APPLYING NUMERICAL CONTROL TO ANALYZE THE PULL-IN STABILITY OF MEMS SYSTEMS

by

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The micro-electro-mechanical system is widely used for energy harvesting and thermal wind sensor, its efficiency and reliability depend upon the pull-in instability. This paper studies a micro-electro-mechanical system using He-Liu [34] formulation for finding its frequency-amplitude relationship. The system periodic motion, pull-in instability and pseudo-periodic motion are discussed. This paper offers a new window for security monitoring of the system reliable operation.

Key words: *micro-electro-mechanical systems, pull-in instability, He's frequency formulation, dynamic characteristic*

Introduction

In the last decades, the micro-electro-mechanical system (MEMS) has seen its wide applications in various fields such as in microdevices [1], microsensors [2, 3], microgenerators [4-6], especially the applications of the MEMS systems to energy harvesting devices and thermal wind sensors have been caught much attention in both industry and academy.

The primary cause for MEMS utilization is its micro construction, exquisite sensitivity, and low power consumption. However, the pull-in instability [7-9] of the system is a relatively under-acknowledged issue that can lead to unreliability, so we place special emphasis on this. As an example, Zhang *et al*. [10] presented a dynamic pull-in instability using a mass-spring system, and a sudden collapse might occur due to the interaction of kinetic and potential energy. Tian *et al.* [11, 12] suggested the fractal MEMS system to overcome the pull-in instability, He *et al.* [13] provides a simple method for its security monitoring in a fractal MEMS system. Beside the pull-in instability, some researchers also found the pull-down instability in quadratic non-linear oscillators [14].

This paper aims to the design of the MEMS micro-actuator. An electric current runs through a conductor, and the Biot-Savat theorem dictates that every line produces a magnetic field. According to Lorenz's theorem, the wire located on the contrary end will be exposed to a magnetic force, an electric current creates a force, articulated:

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$$f=\frac{\mu_o i_1 i_2 l}{2\pi R}=\frac{\mu_o i_1 i_2 l}{2\pi(b-\tilde{x})} \tag{1}$$

where $\mu_o = 4\pi 10^{-7}\,NA^2$ is the magnetic coefficient, i_1 – the internal flow in the movable wire, i_2 – the internal flow in the fixed wire, l – the length of the wire, R – the original distance between the movable part of the wire and the fixed wire, and $\tilde{x}$ – the displacement at time of $\tilde{t}$.

The spring system produces a non-linear restoring force:

$$F = k\tilde{x} + k_1\tilde{x}^3 \tag{2}$$

where k is the elastic coefficient and k_1 – the coefficient of the non-linear term.

In accordance with Newton's Second law, the governing equation of movement for the wire particle is expressed in the form of the following differential equation:

$$m\frac{\mathrm{d}^2\tilde{x}}{\mathrm{d}\tilde{t}^2}+k\tilde{x}+k_1\tilde{x}^3-\frac{\mu_o i_1 i_2 l}{2\pi(b-\tilde{x})}=0 \tag{3}$$

where m is the rod mass connected to the spring system, $\tilde{x}(t)$ – the horizontal displacement at time $\tilde{t}$. Taking $x=\tilde{x}/b,\ t=\tilde{t}\,\omega_o,\ \omega_o{}^2=k/m,\ K=(\mu_o i_1 i_2 l)/[2\pi(b-\tilde{x})]$, and $\varepsilon=(k_1 b^2)/(m\omega_o^2)$, eq. (3) can be reduced to the following form:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}+x+\varepsilon x^3-\frac{K}{1-x}=0,\quad x(0)=0\quad\text{and}\quad\frac{\mathrm{d}x}{\mathrm{d}t}(0)=0 \tag{4}$$

where K is the voltage parameter of the system.

