ASSESSMENT OF REGIONAL WATER RESOURCE CARRYING CAPACITY BY THE CONNECTION NUMBER OF SET PAIR ANALYSIS

by

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Original scientific paper
https://doi.org/10.2298/TSCI2403287J

In order to evaluate regional water resource carrying capacity, the subtraction set pair analysis is adopted to calculate the connection number, and an evaluation model of the regional water resource carrying capacity based on quadratic subtraction set pair potential is established. The model is applied to assess the water resource carrying capacity of 16 cities in Anhui province, China, 2015. The results show that the water resource carrying capacity was sever throughout the province in 2015. Specifically, the carrying capacity in southern part of Anhui province was better than that in the northern part. The main factor of water resource carrying status varied in different cities.

Key words: *water resource carrying capacity, connection number, Anhui, set pair analysis, subtraction set pair potential*

Introduction

Water resources carrying capacity (WRCC) refers to the population size, economic and social development scale that water resources can continuously support in certain region. The WRCC is closely related to water resource system, ecological environment, economy and society, thus is an important indicator for regional water resource security measurement [1]. The WRCC reflects that the balanced relationship between regional water resources carrying support and pressure forces under the action of regulation force control [2], and is crucial to judge regional water resources security [3, 4]. One of the difficulties is how to establish a reasonable regional WRCC evaluation method according to the proximity relationship between evaluation object and the carrying grade standard. At present, the main assessment method includes fuzzy comprehensive evaluation and the principal component analysis [5, 6]. However, these two methods have limitations in dealing with the uncertainty of proximity between the evaluation index and the evaluation grade standard [7-9]. Zhao [10] proposed the pair potential of connection number on the basis of connection number. The set pair analysis connection number, especially the subtraction set pair potential method, has been applied in the environment and resources evaluation [2]. Luo and Yang [11] built a logistic set pair analysis model

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by combining set pair analysis theory and logistic model to assess of the suitability of human settlements. In order to further allocate the uncertainty information remaining in the difference degree to the same and opposite degree, this paper applies the subtraction set pair potential to measure connection number twice and establishes water resource carrying capacity assessment method based on the quadratic subtraction set pair potential method (QSSPP), so as to further improve the reliability of the evaluation results and carry out the corresponding empirical study of WRCC.

Establishment of regional WRCC evaluation method based on quadratic subtraction set pair potential

The connection number was used to analyze the proximity information among the index sample, index subsystem, index value, and grade standard of regional WRCC. The connection number is described from three aspects: same relationship degree, difference relationship degree and opposition relationship degree [2, 4, 10]. The QSSPP method for evaluating regional WRCC is constructed based on the connection number and quadratic set pair potential. The establishment process includes six steps.

Step 1. For analytic hierarchy process to select indicators, the index system $\{x_j|j = 1, 2, \ldots, n_j\}$ and grade standard $\{s_{kj}|k = 1, 2,\ldots, n_k; j = 1, 2, \ldots, n_j\}$ of regional WRCC are constructed considering carrying support, pressure and regulation force of water resources system. The corresponding evaluation index sample data set is denoted as $\{x_{ij}|I = 1, 2, \ldots, n_i; j = 1, 2, \ldots, n_j\}$, where x_j is the j^{th} indicator in the carrying capacity evaluation index system. n_i, n_j, n_k are the number of evaluation samples, evaluation indicators and grades of evaluation criteria, respectively. The analytic hierarchy process is used to calculate each subsystem and index weight $\{w_j|j = 1, 2, \ldots, n_j\}$, where w_j is the weight of the j^{th} indicator [1, 4, 11].

Step 2. Calculate the connection number of the regional WRCC indicators. To be specifically, it is calculated according to numbers of each index that fall into grade 1, grade 2, and grade 3, respectively [2, 4]:

$$u_{1i} = v_{1i1} + v_{1i2}I + v_{1i3}J = a + bI + cJ \tag{1}$$

where u_{1i} is the ternary connection number of sample i ($i = 1, 2, \ldots, n_i$), a, b, and c correspond to the same degree, difference degree and opposite degree, respectively. Equation (1) satisfies $a,b,c \in [0, 1]$ and $a + b + c = 1$. I, J are the coefficient of difference degree and opposition degree, respectively, v_{1ik} is the connection number component of u_{1i}, including same, difference, and opposite degree, as shown in eq. (2) [2, 4].

$$v_{1i1} = \sum_{j=1}^{n_a} w_j, \quad v_{1i2} = \sum_{j=n_a+1}^{n_a+n_b} w_j, \quad v_{1i3} = \sum_{j=n_a+n_b+1}^{n_a+n_b+n_c} w_j \tag{2}$$

where w_j is the weight of the j^{th} indicator which can be optimized by fuzzy analytic hierarchy process based on accelerated genetic algorithm, n_a, n_b, and n_c are numbers of indicators among the n_j indicators of sample i that fall in grade 1, grade 2, and grade 3, $n_a + n_b + n_c = n_j$.

