BIVARIATE AND TWO-PHASE DEGRADATION MODELING AND RELIABILITY ANALYSIS WITH RANDOM EFFECTS

by

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The paper aims at predicting the remaining useful life of highly reliable and long-life products with multiple and multi-stage characteristics in the degradation process. Considering the unit-to-unit variability among the product units, a new bivariate and two-phase Wiener process model with random effects is established. Schwarz Information Criterion is used to identify the change points of the degradation model, and the analytical expressions of life and remaining useful life are given by the concept of first hitting time. Furthermore, the appropriate Copula function is selected to describe the correlation between the two quality characteristics based on Akaike Information Criterion. A bivariate degradation model is established and the unknown parameters of the model are estimated by Markov Chain Monte Carlo method. Finally, the applicability and effectiveness of the proposed method are verified by the comparative analysis of turbine engine.

Key words: *two-phase Wiener process, bivariate Wiener process, remaining useful life estimation*

Introduction

With the development of science and technology, highly reliable and long-life products have been widely applied in engineering [1-4]. Due to the impact from a variety of random factors, the performance of these products will inevitably degrade, leading to product failure. For such products, the degradation model based on stochastic process is a good choice for life prediction [5], among which Wiener process-based model has been widely applied due to its good mathematical properties [6, 7].

In recent years, it was assumed that the product degradation process presents a single stage variation in the majority of Wiener degradation model. However, in engineering practice, because of the wear of the product, overload operation, environment change and other factors, there exist many different kinds of faults in the product degradation process. Most of these faults are not equal to the functional failure, which only exhibit the probability of degradation paths with two-phase or multi-phase pattern [8, 9]. Wang *et al.* [10] studied the trajectory change in the degradation process of display devices, and explained that the degradation trajectory change was a common phenomenon during the degradation process.

Modern reliability products usually have complex structure and diverse functions, which means that they may have two or more performance characteristics. Generally, the performance characteristics, which degrades over time, are not mutual independent. Therefore, it

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is important to find suitable joint distribution function to model the dependency of performance characteristics, which can predict the reliability of products accurately. In recent years, Copula function has been widely applied to estimate the remaining useful life of highly reliable products with multiple relevant performance characteristics [11, 12].

For meeting the requirements in engineering practice, this paper proposes a reliability model for bivariate and two-stage Wiener process with random effects. Wiener process with random effect is used to describe the measurement error and individual difference in the degradation process. Copula function is used to describe the correlation between the two performance characteristics, and unknown parameters are estimated by Markov chain Monte-Carlo (MCMC) method [13]. Finally, the data of turbine engine is taken as an example to verify the effectiveness of the proposed method and the degradation model.

Bivariate and two-phase degradation modelling with random effects

The bivariate and two-stage degradation process model is mainly focused on products with two performance indexes and two stages of degradation process. It is assumed that there is only one change point in the degradation process, the occurrence time of the change point is fixed, and the degradation amount at the change point is known when the single performance degradation process is described. Due to the correlation between the two performance indexes, the Copula function is used to describe the correlation.

Two-stage Wiener process degradation modelling with random effects

The paper denotes the degradation process of the equipment by $X(t)$. For the single-performance and single-stage Wiener degradation process [14], the model can be expressed:

$$X(t) = X(0) + \lambda t + \sigma B(t) \tag{1}$$

where $X(0)$ is the initial degradation amount, and $X(0) = 0$, λ – the drift coefficient, σ – the diffusion coefficient of the degradation process, and $B(t)$ – the standard Brownian motion.

Based on the previous model, the two-stage Wiener degradation process with change points is represented:

$$X(t) = \begin{cases} x(0) + \lambda_1 t + \sigma_1 B(t), & 0 < t \le \tau \\ x(\tau) + \lambda_2 (t-\tau) + \sigma_2 B(t-\tau), & t > \tau \end{cases} \tag{2}$$

where τ is the occurrence time of change point in the degradation process, $X(\tau)$ – the degradation amount at the change point, λ_1, λ_2 – the drift coefficient, σ_1 – the diffusion coefficient of the first-stage degradation process, and σ_2 – the diffusion coefficient of the second-stage degradation process.

