding policies to solve the problem of the continuous increase of the elderly population, it is very important to accurately predict the number of the elderly population. The prediction of small sample data is more important, and grey models are more suitable for predicting small sample data. As is well known, sometimes the data is not perfect due to various reasons. In this case, grey prediction models are clearly better suited to predict China's elderly population.

In Deng's theory [1] of gray system, the grey prediction model has a crucial part. Because they are built on the basis of grey systems, such models are frequently referred to as grey models (GM). Due to their excellent accuracy, grey prediction models are being employed extensively across many facets of society [2-4].

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The far more popular and significant grey model is the one known as the basic GM (1, 1) (Base-G), where all these numbers inside the parenthesis mean the following: the first "1" is the number that stands for *first order* of this model while the second "1" is the number that represents *univariate*. As a significant model, the GM (1, 1) comes with lots of drawbacks. Numerous academics have refined this model in an effort to greatly raise forecast accuracy. In this regard, Wu *et al*. [5] was successful in increasing the flexibility of the grey prediction model through following a certain procedure which requires including fractional-order cumulative processes into the Base-G model, and Anjum *et al*. [6] study a two-scale population growth model in a closed system. By considering and further relying on the Xie *et al*. approach, Ma *et al*. [7] suggested a discrete Base-G model and created a generalized model which is based on the Simpson method, which aims to implement the Simpson formula for a successful and efficient approach to maximize the Base-G's background values [8]. To significantly enhance the grey prediction model background value, Wei *et al*. [9] employed linear interpolation, and so on [10]. Several scientists have observed that the Base-G model is not applicable in every kind of time series as a result of its extensive use. Moreover, because the Base-G model has been implemented on the basis of homogeneous exponential functions, it can be extremely challenging to have an accurate description of the time series using a roughly homogeneous exponential function. Cui *et al*. [12] presented a new model known as the NGM(1,1, k) model for overcoming such a problem, opening up another path for the enhancement of the Base-G model [12]. The scientific findings of Cui *et al*. [12] served as the basis for the new GM(1,1, t) model that was first put forward by Qian *et al*. [13]. Through swapping out the time power term α, this new model may fit a variety of time series, enabling reliable prediction.

To improve the model ability to forecast outcomes, an improved GM(1,1, t^α) model was developed. One main feature of this model is that it does not only address the shortcomings of the prior model but also includes a fractional order accumulation operator. In contrast, few researchers provide efficient algorithms for finding the accumulation order z of fractional gray prediction models [13]. To address this shortcoming, we employ a novel approach by utilizing the quantum genetic algorithm in solving for the parameters that cause the FGM(1,1, t^α) model to have a significantly higher prediction accuracy. For the cases presented in this paper, we will confirm the FGM(1,1, t^α) model efficacy. For examining the demographic aging in China, we contribute by developing and proposing a new fractional gray prediction model that also includes a temporal power term, designated by the FGM(1,1, t^α) model.

Definition of fractional order accumulation

Fractional accumulated generative operation (FAGO) is covered in this section, which may lessen the unpredictability of the raw data in the gray theory. The FAGO is defined as follows.

Definition 1. Let $X^{(0)}=\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\}$ represent the initial sequence and $X^{(z)}(z>0)$ represent the r-th accumulated generating operation (z-AGO) sequence of $X^{(0)}$, where:

$$x^{(z)}(k)=\sum_{i=1}^{k}x^{(z-1)}(i),\quad k=1,2,\cdots,n$$

Designate A^z to z-AGO matrix which fulfills $X^{(z)} = X^{(0)}A^z$, and:

$$A^z = \begin{pmatrix} \begin{bmatrix} z \\ 0 \end{bmatrix} & \begin{bmatrix} z \\ 1 \end{bmatrix} & \begin{bmatrix} z \\ 2 \end{bmatrix} & \cdots & \begin{bmatrix} z \\ z-1 \end{bmatrix} \\ 0 & \begin{bmatrix} z \\ 0 \end{bmatrix} & \begin{bmatrix} z \\ 1 \end{bmatrix} & \cdots & \begin{bmatrix} z \\ n-2 \end{bmatrix} \\ 0 & 0 & \begin{bmatrix} z \\ 0 \end{bmatrix} & \cdots & \begin{bmatrix} z \\ n-3 \end{bmatrix} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \begin{bmatrix} z \\ 0 \end{bmatrix} \end{pmatrix}_{n\times n}$$