Step 3. Calculate the index value connection number of the regional WRCC evaluation samples. The connection number of evaluation index value u_{2ijk} ($i = 1, 2, \ldots, n_i$; $j = 1, 2, \ldots, n_j$; $k = 1, 2, 3$) is measured according to the similarity between the sample value x_{ij} and the evaluation standard grade s_{kj}:

$$u_{2ij1}=1$$
$$u_{2ij2}=1-\frac{2(s_{1j}-x_{ij})}{s_{1j}-s_{0j}} \text{ (Positive indicator } s_{0j}< x_{ij}\leqslant s_{1j}\text{, or negative indicator } s_{0j}> x_{ij}\leqslant s_{1j}) \quad (3)$$
$$u_{2ij3}=-1$$

$$u_{2ij1}=1-2(x_{ij}-s_{1j})/(s_{2j}-s_{1j})$$
$$u_{2ij2}=1, \text{(Positive indicator } s_{1j}< x_{ij}\le s_{2j}\text{, or negative indicator } s_{1j}> x_{ij}\ge s_{2j}) \quad (4)$$
$$u_{2ij3}=1-(s_{2j}-x_{ij})/(s_{2j}-s_{1j})$$

$$u_{2ij1}=-1$$
$$u_{2ij2}=1-\frac{2(x_{ij}-s_{2j})}{s_{3j}-s_{2j}} \text{ (Positive indicator } s_{2j}< x_{ij}\leqslant s_{3j}\text{, or negative indicator } s_{2j}> x_{ij}\ge s_{3j}) \quad (5)$$
$$u_{2ij3}=1$$

where the positive index value x_{ij} (the negative index value) increases (decreases) with the increase of grade k of the evaluation standard, such as the indicator urbanization rate (water resources per capita), s_{1j} and s_2 are the critical values of evaluation indicators between grade 1 and grade 2 of the evaluation grade standard, and between grade 2 and grade 3, respectively, s_{0j} and s_{3j} are the other critical values of the evaluation criteria of grade 1 and grade 3 of each index, i = 1, 2, …, n_i, j = 1, 2, …, n_j. The relative membership degree of the sample value x_{ij} belonging to the fuzzy set *evaluation standard level k* can be expressed as in eq. (6) [2, 4]:

$$v_{2ijk}^{*}=0.5+0.5u_{2ijk} \quad (i = 1, 2, \dots, n_i;\ j = 1, 2, \dots, n_j;\ k = 1, 2, \dots, n_k) \quad (6)$$

From the normalized eq. (6), the connection number component v_{2ijk} of the WRCC evaluation sample is obtained [2, 4]:

$$v_{2ijk}=v_{2ijk}^{*}/\sum_{k=1}^{3}v_{2ijk}^{*} \quad (7)$$

The connection number of u_{2ij} can be composed by the connection number component v_{2ijk}, as shown in eq. (8) [2, 4]:

$$u_{2ij}=v_{2ij1}+v_{2ij2}I+v_{2ij3}J \quad (8)$$

where I, J are the coefficient of difference degree and opposite degree, respectively. Thus, the index value connection number u_{2i} of WRCC evaluation sample i can be obtained, as show in eq. (9) [2, 4]:

$$u_{2i}=v_{2i1}+v_{2i2}I+v_{2i3}J=\sum_{j=1}^{n_j}w_j v_{2ij1}+\sum_{j=1}^{n_j}w_j v_{2ij2}I+\sum_{j=1}^{n_j}w_j v_{2ij3}J \quad (i = 1, 2, \dots, n_i) \quad (9)$$

where I, J are the coefficient of difference degree and opposite degree, respectively.