In engineering practice, the individual difference means that the same batch of products are influenced by different external and internal factors. Because there are differences in the performance degradation process between different individuals, the introduction of individual differences in the degradation model is necessary. We assume that the drift coefficients λ_1 and λ_2 in the two-stage degradation process are random parameters, and distributions for the aforementioned parameters are specified as $\lambda_1 \sim N(\mu_{\lambda_1}, \sigma_{\lambda_1}^2)$, $\lambda_2 \sim N(\mu_{\lambda_2}, \sigma_{\lambda_2}^2)$, the diffusion coefficients σ_1 and σ_2 are determined parameters, and $\sigma_1 = \sigma_2 = \sigma$. Measurement errors are inevitable, which are influenced by factors of the sensor accuracy and operating environmental noise. Therefore, it is necessary to introduce the uncertainty of measurement error in the process of establishing the model. In order to represent the measurement error, the degradation process $\{Y(t), \mathrm{t} > 0\}$ is:

$$Y(t) = X(t) + \varepsilon \tag{3}$$

where ε is the measurement error, and $\varepsilon \sim N(0, d^2)$.

Suppose ω is the failure threshold of the performance degradation process. The T is defined as the failure time of degradation process when the degradation reaches the failure threshold for the first time:

$$T = \inf\{t : Y(t) \geq \omega \,|\, Y(0) < \omega\} \tag{4}$$

where *inf* denotes the infimum. Based on these assumptions, the probability density function and distribution function of the first hitting time of the performance index degradation process considering the individual difference of the drift coefficient and the measurement error are:

$$f(t) = \begin{cases} \dfrac{(\sigma_{\lambda_1}^2 t + \sigma^2)\omega + \mu_{\lambda_1} d^2}{\sqrt{2\pi(\sigma_{\lambda_1}^2 t^2 + \sigma^2 t + d^2)^3}} \exp\left\{\dfrac{\omega - \mu_{\lambda_1} t}{2(\sigma_{\lambda_1}^2 t^2 + \sigma^2 t + d^2)}\right\}, & 0 < t \leq \tau \\ \dfrac{[\sigma_{\lambda_2}^2 (t-\tau) + \sigma^2][\omega - X(\tau)] + \mu_{\lambda_2} d^2}{\sqrt{2\pi[\sigma_{\lambda_2}^2 (t-\tau)^2 + \sigma^2 (t-\tau) + d^2]^3}} \exp\left\{\dfrac{[\omega - X(\tau)] - \mu_{\lambda_2}(t-\tau)}{2[\sigma_{\lambda_2}^2 (t-\tau)^2 + \sigma^2 (t-\tau) + d^2]}\right\}, & t > \tau \end{cases} \tag{5}$$

$$F(t) = \begin{cases} \phi\left(\dfrac{\mu_{\lambda_1} t - \omega}{\sqrt{\sigma_{\lambda_1}^2 t^2 + \sigma^2 t + d^2}}\right) + \dfrac{\sigma^2}{\sqrt{\sigma^4 - 4\sigma_{\lambda_1}^2 d^2}} \cdot \\ \cdot \exp\left[\dfrac{2\omega(\mu_{\lambda_1}\sigma^2 + \sigma_{\lambda_1}^2 \omega) + 2\mu_{\lambda_1}^2 d^2}{\sigma^4 - 4\sigma_{\lambda_1}^2 d^2}\right] \cdot \\ \cdot \phi\left[\dfrac{\mu_{\lambda_1}(\sigma^2 t + 2d^2) + \omega(2\sigma_{\lambda_1}^2 t + \sigma^2)}{\sqrt{(\sigma^4 - 4\sigma_{\lambda_1}^2 d^2)(\sigma_{\lambda_1}^2 t^2 + \sigma^2 t + d^2)}}\right], \quad 0 < t \leq \tau \\ \phi\left\{\dfrac{\mu_{\lambda_2}(t-\tau) - [\omega - x(\tau)]}{\sqrt{\sigma_{\lambda_2}^2 (t-\tau)^2 + \sigma^2 (t-\tau) + d^2}}\right\} + \dfrac{\sigma^2}{\sqrt{\sigma^4 - 4\sigma_{\lambda_2}^2 d^2}} \cdot \\ \cdot \exp\left\{\dfrac{2[\omega - x(\tau)](\mu_{\lambda_2}\sigma^2 + \sigma_{\lambda_2}^2 \omega) + 2\mu_{\lambda_2}{}^2 d^2}{\sigma^4 - 4\sigma_{\lambda_2}^2 d^2}\right\} \cdot \\ \cdot \phi\left\{\dfrac{\mu_{\lambda_2}[\sigma^2 (t-\tau) + 2d^2] + [\omega - x(\tau)][2\sigma_{\lambda_2}^2 (t-\tau) + \sigma^2]}{\sqrt{(\sigma^4 - 4\sigma_{\lambda_2}^2 d^2)[\sigma_{\lambda_2}^2 (t-\tau)^2 + \sigma^2 (t-\tau) + d^2]}}\right\}, \quad t > \tau \end{cases} \tag{6}$$