Equation (4) is called MEMS oscillator. The singularity at $x = 1$ makes it much complex [15] and different from some well-known oscillators like the Helmholtz-Duffing oscillator [16], van der Pol-Duffing oscillator [17], and the spring-pendulum system [18]. When the excitation parameter K is beneath the threshold value K^* [19-23], MEMS oscillator will display a periodic behavior. Upon K exceeding an established threshold, the pull-in instability will ensue. The following transcendental equation is obtained by simplifying the simulation equation for the case when $\varepsilon = 0$ [22, 23]:

$$\left(\frac{1+\sqrt{1-4K}}{2}\right)^2+2K\ln\left|1-\frac{1+\sqrt{1-4K}}{2}\right|=0 \tag{5}$$

The threshold value is $K^* = 0.203632188...$ [22, 23]. If $K_1 < K^*$, the answer to eq. (4) is periodic. If K_2 is larger than K^*, eq. (4) will bring about the pull-in instability. In fact, it is hard to identify the threshold value and there is an uncertain area of $K^*\in(K_1,K_2)$ in the application process, beyond that the system could demonstrates either periodic properties or be unstable [24].

The amplitude can be expressed in terms of the smallest positive root $s_K\in(0,1)$ of the function:

$$f_K(s)=-s^2-2K\ln(1-s)=0 \tag{6}$$

Through elliptic integration, the magnitude and rate of the phenomenon can be quantified numerically [22].

$$\omega_K = \frac{\pi}{\int_0^{s_K} \frac{\mathrm{d}s}{\sqrt{-s^2 - 2K\ln(1-s)}}} \tag{7}$$

Pull-in instability

When the initial conditions are applied, the approximate solutions are difficult to be obtained by some matured analytical methods, such as the variational iterative method [25, 26] or the homotopy perturbation method [27-33]. To solve this conundrum, we introduce the following transformation:

$$x = A - u, \quad A < 1 \tag{8}$$

According to eq. (4), the direction of acceleration for a linear spring sinusoidal vibration is inversely related to its horizontal displacement in all its repeated motions. Let's consider that the differential equations are in the broadest sense:

$$\frac{\mathrm{d}^2u}{\mathrm{d}t^2} + P(u) = \frac{\mathrm{d}^2u}{\mathrm{d}t^2} + \left[\frac{P(u)}{u}\right]u = 0 \tag{9}$$

where $\mathrm{d}^2u/\mathrm{d}t^2$ is the acceleration and u presents displacement. The relation between frequency and amplitude can be expressed:

$$\omega^2 = \frac{P(u)}{u} \tag{10}$$

The function $P(u)$ for the MEMS system can be denoted:

$$P(u) = x + \varepsilon x^3 - \frac{K}{1-x} = A - u + \varepsilon(A-u)^3 - \frac{K}{1-A+u} \tag{11}$$

Subsequently, the criterion for evaluating the period is that $P(u)/u$ is positive for $t > 0$.

Determination of pull-in instability

Using Taylor series, eq. (4) can be reduced to the following form:

$$\frac{\mathrm{d}^2u}{\mathrm{d}t^2} + u - \varepsilon(A-u)^3 + \frac{K}{1-A}\left[-\frac{u}{1-A} + \frac{u^2}{(1-A)^2} - \frac{u^3}{(1-A)^3} + \cdots\right] + \frac{K}{1-A} - A = 0 \tag{12}$$

We set:

$$\frac{K}{1-A} - A = 0 \tag{13}$$

Solving A from eq. (13), we can obtain:

$$A = \frac{1 - \sqrt{1-4K}}{2} \tag{14}$$

Equation (4) shows that if $K = 0$, the single non-linear oscillator is modified to the Duffing oscillator and $P(u) = u + \varepsilon u^3$,which is $P(u)/u = 1 + \varepsilon u^2 > 0$. Therefore, when $\varepsilon > 0$, a periodic solution is present.

If the values of $P(u)/u$ and t meet the requirement of being greater than zero, eq. (8) will contain a periodic solution:

$$\frac{P(u)}{u} > 0, \qquad t > 0 \tag{15}$$

For MEMS systems, the foundation of pull-in instability judgement is dependent on aperiodic judgement. Consequently, we can obtain the pull-in instability condition with necessary conditions like this:

$$P(u) = x + \varepsilon x^3 - \frac{K}{1-x} = A - u + \varepsilon(A-u)^3 - \frac{K}{1-A+u} < 0 \tag{16}$$

Under the previous condition, the MEMS system will be rendered unstable. By utilizing Legendre polynomials, we can determine the area of pull-in instability, then integrate eq. (4) from zero to one, followed by incorporating a weight function u^6 then we can obtain the result:

$$\int_0^1 u^5\left[A - u + \varepsilon(A-u)^3 - \frac{K}{1-A+u}\right]\mathrm{d}u = \int_0^1 \frac{1-\sqrt{1-4K}}{2}u^5 - u^6 + \varepsilon u\left(\frac{1-\sqrt{1-4K}}{2} - u\right)^3 -$$

$$- \frac{Ku^5}{1 - \dfrac{1-\sqrt{1-4K}}{2} + u}\mathrm{d}u = \frac{1}{6} + \frac{1}{8}\varepsilon - \frac{96}{35}K < 0 \tag{17}$$