With:

$$\begin{bmatrix} z \\ 1 \end{bmatrix} = \frac{z(z+1)\cdots(z+i-1)}{i!} = \begin{pmatrix} z+i-1 \\ i \end{pmatrix} = \frac{(z+i-1)!}{i!(z-1)!}, \quad \begin{bmatrix} 0 \\ i \end{bmatrix} = 0, \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 1$$

Definition 2. The definition of the inverse accumulated generation is:

$$x^{(z-1)}(k) = x^{(z)}(k) - x^{(z)}(k-1), \quad k = 1, 2, \cdots, n$$

Designate D^r to z^{th} inverse accumulated generating operation (z-IAGO) matrix, which fulfils $X^{(0)} = X^{(z)}D^z$, with:

$$\begin{bmatrix} -z \\ 1 \end{bmatrix} = \frac{-z(-z+1)\cdots(-z+i-1)}{i!} = (-1)^i \frac{z(z-1)\cdots(z-i+1)}{i!} = (-1)^i \begin{pmatrix} z \\ i \end{pmatrix}, \quad \begin{bmatrix} -z \\ i \end{bmatrix} = 0, \quad i > z$$

and

$$D^z = \begin{pmatrix} \begin{bmatrix} -z \\ 0 \end{bmatrix} & \begin{bmatrix} -z \\ 1 \end{bmatrix} & \begin{bmatrix} -z \\ 2 \end{bmatrix} & \cdots & \begin{bmatrix} -z \\ z-1 \end{bmatrix} \\ 0 & \begin{bmatrix} -z \\ 0 \end{bmatrix} & \begin{bmatrix} -z \\ 1 \end{bmatrix} & \cdots & \begin{bmatrix} -z \\ n-2 \end{bmatrix} \\ 0 & 0 & \begin{bmatrix} -z \\ 0 \end{bmatrix} & \cdots & \begin{bmatrix} -z \\ n-3 \end{bmatrix} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \begin{bmatrix} -z \\ 0 \end{bmatrix} \end{pmatrix}_{n\times n}$$

In fact, Qian *et al*. [13] did not give a prediction formula when they proposed the GM(1,1, t^α) model because this whitening differential equation is hard to solve.

The fractional GM(1,1, t^α) model and FGM(1,1, t^α) model

The time response function for the GM(1,1, t^α) model is difficult to solve. In this section, we will address this issue and construct a novel fractional GM(1,1, t^α) model(shortened as FGM(1,1, t^α) or FGM).

The FGM(1,1, t^α) model

The linear differential equation:

$$\frac{\mathrm{d}x^{(z)}(t)}{\mathrm{d}t} + ax^{(z)}(t) = bt^{\alpha} + c, \quad z > 0, \quad \alpha \ge 0 \tag{1}$$

is also known as the FGM(1,1, t^α) model (or FGM), which stands for the whitening equation of the fractional grey model with time power term. By integrating eq. (1) using the interval time $[k-1, k]$ as a boundary condition, we get:

$$\int_{k-1}^{k} \mathrm{d}x^{(z)}(t) + a\int_{k-1}^{k} x^{(z)}(t)\mathrm{d}t = b\int_{k-1}^{k} t^{\alpha}\mathrm{d}t + c \tag{2}$$

Based on the trapezoid formula and $y^{(z)}(k) = 0.5[x^{(z)}(k) + x^{(z)}(k-1)]$, k = 2, 3,…, n, eq. (2) turns to be:

$$x^{(z)}(k) - x^{(z)}(k-1) + ay^{(z)}(k) = b\frac{k^{1+\alpha} - (k-1)^{1+\alpha}}{1+\alpha} + c \tag{3}$$