Step 4. Calculate the average connection number u_i of sample i. The distribution $\{v_{ik}|k = 1\sim3\}$ of the same, difference and opposite degree of the connection number u_i should be as close as possible to the distribution of $\{v_{1ik}|k = 1\sim3\}$ of the index numbers connection number in eq. (2). Meanwhile, the index value connection number of the evaluation sample in eq. (9), with as little information required as possible. According to the principle of minimum relative entropy [2, 4]:

$$v_{ik} = (v_{1ik} v_{2ik})^{0.5} / \sum_{k=1}^{3} (v_{1ik} v_{2ik})^{0.5}, \quad u_i = v_{i1} + v_{i2} I + v_{i3} J \quad (i = 1,2,\ldots, n_i) \tag{10}$$

Step 5. Derive WRCC evaluation grade value of sample i using eigenvalue method:

$$h(i) = \sum_{k=1}^{3} v_{ik} k \tag{11}$$

To facilitate analysis and comparison of the evaluation results and improve the reliability of the evaluation results, the attribute recognition method is used to infer the WRCC evaluation grade value of sample *i* corresponding to the average connection number simultaneously [2, 4].

$$g(i) = \min_{k^*} \{k^* \mid \sum_{k=1}^{k^*} v_{ik} > \lambda\} \tag{12}$$

where λ is confidence which is generally within [0.50, 0.70].

Step 6. The connection number value is calculated by using subtraction set pair potential method twice, and then the evaluation grade of regional WRCC is determined. Jin *et al.* [2] proposed to use the subtraction set pair potential to judge the evaluation level of WRCC and the subtraction set pair potential $s_1(u)$ of the ternary connection number was defined:

$$s_1(u) = a + ba - c - bc \tag{13}$$

Equation (13) means that the uncertainty term bI in the ternary connection number $u = a + bI + cJ$ of eq. (1) is decomposed into the same degree term $(a + ba)$ and the opposite degree term $(c - bc)$ according to the ratio of deterministic relation values $a/(a + b + c)$ and $b/(a + b + c)$. After the subtraction set pair potential operation in this way, it is obvious that the uncertainty term $b(1 - a - c)I = bbI$ is still left in the difference degree term of the ternary connection number of eq. (1).

The semi-partial subtraction set pair potential of the ternary connection number in eq. (1) is [2, 4]:

$$s_2(u) = a + ba/(a+b) - c - bc/(b+c) \tag{14}$$

There is a new ternary relation corresponding to eq. (13):

$$u' = (a + ba) + bbI' + (c + bc)J \tag{15}$$

where the uncertainty degree of the residual difference degree coefficient I' is smaller than that of the original ternary connection number $u = a + bI + cJ$ of eq. (1). A reasonable assumption is that the uncertainty degree of the remaining difference degree coefficient I' varies within the interval [–0.5, 0.5]. Therefore, the subtraction set pair potential calculation formula of eq. (13) can be referred to and the subtraction set pair potential formula can be written as in eq. (15).

$$s_3(u) = (a+ba) + 0.5bb(a+ba)/(a+ba+bb+c+bc) - (c+bc) - \\ -0.5bb(c+bc)/(a+ba+bb+c+bc) \tag{16}$$

Since $a + b + c = 1$, then in the denominator of eq. (16), $(a + ba + bb + c + bc) = [a + c + b\,(a + b + c)] = 1$. Equation (16) can be simplified:

$$s_3(u) = (a-c)(1+b)(1+0.5bb) \tag{17}$$

Equation (17) is named as quadratic subtraction set pair potential (QSSPP). The QSSPP in eq. (17) further determines the uncertainty of the difference degree term than eq. (13). Therefore, the QSSPP is more reasonable in reflecting the relative deterministic trend of the research object expressed by the ternary relation number of eq. (1) at the current level of macro expectation.

Since there is $(a + b + c = 1)$ in eq. (1), it is obvious that when a=1, the QSSPP $s_3(u)$ in eq. (17) takes the maximum value 1, while when $c = 1$, $s_3(u)$ takes the minimum value –1. Therefore, according to the *equipartition principle* [4, 10], the value interval [–1, 1] of the $s_3(u)$ can be divided into three potential levels: counter potential $s_3(u) \in [-1.0, -0.333)$, balanced potential $s_3(u) \in [-0.333, 0.333]$, and homo-potential $s_3(u) \in (0.333, 1.0]$ correspond to grade 3, grade 2, and grade 1 of the evaluation results of regional WRCC.