Bivariate Wiener process degradation modelling

Facing with the complex system, it is not reasonable to use a single variable to describe the reliability of products. When multiple indexes are used to describe the reliability of a complex system, the correlation between multiple indexes must be considered. Moreover, the correlation between indexes is often not a simple linear relationship. Therefore, Copula function is introduced to describe the correlation between variables.

The Copula function was first proposed by Sklar [15], who proposed the idea so that multivariate joint distribution function can be cconstructed by associating the marginal distributions of the variables. Under a situation with two dimensions, the Copula function can be defined as a 2-D joint distribution function in the space of $[0, 1]^2$, in which the marginal distribution of each variable is uniformly distributed within the interval [0, 1].

$$F(x_1, x_2) = C[F_1(x_1), F_2(x_2); \theta] \tag{7}$$

where $F(x_1, x_2)$ is the joint distribution function of x_1 and x_2, $F_1(x_1)$ – the marginal distribution function of the variable x_1, and $F_2(x_2)$ – marginal distribution function of the variable x_2, $C(\bullet)$ – a Copula function, and θ – the correlation parameter of the Copula function. In this paper, the Copula function is used to describe the correlation between the two degradation performances. Table 1 shows three common types of Copula functions.

Table 1. Copula function types

Copula type	Copula distribution function $C(u_1, u_2; \theta)$	The value range of θ
Plackett [16]	$\dfrac{S - \sqrt{S^2 - 4u_1u_2\theta(\theta-1)}}{2(\theta-1)}, \quad S = 1 + (\theta-1)(u_1+u_2)$	$\theta \in (0, \infty) \setminus \{1\}$
Frank [16]	$-\dfrac{1}{\theta}\ln\left[1 + \dfrac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)}{e^{-\theta}-1}\right]$	$(-\infty, \infty) \cap (\theta \neq 0)$
Gaussian [16]	$\phi_2[\phi^{-1}(u_1), \phi^{-1}(u_2); \theta]$	[–1,1]

A bivariate dependent degradation model of the system is developed using the Copula function. It is assumed that $T^{(1)}$ and $T^{(2)}$ stand for the failure time of two performance characteristics, respectively. The reliability function is:

$$\begin{aligned} R(t) = P[T^{(1)} > t, T^{(2)} > t] = 1 - P[T^{(1)} \le t] - P[T^{(2)} \le t] + P[T^{(1)} \le t, T^{(2)} \le t] = \\ = R^{(1)}(t) + R^{(2)}(t) - 1 + C[F^{(1)}(t), F^{(2)}(t); \theta] \end{aligned} \tag{8}$$

Choosing different Copula functions will lead to different results. According to the actual situation, it is important to choose the appropriate Copula function.

Akaike information criterion (AIC) [16] is to evaluate the quality of model, which has a wide range of applicability. So, the paper utilizes the AIC function to select the appropriate Copula function, it is defined:

$$AIC = -2\ln[L(\hat{\theta})] + 2m \tag{9}$$

where $L(\hat{\theta})$ is the likelihood function of the model, and m is the number of unknown parameters in the likelihood function.