The judgment basis of pull-in instability is:

$$K > K_1 = \frac{35}{576} + \frac{35}{768}\varepsilon \tag{18}$$

Meeting the requirements of eq. (18) guarantees the pull-in instability. As an illustration $\varepsilon = 1$, $K_1 = 35/576 + 35/768\varepsilon = 0.10633681$. When the excitation parameter K is bigger than K_1, the approximate solutions that aligns accurately with the reference results. When $K = 0.35$, the pull-in instability can be forecasted, as depicted in fig. 1(c).

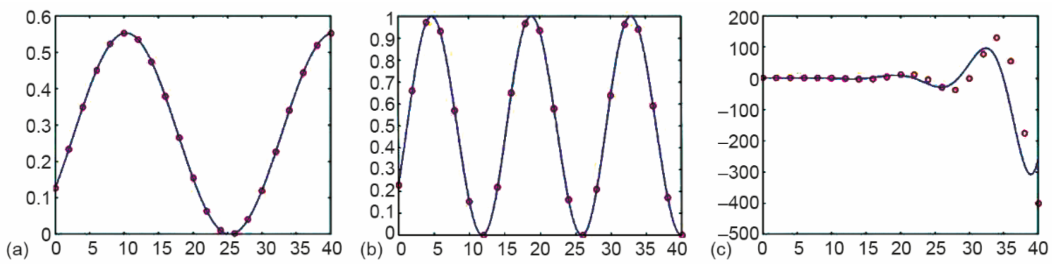

Figure 1. The pull-in instability be depicted when choose different value *K*; (a) *K* = 0.2, (b) *K* = 0.25, and (c) *K* = 0.35

Evaluation of pull-in instability

In this section, He-Liu [34] formulation for finding the frequency-amplitude relationship is used:

$$\omega^2 = \frac{\int_0^A \lambda(u)P(u)\mathrm{d}u}{\int_0^A \lambda(u)u\mathrm{d}u} \tag{19}$$

where $\lambda(u)$ is the weight function. He-Liu [34] formulation of eq. (19) is an extension of Ji-Huan He's frequency formulations [35, 36], which read:

$$\omega^2 = \left.\frac{P(u)}{u}\right|_{u=\frac{\sqrt{3}}{2}A} \quad \text{and} \quad \omega^2 = \left.\frac{P(u)}{u}\right|_{u=\frac{A}{2}}$$

various modifications were also appeared in literature [37-42]. Here we reveal the formulation [33] is the most suitable mathematical tool to MEMS oscillators.

By the formulation, we have:

$$\omega^2 = \frac{\int_0^A \lambda(u)\left[A-u+\varepsilon(A-u)^3-\frac{K}{1-A+u}\right]\mathrm{d}u}{\int_0^A \lambda(u)u\mathrm{d}u} =$$

$$= \frac{\int_0^A \lambda(u)\left(\frac{u\{[(\sqrt{1-4K}-3)K-\sqrt{1-4K}+1]\varepsilon-2\sqrt{1-4K}\}-K(2K+\sqrt{1-4K}-1)\varepsilon-2u^2}{\sqrt{1-4K}+2u+1}\right)\mathrm{d}u}{\int_0^A \lambda(u)u\mathrm{d}u} \tag{20}$$

We get an approximate periodic solution:

$$x(t) = A[1-\cos(\omega t+\sigma)] = 2A\sin^2\left(\frac{\omega t+\sigma}{2}\right) \tag{21}$$

The amplitude can be worked out through the eq. (14), and the final approximate solution is:

$$x(t) = (1-\sqrt{1-4K})\sin^2\left(\frac{\sqrt{\omega^2}t+\sigma}{2}\right) \tag{22}$$