In order to further compare the numerical closeness among the QSSPP $s_3(u)$, the subtraction set pair potential $s_1(u)$ and the semi-partial subtraction set pair potential $s_2(u)$, the average absolute error is derived by:

$$d_1 = \sum_{m=1}^{M} |\, s_3(u_m) - s_1(u_m)\,|/M \tag{18}$$

$$d_2 = \sum_{m=1}^{M} |\, s_3(u_m) - s_2(u_m)\,|/M \tag{19}$$

where M is the number of random simulations of the ternary connection number. When M is 10^3, 10^4, 10^5, and 10^6, d_1 is 0.021, 0.022, 0.022, and 0.022, and d_2 is 0.032, 0.034, 0.034, and 0.034, respectively. It indicates that the $s_3(u)$ in eq. (17) is numerically very close to the $s_1(u)$ in eq. (13) and $s_2(u)$ in eq. (14). The three can be used to determine the overall development trend of the deterministic level of the macroscopic set pair system.

A linear relationship between the $s_3(u)$ and the regional WRCC evaluation level value $g(u)$ is assumed. If $s_3(u)$ takes 1 and –1, then $g(u)$ takes grade 1 and 3 respectively. If the value of $s_3(u)$ varies between [–1, 1], then the value of $g(u)$ varies between [1, 3], respectively. The calculation formula for estimating $g(u)$ from $s_3(u)$ can be described:

$$g(u) = 2 - s_3(u) \tag{20}$$

Application of QSSPP in the evaluation of WRCC in Anhui

The mentioned QSSPP method is used to comprehensively evaluate the status of WRCC in Anhui and 16 prefectural cities in 2015. The evaluation grade standards and index weights of WRCC in Anhui are shown in [4]. Sample values $\{x_{ij}\}$ of evaluation indicators of prefecture-level cities in Anhui in 2015 are obtained according to Statistical Yearbook of Anhui Province and Water Resources Bulletin of Anhui Province [12]. The average connection number of 16 cities in Anhui in 2015 can be measured by substituting eqs. (1) to (10), and then using eqs. (17) and (20) to obtain the quadratic subtraction set pair potential and evaluation grade values, as shown in tab. 1.

Table 1. Set pair potential of the connection number and the grad of WRCC in various cities of Anhui Province

Code	City	The average connection number [4]	QSSPP	Subtraction set pair potential [4]	Evaluation level from eigenvalue method, attribute recognition method [4]; QSSPP method
1	Hefei	0.1512 + 0.5137I + 0.3351J	–0.32	–0.28	2.18, 2; 2.32
2	Huaibei	0.1210 + 0.2141I + 0.6649J	–0.68	–0.66	2.54, 3; 2.68
3	Bozhou	0.0832 + 0.3742I + 0.5425J	–0.68	–0.63	2.46, 3; 2.68
4	Suzhou	0.1451 + 0.3263I + 0.5286J	–0.54	–0.51	2.38, 3; 2.54
5	Bengbu	0.0303 + 0.5642I + 0.4054J	–0.68	–0.58	2.38, 2; 2.68
6	Fuyang	0.0690 + 0.3493I + 0.5817J	–0.73	–0.69	2.51, 3; 2.73
7	Huainan	0.0318 + 0.3866I + 0.5816J	–0.82	–0.76	2.55, 3; 2.82
8	Chuzhou	0.2067 + 0.6983I + 0.0950J	0.24	0.19	1.89, 2; 1.76
9	Lu’an	0.3545 + 0.6331I + 0.0124J	0.67	0.56	1.66, 2; 1.33
10	Ma’anshan	0.1050 + 0.4751I + 0.4199J	–0.52	–0.46	2.31, 2; 2.52
11	Wuhu	0.1136 + 0.8181I + 0.0683J	0.11	0.08	1.95, 2; 1.89
12	Xuancheng	0.5234 + 0.4766I + 0J	0.86	0.77	1.48, 1; 1.14
13	Tongling	0.3435 + 0.5545I + 0.1020J	0.43	0.38	1.76, 2; 1.57
14	Chizhou	0.6507 + 0.3398I + 0.0095J	0.91	0.86	1.36, 1; 1.09
15	Anqing	0.3983 + 0.5771I + 0.0245J	0.69	0.59	1.63, 2; 1.31
16	Huangshan	0.7558 + 0.2155I + 0.0286J	0.90	0.88	1.27, 1; 1.10