Parameter estimation

Suppose that there are N test units, and each unit is measured M times. Two performance indicators are measured each time. In a degradation test, let $y_{i,j}^{(k)}$ be the degradation observation of the ith unit at the measurement time $t_{i,j}$, $i(i=1,2,\cdots N)$, $j(j=1,2,\cdots M)$, $k(k=1,2)$, and assume that $\Delta t_{i,j}^{(k)} = t_{i,j}^{(k)} - t_{i,j-1}^{(k)}$, $\Delta y_{i,j}^{(k)} = y_{i,j}^{(k)} - y_{i,j-1}^{(k)}$ and $t_{i,0}^{(k)} = 0, y_{i,0}^{(k)} = 0.$

Assume that the time of occurrence of the change point is known and is measured at the moment of sampling. Let:

$$\Delta y_{1i,j}^{(k)} = \left\{ y_{i,0}^{(k)}, y_{i,1}^{(k)}, \cdots, y_{i,\tau_i^{(k)}}^{(k)} \right\}$$

denotes the first-stage degradation increment at time:

$$\Delta t_{1i,j}^{(k)} = \left\{ t_{i,0}, t_{i,1}, \cdots, t_{i,\tau_i^{(k)}} \right\}$$

and let:

$$\Delta y_{2i,j}^{(k)} = \left\{ y_{i,\tau_{i+1}^{(k)}}^{(k)}, y_{i,\tau_{i+2}^{(k)}}^{(k)},, \cdots, y_{i,m}^{(k)} \right\}$$

represents the second-stage degradation data at time:

$$\Delta t_{2i,j}^{(k)} = \left\{ t_{i,\tau_{i+1}^{(k)}}, t_{i,\tau_{i+2}^{(k)}}, \cdots, t_{i,m} \right\}.$$

According to the independent increment properties of the Wiener process:

$$\Delta y_{1i,j}^{(k)} = \left\{ y_{i,0}^{(k)}, y_{i,1}^{(k)}, \cdots, y_{i,\tau_i^{(k)}}^{(k)} \right\}$$

follows the multivariate normal distribution, and its distribution expectation and the covariance matrix are:

$$E[\Delta y_{i,j}^{(k)}] = \begin{cases} \mu_{\lambda_1}^{(k)} \Delta t_{1i,j}^{(k)} & 0 < t \le \tau_i^{(k)} \\ \mu_{\lambda_2}^{(k)} \Delta t_{2i,j}^{(k)} & t > \tau_i^{(k)} \end{cases} \tag{10}$$

$$\mathrm{Cov}[\Delta y_{i,j}^{(k)}] = \begin{cases} \sigma_{\lambda_1}^{(k)} \Delta t_{1i,j}^{(k)} [\Delta t_{1i,j}^{(k)}]^T + [\sigma^{(k)}]^2 K_{1i}^{(k)} + [d^{(k)}]^2 P_{1i}^{(k)} & 0 < t \le \tau_i^{(k)} \\ \sigma_{\lambda_2}^{(k)} \Delta t_{2i}^{(k)} [\Delta t_{1i,j}^{(k)}]^T + [\sigma^{(k)}]^2 K_{2i}^{(k)} + [d^{(k)}]^2 P_{2i}^{(k)} & t > \tau_i^{(k)} \end{cases} \tag{11}$$

where

$$K_{1i}^{(k)} = \mathrm{diag}[\Delta t_{1i}^{(k)}], \quad K_{2i}^{(k)} = \mathrm{diag}[\Delta t_{2i}^{(k)}],$$

$$P_{1i}^{(k)} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ 0 & -1 & 2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 & 2 \end{bmatrix}_{\tau_i^{(k)} \cdot \tau_i^{(k)}}, P_{2i}^{(k)} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ 0 & -1 & 2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 & 2 \end{bmatrix}_{[m-\tau_i^{(k)}]\cdot[m-\tau_i^{(k)}]}$$