The difference eq. (3) is the FGM model discrete formulation, which is used for parameters estimation. Once given the fractional order z and α, the FGM model would essentially have a linear formulation, therefore all remaining variables that are regarded as linear (a, b, c) may be approximated by using the approach of the least square, where the equation for the solution is stated:

$$\hat{\vartheta} = (\hat{a}, \hat{b}, \hat{c}) = (\Delta^T\Delta)^{-1}\Delta^T\Theta \tag{4}$$

$$\Delta = \begin{pmatrix} -y^{(z)}(2) & \dfrac{2^{1+\alpha} - 1}{1+\alpha} & 1 \\ -y^{(z)}(3) & \dfrac{3^{1+\alpha} - 2^{1+\alpha}}{1+\alpha} & 1 \\ \vdots & \vdots & \vdots \\ -y^{(z)}(n) & \dfrac{n^{1+\alpha} - (n-1)^{1+\alpha}}{1+\alpha} & 1 \end{pmatrix}, \quad \Theta = \begin{pmatrix} x^{(z)}(2) - x^{(z)}(1) \\ x^{(z)}(3) - x^{(z)}(2) \\ \vdots \\ x^{(z)}(n) - x^{(z)}(n-1) \end{pmatrix}$$

Theorem 1. The time response function of the FGM(1,1, t^α) model can be represented by:

$$\hat{x}^{(z)}(k) = \left[x^{(0)}(1) - \frac{\hat{c}}{\hat{a}} \right] e^{-\hat{a}(k-1)} + \frac{\hat{c}}{\hat{a}} + \frac{\hat{b}}{6} e^{-\hat{a}(k-1)}$$

$$\sum_{i=1}^{k-1} \left[i^{\alpha} e^{\hat{a}(i-1)} + 4(i+0.5)^{\alpha} e^{\hat{a}(i-0.5)} + (i+1)^{\alpha} e^{\hat{a}i} \right], \quad k = 2,3,\cdots,n \tag{5}$$

and the restored values of $\hat{x}^{(0)}(k), k = 2,3,\cdots,n$ are described:

$$\hat{X}^{(0)} = \left\{ \hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \cdots, \hat{x}^{(0)}(n) \right\} = \hat{X}^{(z)} D^z = \left\{ \hat{x}^{(z)}(1), \hat{x}^{(z)}(2), \cdots, \hat{x}^{(z)}(n) \right\} D^z \tag{6}$$

The objective of optimal z and α values should provide the proposed model with the highest accuracy in a given sample. So we just need to create an optimization problem with the goal of minimizing errors in proposed model by changing the values of z and α:

$$\min \ f(\alpha, \mathrm{z}) = \frac{1}{n-1} \sum_{t=2}^{n} \left| \frac{\hat{x}^{(0)}(t) - x^{(0)}(t)}{x^{(0)}(t)} \right| \times 100\%$$

$$s.t. \begin{cases} \hat{\vartheta} = (\hat{a}, \hat{b}, \hat{c}) = (\Delta^T \Delta)^{-1} \Delta^T \Theta \\ \hat{x}^{(z)}(k) = \left[x^{(0)}(1) - \dfrac{\hat{c}}{\hat{a}} \right] e^{-\hat{a}(k-1)} + \dfrac{\hat{c}}{\hat{a}} + \dfrac{\hat{b}}{6} e^{-\hat{a}(k-1)} \\ \displaystyle\sum_{i=1}^{k-1} \left[i^{\alpha} e^{\hat{a}(i-1)} + 4(i+0.5)^{\alpha} e^{\hat{a}(i-0.5)} + (i+1)^{\alpha} e^{\hat{a}i} \right], \quad k = 2,3,\cdots,n \\ \hat{X}^{(0)} = \left\{ \hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \cdots, \hat{x}^{(0)}(n) \right\} = \hat{X}^{(z)} D^z = \left\{ \hat{x}^{(z)}(1), \hat{x}^{(z)}(2), \cdots, \hat{x}^{(z)}(n) \right\} D^z \end{cases} \tag{7}$$

Intelligent algorithms are often used to solve these planning problems. The optimal parameters z and α are obtained directly by quantum genetic algorithm.