In order to select an appropriate $\lambda(u)=(1-u)u^3$ for the MEMS system, we set $A=1$ so that:

$$\omega^2 = \frac{\int_0^1 (1-u)u^3 \left(\frac{u\{[(\sqrt{1-4K}-3)K-\sqrt{1-4K}+1]\varepsilon - 2\sqrt{1-4K}\} - \atop -K(2K+\sqrt{1-4K}-1)\varepsilon - 2u^2}{\sqrt{1-4K}+2u+1} \right) \mathrm{d}u}{\int_0^1 (1-u)u^4 \mathrm{d}u}$$

$$= 42\left(-\frac{K}{4}+\frac{\varepsilon}{56}+\frac{1}{30}\right) > 0 \tag{23}$$

The essential requirements for MEMS systems which have an approximate periodic solution is:

$$K < K_2 = \frac{2}{15} + \frac{1}{14}\varepsilon \tag{24}$$

Choosing different values of the excitation parameter $K < K_2$, the approximate periodic solutions corresponding to ω are in a good agreement with the reference numerical solutions within a certain range. When $K = 0.05$, we can predict the periodic phenomenon of the system, as shown in fig. 2. When $\varepsilon = 1$ and $K = 0.25$, the period is calculated accordingly.

$$T = \frac{2\pi}{2\omega} = \frac{\pi}{\sqrt{-\frac{21K}{2}+\frac{3\varepsilon}{4}+\frac{7}{5}}} = 6.16117 \tag{25}$$

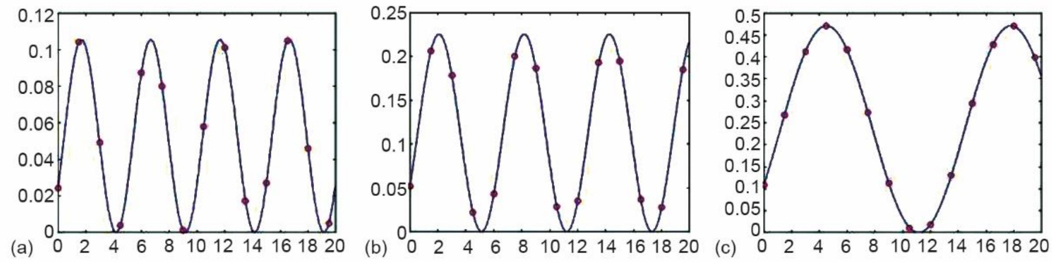

Figure 2. The approximate periodic solutions compare with the reference numerical solutions when choose different value *K*; (a) *K* = 0.05, (b) *K* = 0.1, and (c) *K* = 0.18

When $K \in [K_1, K_2]$, there is an uncertain region, then MEMS may be in a state of periodic motion or a pull-in unstable state. There may be no obvious separation region to distinguish whether the MEMS system is in a state of periodic motion or a pull-in unstable state, or there may be a pseudo-pull-in unstable state. Typically, the MEMS system has cyclical motion, but after a cycle of periodic motion, the system will slowly becomes pull-in unstable. As shown in fig. 3, when $S = 100$, $K = 4.6180556$ are selected, the pseudo-periodic motion becomes pull-in instability.

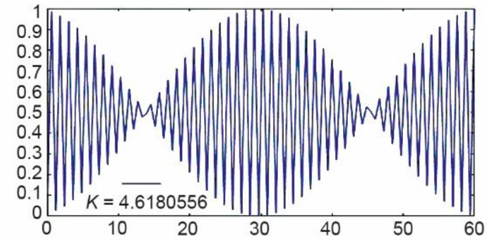

Figure 3. The pseudo-periodic motion becomes pull-in instability when *K* = 4.6180556

Establishing $P(u)/u > 0$ is less challenging if $P(u)$ is a function of eq. (9). In the event that $P(u)$ is not an odd function, eq. (19) is still generally accurate, yet its even term will significantly influence the frequency properties of

MEMS system oscillator. An odd term that is not included in eq. (9) that will cause the solution to be non-repeating, which can cause unsteadiness. As can be seen from fig. 3, when ε is larger, the spring potential is less stable, but when $\varepsilon = 100$, the energy consumed by the spring potential takes a longer time.

Conclusion

This paper shows an oscillation in the pseudo-pull-in stabilization in the periodic movement, but it will enter the phase of the pull-in instability gradually. With the help of mathematical concepts, we improve the handling method for MEMS systems. This simple approximation method quickly grasps the oscillator dynamic features and has a certain value for the development of MEMS devices.

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