The probability density function (PDF) of $\Delta y_{i,j}^{(k)}$ is:

$$f_{\Delta Y^{(k)}}[\Delta y^{(k)}] = \begin{cases} (2\pi)^{-\frac{\tau^{(k)}}{2}} \left|\Sigma_i^{(k)}\right|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\{\Delta y_i^{(k)} - E[\Delta y_i^{(k)}]\}^T [\Sigma_i^{(k)}]^{-1} \{\Delta y_i^{(k)} - E[\Delta y_i^{(k)}]\}\right) \\ \qquad 0 < t \le \tau_i^{(k)} \\ (2\pi)^{-\frac{m-\tau^{(k)}}{2}} \left|\Sigma_i^{(k)}\right|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\{\Delta y_i^{(k)} - E[\Delta y_i^{(k)}]\}^T [\Sigma_i^{(k)}]^{-1} \{\Delta y_i^{(k)} - E[\Delta y_i^{(k)}]\}\right) \\ \qquad t > \tau_i^{(k)} \end{cases} \tag{12}$$

According to the relevant knowledge of Copula function, the joint probability density function of $\Delta y^{(1)}$ and $\Delta y^{(2)}$ is:

$$f[\Delta y^{(1)}, \Delta y^{(2)}] = c\{F[\Delta y^{(1)}], F[\Delta y^{(2)}]; \theta\} f_{\Delta Y^{(1)}}[\Delta y^{(1)}] f_{\Delta Y^{(2)}}[\Delta y^{(2)}] \tag{13}$$

Determining the estimated value of the model parameters is the premise of reliability analysis using the bivariate two-phase Wiener process model. Based on Bayesian theorem, the joint posterior distribution of unknown parameters is:

$$\pi(\Theta|\Delta y) \propto L(\Delta y|\Theta)\pi(\Theta)$$

where $\pi(\Theta) = \pi[\mu_{\lambda_1}^{(1)}, \sigma_{\lambda_1}^{(1)}, \mu_{\lambda_2}^{(1)}, \sigma_{\lambda_2}^{(1)}, \sigma^{(1)}, d^{(1)}, \mu_{\lambda_1}^{(2)}, \sigma_{\lambda_1}^{(2)}, \mu_{\lambda_2}^{(2)}, \sigma_{\lambda_2}^{(2)}, \sigma^{(2)}, d^{(2)}, \theta]$ is the joint prior distribution of related parameters. The parameters to be estimated are Θ=$[\Theta^{(1)}, \Theta^{(2)}, \theta], [\Theta^{(k)} = \mu_{\lambda_1}^{(k)}, \sigma_{\lambda_1}^{(k)}, \mu_{\lambda_2}^{(k)}, \sigma_{\lambda_2}^{(k)}, \sigma^{(k)}, d^{(k)} (k=1,2)]$. It is difficult to obtain several model parameters estimates directly by optimizing the log-likelihood function, and two change points in the model also bring about difficulties to solve the parameters of the model. For the above problems, this paper uses the Gibbs sampling method [13] in the MCMC algorithm to estimate the unknown parameters of the model. There are many ways to specify the prior distribution of the model which will not be introduced here because they are not the focus of this article. In this paper, the prior distribution of all parameters is assumed to be normal distribution, and the parameters will be estimated only by the degradation observations.

Change-point detection

The SIC function was proposed by Schwarz in 1978 [17], which has been applied to determine whether the model has a change point problem. The principle is that if there is a change point in the sequence, the entropy of the samples is greater than that of the samples which have no change point. The SIC function is defined [17]:

$$SIC = -2\ln L(\theta) + p\ln m$$

where $L(\theta)$ is the maximum likelihood function of the model, p – the number of free parameters in the model, and m – the sample size. The assumptions which are made based on the SIC function include the following two hypotheses.

Original hypothesis H_0: There is no change point in the model if each parameter is equal. The $SIC(m)$ based on original hypothesis H_0 is:

$$SIC(m) = m\ln 2\pi + m\ln\frac{1}{m}\sum_{1}^{m}(\Delta x_i - \Delta\bar{x})^2 + m + (2-m)\ln m$$

where

$$\Delta\bar{x} = \frac{1}{m}\sum_{1}^{m}\Delta x_i$$

Alternative hypothesis H_1: There is a change point $\tau_i^{(k)}$, which divides the degradation process into two stages. The value of $SIC(k)$ based on the alternative hypothesis H_1 is [17]:

$$SIC(k) = m\ln 2\pi + k\ln\frac{1}{k}\sum_{1}^{k}(\Delta x_i - \Delta\overline{x_1})^2 + 4\ln m + (m-k)\ln\frac{1}{m}\sum_{k+1}^{m}(\Delta x_i - \Delta\overline{x_2})^2 - m$$

where

$$\Delta\overline{x_1} = \frac{1}{k}\sum_{1}^{k}\Delta x_i, \quad \Delta\overline{x_2} = \frac{1}{m-k}\sum_{k+1}^{m}\Delta x_i$$