The quantum genetic algorithm

The parameters of grey prediction models were frequently solved using the genetic algorithm, although the approach requires a lot of research time and goes through many iterations. Compared to genetic algorithms, grey prediction models require less time to solve the parameters. This has been the rationale for the selection of the quantum genetic algorithm (QGA) to resolve the model parameters put out in this work. The QGA include polymorphic encoding gene cubes and full quantum algorithms. More prevalent and efficient than the generalized genetic algorithm is its dynamic control mechanism of the rotational angle and quantum cross reactor. The following represents the key elements of the QGA.

– Binary coding has been used in QGA, which itself is premised on the notion of qubits. Genes are stored and expressed by QGA using a single or multiple qubits to create chromosomes:

$$q_i^t = \begin{bmatrix} \tau_{11}^t & \cdots & \tau_{1n}^t & \cdots & \tau_{m1}^t & \cdots & \tau_{mn}^t \\ \upsilon_{11}^t & \cdots & \upsilon_{1n}^t & & \upsilon_{m1}^t & \cdots & \upsilon_{mn}^t \end{bmatrix} \tag{8}$$

In the formula of q_i^t, i represents the i^{th} generation chromosome, n represents the quantum bit number contained in the chromosome of the t^{th} individual, $|\tau_i|^2 + |\upsilon_i|^2 = 1$, $i = 1, 2, \cdots, n$ and m represents the number of genes in the chromosome.

– The fitness function used to assess the chromosomes quality is the mean absolute percentage error (MAPE). The highest level of genetic algebra used during calculation is T = 400. Moreover, this genetic update process stops once the genetic algebra reaches this value $t > T$.

Quantum rotating gate $U(\Delta A)$ has been primarily utilized for updating and evolving chromosomes in QGA, where the term $U(\Delta A)$ is represented through the following expression:

$$U(\Delta A) = \begin{bmatrix} \cos(\Delta A) & -\sin(\Delta A) \\ \sin(\Delta A) & \cos(\Delta A) \end{bmatrix} \tag{9}$$

where ΔA is the rotation angle, which is represented through the following expression:

$$\begin{bmatrix} \hat{\tau}_i \\ \upsilon_i \end{bmatrix} = \begin{bmatrix} \cos(\Delta A_i) & -\sin(\Delta A_i) \\ \sin(\Delta A_i) & \cos(\Delta A_i) \end{bmatrix} \begin{bmatrix} \tau_i \\ \upsilon_i \end{bmatrix} \tag{10}$$

Steps in solving the FGM(1,1, t^α) model

The first step is to utilize the quantum genetic algorithm to get the FGM model z and α terms that are representative of the optimal parameters.

The second step in a particular initial sequence requires calculating z-AGO series for the particular proposed time series by implementing the governing relation $X^{(z)} = X^{(0)} A^z$, and calculating the background values using $y^{(z)}(k)$.

In the third step, the initial sequence is swapped out for the background values and z-AGO series as presented in eq. (6), after which all relevant variables in this step (a, b, c) are calculated through computing the linear system in eq. (4).

As for the fourth step, this involves substituting all the variables $(\hat{a}, \hat{b}, \hat{c})$ in the relevant relation and then computing the z-AGO series $\hat{x}^{(z)}(k)$. Afterward, calculating the values anticipated of $\hat{x}^{(0)}(k)$ by utilizing eq. (6).

Finally, the last step focuses on the assessment indices of modeling accuracy, where MAPE would be employed to assess the model accuracy.

$$MAPE = \frac{1}{N-1} \sum_{t=2}^{N} \left| \frac{\hat{x}^{(0)}(t) - x^{(0)}(t)}{x^{(0)}(t)} \right| \times 100\% \tag{11}$$

Application

Numerical results

The NGM(1,1, k) [12], ARGM(1,1) [13], GM(1,1), DGM(1, 1), and the TDPGM(1, 1) [14] models are some examples of regularly employed prediction models with which the FGM(1,1, t^α) model outcome are collated, tab. 1. Some results of these models are shown in [15], and fig. 1 plots the related MAPE and APE. The parametric relationship between r and α, that can be regarded as key parameters in the FGM model, and MAPE is also plotted in fig. 2.