Table 1 shows that the subtraction set pair potential of the average connection number is close to the results of QSSPP for WRCC evaluation in prefecture-level cities in 2015. When the subtraction set pair potential is less than zero (greater than grade 2) and in the opposite potential, the QSSPP value is smaller than the subtraction set pair potential value, and vice versa. The QSSPP further utilizes the remaining uncertainty information of the difference degree term after the subtraction set pair potential extracted the deterministic overall development trend information expressed by the connection number. The correction amplitude of QSSPP is related to the absolute value and difference degree of subtraction set pair potential ($0.5bbI$). For instance, the absolute value of subtraction set against potential in Anqing City was 0.59, and the difference degree value was 0.5771. The correction range of QSSPP reached 0.098 and the corresponding evaluation grade value decreased from 1.63 to 1.31, with a significant correction range. However, although the absolute value of 0.66 in Huaibei City is large, the difference degree value of 0.2141 is small. The correction range of QSSPP is only

0.015, and the corresponding evaluation grade value increases from 2.54 to 2.68, with a small correction range. Similarly, the difference degree value of 0.8181 is relatively large in Wuhu, but the absolute value of the subtraction set pair potential is relatively small at 0.08. The correction amplitude of QSSPP is only 0.027, and the corresponding evaluation grade value is reduced from 1.95 to 1.89, with a relatively small correction amplitude.

The quadratic subtractive set pair potential corresponding to the average connection number of the water resources carrying support, pressure and regulation force in 16 cities of Anhui Province in 2015 is substituted into eq. (20), so as to obtain the corresponding evaluation grade value, as shown in fig. 1. Hefei city has the highest water resources carrying support force. Although the regulation force is loadable state and the pressure force is partial loadable state, the WRCC of Hefei is critical or close to the boundary of overload. The 2nd to 7th overload cities are all from northern Anhui province. The main reason is that the corresponding water resources carrying support force is in overload state and the regulation force is also relatively weak. Chuzhou and Ma'anshan are in critical state of water resources carrying support force. As the pressure of Chuzhou is significantly lower than Ma'anshan and the WRCC state of Chuzhou is obviously better than Ma'anshan. The supporting capacity and regulating capacity of water resources support and regulation force in No. 9 and 11-16 cities are loadable or slightly loadable. The WRCC in these cities is in loadable or slightly loadable state, especially in Xuancheng (12th), Chizhou (14th) and Huangshan (16th). The WRCC evaluation grade value line of series 4 is relatively close to the water resources carrying support evaluation grade value line of series 1, indicating that the water resources carrying support subsystem is the main factor affecting the water resources carrying status.

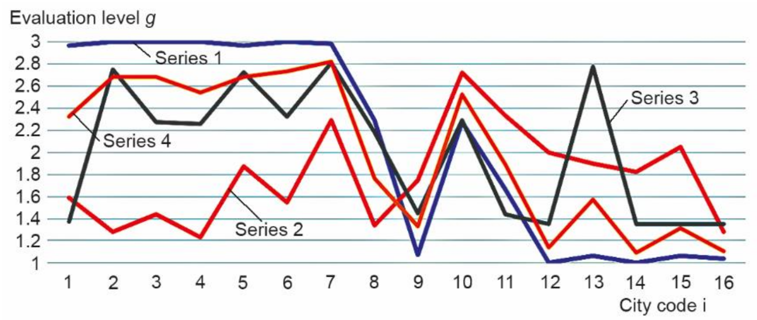

Figure 1. Evaluation level values of water resource carrying capacity and its subsystems in various cities of Anhui Province in 2015 (Note: Series 1, 2, 3, and 4 represent the evaluation level values of water resource carrying support, pressure, regulation force subsystems, and the carrying capacity system, respectively)

Conclusion

This study establishes a regional WRCC evaluation method, QSSPP, to explore the application of the set pair analysis connection number method in the evaluation of regional WRCC. The subtraction set pair potential is proposed to calculate the connection number twice to further excavate the uncertainty information of the difference degree term and the information about the proximity uncertainty between the evaluation sample index and the evaluation grade standard. The correction range of QSSPP is related to the absolute value and difference degree of subtraction set pair potential. The range of correction is 0.10 in some cases and 0.01 in others. The average range of correction is about 0.02. The application of WRCC evaluation in the 16 cities in Anhui province shows that the water resource carrying capacity was relatively severe and the southern cities have better WRCC than the northern cities. The QSSPP provides an efficient method in multi-indication evaluation of water resources regimes.

Acknowledgment

This work was supported by the Project of National Natural Foundation of China (No. 52179001 and U2243228) and China Postdoctoral Science Foundation (No. 2022M713119).

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Paper submitted: April 23, 2023
Paper revised: May 11, 2023
Paper accepted: May 28, 2023