If

$$SIC(m) > \min_{2<k\le m-2} SIC(k)$$

the original hypothesis H_0 will be rejected, which means there is a change point. At the same time, the estimated change point value $\hat{\tau} = t_k$ is:

$$SIC(k) = \min_{2<k\le m-2} SIC(k)$$

Example verification

Reliability optimization design is a hot topic in engineering applications [18, 19]. In order to verify the effectiveness of the proposed method, this paper uses the FD001 data set of NASA's C-MAPSS data set. The FD001 data set simulates the degradation data of each station of the machine during the recession of the high-pressure compressor. Each engine is equipped with 21 sensors. In order to improve the accuracy of remaining life prediction and calculate conveniently, this paper selects the data collected from No. 9 sensor and No. 14 sensor of the No. 11 engine for modeling.

A large amount of random white noise is introduced into this data. Because the noise source is complex and unavailable, it is difficult to use the original observation data directly. In this paper, the moving average filtering [20] is used to eliminate the high frequency fluctuation of degradation path. An alternative promising filtering is to use the tropical algebra [21, 22], which will be discussed in future.

Firstly, the two quality characteristics are detected by the SIC function to determine the occurrence time of the change point. Then, the corresponding SIC values are calculated respectively and the values of change point time are obtained to be $\tau^{(1)} = 185$(cycle), $\tau^{(2)} = 164$(cycle).

Next, the parameter vectors $\Theta^{(1)}$ and $\Theta^{(2)}$ are estimated according to the MCMC algorithm [23], as shown in tab. 2.

Table 2. Estimation of parameters

Parameter	$\mu^{(1)}$	$\sigma^{(1)}$	$\mu^{(1)}$	$\sigma^{(1)}$	$\sigma^{(1)}$	$d^{(1)}$
Value	0.1492	0.003743	1.2061	0.001571	0.1994	0.003004
Parameter	$\mu^{(2)}$	$\sigma^{(2)}$	$\mu^{(2)}$	$\sigma^{(2)}$	$\sigma^{(2)}$	$d^{(2)}$
Value	0.1001	0.004499	0.6461	0.002008	0.2036	0.002981

It can be seen from tab. 2 that there are obvious differences between the first and second stage model parameters for both PC1 and PC2, which shows that the rate of degradation has changed at different stages. In addition, the corresponding Copula parameters are given for different Copula functions. In order to compare the fitting performance of each model, the corresponding AIC values are given in tab. 3.

Table 3. The AIC for the degradation models

Copula function	Gaussian	Frank	Plackett
θ value	0.8099	2.5503	16.1457
AIC	–132.3824	–262.6132	–89.1166

From tab. 3, it can be seen that the value of AIC for Frank Copula function is smaller than the other two AIC values. According to the fitting results, Frank Copula is more suitable for describing the degradation modeling of turbine engines.

Conclusion

In this paper, aiming at several common problems of degradation equipment in engineering, we establish corresponding degradation models and perform remaining life prediction. To reflect the unit-to-unit variability, random effects are incorporated into a two-phase Wiener process model with measurement errors. In addition, the paper assumes that a product has two quality characteristics, and the dependence of them is described by the Copula function. Then, we establish the bivariate Wiener process model and derive the PDF analytic expression of a two-phase Wiener process model with random effects and measurement errors under the concept of first hitting time. The paper also evaluates product remaining useful life based on the Winner process. Gibbs sampling in MCMC algorithm is used to gain the estimators of unknown parameters of the models. Finally, the effectiveness of the proposed models and methods are verified by the comparative analysis of turbine engine example. The future research frontiers are the stochastic process modeling for renewable energy with time variability [24], the stochastic fractal to reliability optimization [25], fractal degradation models [26] for nano/micro devises [27, 28].

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