Table 1. Numerical results by the TDPGM (1, 1), GM (1,1), NGM (1,1, *k*), DGM (1, 1), ARGM (1, 1), and FGM (1,1, t^{α}) model

Year	Data	GM (1,1)	NGM (1,1, *k*)	ARGM (1,1)	TDPGM (1,1)	FGM (1,1, t^{α})
2000	8821	8821.00	8821.00	8821.00	8821.00	8821.00
2001	9062	9105.15	9117.26	9070.22	9076.93	9040.27
2002	9377	9350.43	9354.02	9325.83	9376.27	9375.51
2003	9692	9602.32	9599.49	9587.99	9641.87	9640.96
2004	9857	9861.00	9853.99	9856.86	9886.93	9871.84
2005	10055	10126.65	10117.86	10132.62	10125.36	10102.44
2006	10419	10399.45	10391.45	10415.44	10371.82	10353.75
2007	10636	10679.61	10675.10	10705.50	10641.73	10636.73
2008	10956	10967.30	10969.20	11003.00	10951.38	10956.11
2009	11307	11262.75	11274.12	11308.11	11317.89	11313.06
MAPE (%)		0.3922	0.3898	0.4010	0.2598	0.2325
2010	11894	11566.16	11590.26	11621.04	11759.32	11706.76
2011	12288	11877.75	11918.04	11941.99	12294.70	12135.47
2012	12714	12197.72	12257.89	12271.16	12944.07	12597.02
2013	13161	12526.32	12610.24	12608.76	13728.55	13089.20
2014	13755	12863.77	12975.56	12955.00	14670.38	13609.85
2015	14386	13210.31	13354.33	13310.12	15793.00	14157.00
2016	15003	13566.18	13747.04	13674.33	17121.12	14728.86
2017	15831	13931.64	14154.20	14047.87	18680.74	15323.81
2018	16658	14306.95	14576.35	14430.98	20499.29	15940.46
2019	17599	14692.36	15014.04	14823.90	22605.65	16577.53
MAPE (%)		8.1834	7.2322	7.5342	10.7371	2.2071

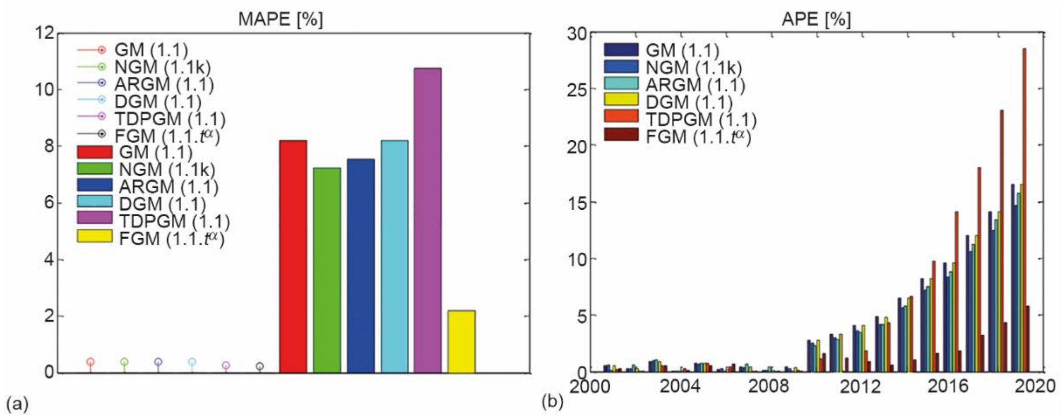

Figure 1. The APE and MAPE of the six prediction model

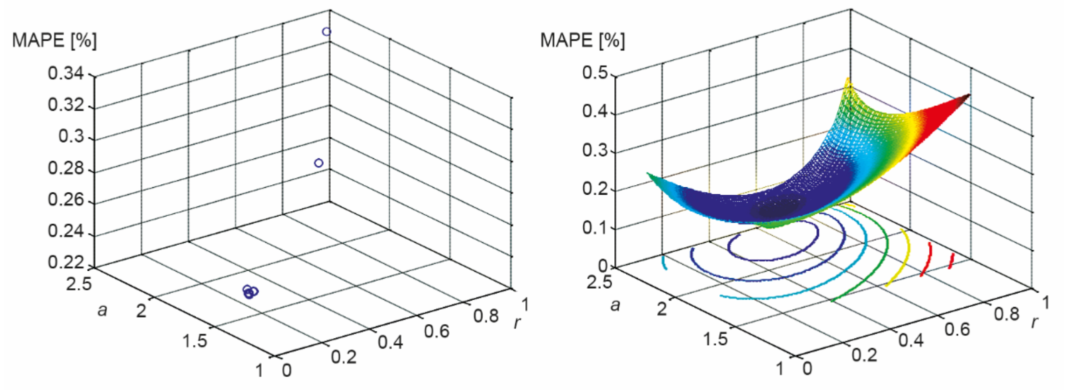

Figure 2 Relationship between FGM (1,1, t^{α}) model parameters *r*, *α*, and MAPE

The parameters $r = 0.16335845$ and $\alpha = 1.53172725$ in the FGM (1,1, t^{α}) model were computed through applying the principles of QGA. According to all prediction data shown in fig. 1, it is clearly observed that the TDPGM (1,1) model shows some variations where an overestimation in the actual quantity, whereas the remaining models GM (1,1), NGM (1,1, *k*), ARGM (1,1), and DGM (1,1) also show some variations that are regarded as an underestimation to the actual values. On the other hand, it is obvious that the FGM model shows more reliable predictions that are very similar to the real numbers. The FGM model has the lowest MAPE and APE among the six prediction models, as evidenced by the APE and MAPE in fig. 1. As a result, this model outperforms all the others under the current scenario conditions.

Forecasting results of China's elderly population from 2020 to 2029

This part employs the FGM model to forecast China's elderly population within the years 2020 and 2029 since its outcomes were the most accurate in predicting performance as seen in the preceding part. The prediction results are plotted in fig. 3. China's elderly population will surpass 30000 (10^4) by the year 2028. In addition, it is clearly obvious that the growth rate of China's elderly population will keep growing over the next several years. This is obviously a very serious situation. China's continually growing elderly population will put a lot of pressure on the government. We all knew that the main reason for China's growing elderly population is that many young couples find it difficult to shoulder the responsibility of raising children. House prices and life pressures cause young couples to be reluctant to have children. Given this state of affairs, the government should control the growth of home values and introduce some effective policies to encourage young couples to have children, in order to cope with the rapidly growing elderly population.

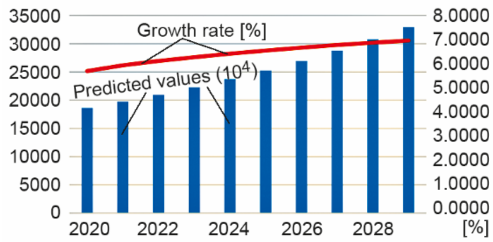

Figure 3. The number and growth rate of China's elderly population (2020-2029)

Conclusions

In order to anticipate China's old population, a novel FGM (1,1, t^α) model was put out in this study. The fractional order accumulation and grey system theory are the foundations upon which this model was built. The numerical outcomes demonstrate that compared to other grey prediction models, this one is better capable of anticipating the size of China's elderly population. China's elderly population is projected using the FGM (1,1, t^α) model from 2020 to 2029. This result shows that the number of China's elderly population will keep growing over the coming few decades, which is very serious for China. If the government cannot timely launch policies to cope with the continuous growth of China's elderly population, then our country will truly become an *elderly country*. To address this predicament, the Chinese government has to act decisively. The FGM (1,1, t^α) model projection findings show that the elderly population of China will keep growing in a manner that is extremely consistent with our country circumstances.

In conclusion, the FGM (1,1, t^α) model presented in this research is a useful tool for estimating the size of China's senior population and may be applied in various situations. For example, forecast the consumption of energy, forecast the emission of harmful gases, *etc.* However, this model is not entirely perfect and still includes several limitations and defects. When calculating the background values, the trapezoidal curve area is replaced by a trapezoidal relation in this model, that results in an inaccuracy. Therefore, it remains difficult to create another similar model that is more optimized and ideal.

Acknowledgment

This work is supported by the Key Project of Philosophy and Social Science Planning of Zhejiang Province (23NDJC055Z), the National Science Foundation of China (11971433), Zhejiang College of Shanghai University of Finance and Economics for Scientific Research Projects at the Provincial and Above Levels, and Humanities and Social Sciences Youth Foundation of Ministry of Education of China (23YJC630054).

References

[1] Deng, J. L., Control Problems of Grey Systems. *System Control Letter, 1* (1982), 5, pp. 288-294
[2] Wu, W. Q., *et al*. Forecasting Short-Term Renewable Energy Consumption of China Using a Novel Fractional Non-linear Grey Bernoulli Model, *Renewable Energy*, *140* (2019), Sept., pp. 70-87
[3] Yan, C., *et al*. Fractional Hausdorff Grey Model and Its Properties, *Chaos, Solitons & Fractals, 138* (2020), Sept., 109915
[4] Wu, L. F., *et al*. Using a Novel Multi-Variable Grey Model to Forecast the Electricity Consumption of Shandong Province in China, *Energy, 157* (2018), 15, pp. 327-335
[5] Wu, L. F., *et al*. Grey System Model with the Fractional Order Accumulation, *Communications in Nonlinear Science and Numerical Simulation, 18* (2013), 7, pp. 1775-1785
[6] Anjum, N., *et al*., Two-scale Fractal Theory for the Population Dynamics, *Fractals, 29* (2021), 7, 2150182
[7] Ma, X., *et al*., The Conformable Fractional Grey System Model, *ISA Transactions, 96* (2020), Jan., pp. 255-271
[8] Ma, X., *et al*., Improved GM (1,1) Model Based on Simpson Formula and Its Applications, *Journal of Grey System, 31* (2019), 4, pp. 33-46
[9] Wei, B. L., *et al*., Optimal Solution for Novel Grey Polynomial Prediction Model, *Applied Mathematical Modelling, 62* (2018), Oct., pp. 717-727
[10] Duan, H., Wang, G., Partial Differential Grey Model Based on Control Matrix and Its Application in Short-Term Traffic Flow Prediction, *Applied Mathematical Modelling*, *116* (2023), Apr., pp. 763-785
[11] Shen, Q. Q., *et al*., A Novel Weighted Fractional GM (1,1) Model and Its Applications, *Complexity 2020* (2020), 6570683

[12] Cui, J., *et al*., Novel Grey Forecasting Model and Its Modeling Mechanism, *Control and Decision 24* (2009), 11, pp. 1702-1706
[13] Qian, W. Y., *et al*., Grey GM (1,1, t$^{\alpha}$) Model with Time Power and Its Application, *Systems Engineering Theory Practice, 32* (2012), 10, pp. 2247-2252
[14] Ma, X., Liu, Z. B., Application of a Novel Time-Delayed Polynomial Grey Model to Predict the Natural Gas Consumption in China, *Journal of Computational and Applied Mathematics, 324* (2017), Nov., pp. 17-24
[15] Wang, L., *et al*., Two Novel Non-linear Multivariate Grey Models with Kernel Learning for Small-Sample Time Series Prediction [J],. *Non-linear Dynamics*, *111* (2023), 9, pp. 8571-8590

Paper submitted: November 1, 2022
Paper revised: May 23, 2023
Paper accepted: May 25, 2023