NON-NEWTONIAN NATURAL-CONVECTION IN A SQUARE BOX SUBMITTED TO HORIZONTAL HEAT FLUX AND MAGNETIC FIELD

by

Redouane NOURI*, Mourad KADDIRI, Youssef TIZAKAST, and Hamza DAGHAB

Industrial Engineering and Surface Engineering Laboratory, FST, Sultan Moulay Slimane University Beni-Mellal, Beni-Mellal, Morocco

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The current work numerically investigates free convection in a square box filled with a non-Newtonian conductive fluid, which is numerically analyzed and is subjected to a steady heat flux at the normal walls, while the horizontal walls are thought of as adiabatic. To examine the impacts of the regulating parameters, including Rayleigh number, behavior index, n, and Hartmann number, on fluid-flow and heat transfer, the governing equations are solved numerically by applying the finite volume method. The results are shown and analyzed in terms of streamlines, isotherms, flow intensity, medium Nusselt, and velocity profiles.

Key words: natural-convection, non-Newtonian fluids, finite volume method, magnetic field

Introduction

Natural-convection, which produces fluid movement due to density variation engendered by an applied temperature gradient, attracts attention owing to its various applications in the industrial field such as the production of electric power, cooling of electronic components, heat exchangers, solar energy collectors, chemical processes, food processing, and material processing [1, 2].

Given the linked physical phenomena that appeared, scientific researchers concentrated particularly on the impact of the magnetic flux on free convection. The magnetic force caused by the presence of magnetic flux can be exploited in applications such as the control of crystal growth. For example, Vives and Peery [3] showed that the consequence of a magnetic flux at the time of the solidification process is characterized primarily by a reduction in superheat escape and an increase in consolidation rate. The focus has been mainly directed towards square cavities, where industrial plants and electronic component cooling are some of the crucial applications that are usually identified by the existence of an intense magnetic flux that reigns in the surrounding space, hence, the usefulness of introducing the magnetic flux in free convection studies. Further, the working fluids are generally non-Newtonian fluids. Many published works investigate the rheological behavior of non-Newtonian fluids in the absence of a magnetic flux [4-7]. Turan *et al.* [8] demonstrated that the mean Nusselt value augments with enhancing values of and reducing values of the power-law index, while the impact of the Prandtl number, is deemed to be negligible. Furthermore, for sufficiently high values of *n*, the Nusselt number, was found to be equal to unity, Nu =1, as heat transfer is mainly assured by conduction.

^{*} Corresponding author, e-mail: redouanech1990@gmail.com

In the existence of a magnetic flux, the literature review reveals different works that investigate the consequences of free convection within square cavities filled with Newtonian fluids under varying boundary conditions [9-17]. In all these works, intensifying the magnetic flux is always found to diminish convective fluid-flow and heat transfer. However, and as mentioned before, the fluids encountered in the industrial files are mostly non-Newtonian, making the study of natural-convection for non-Newtonian fluids under the effect of magnetic field an essential research topic. Yet, the number of investigations conducted does not reflect their importance. For instance, Kefayati [18] used the finite difference lattice Boltzmann method, to examine the effect of magnetic flux on natural-convection for non-Newtonian power-law fluids in a portion of a heated container. The obtained results showed that extending the behavior index in the non-existence of the magnetic flux reduces the heat transfer rate, while introducing the magnetic flux diminishes the observed effect of the behavior index on heat transfer. On the other hand, the influence of the behavior index strengthened with increasing the size of the active heated section. Dimitrienko [19] studied the laminar MHD free convection of a non-Newtonian fluid in a square box under a constant magnetic flux in various directions. This study demonstrated that the angle of inclination has an important influence on flow and heat transfer in addition the magnetic flux. Liao et al. [20] operated a digital study of free convection induced by a thermally driven flow under the impact of an inclined magnetic flux considering the effects of Rayleigh numer, Hartmann number, and magnetic flux angles. Their findings demonstrated that the direction of the applied magnetic flux had a substantial impact on the streamlines and isotherms furthermore, as the Hartmann number increases, as the applied magnetic field grew stronger, the mean Nusselt number and maximum streamline function declined. Makaysi et al. [21] numerically studied the transfer of heat and mass in a square enclosure filled with an electrically conductive non-Newtonian fluid in the presence of an inclined external magnetic flux. The authors observed that the increase in Hartmann number leads to the decrease in the flow intensity, heat, and mass transfer rates dropped for both Newtonian and non-Newtonian fluids. Further, for a weak magnetic flux, decreasing the behavior index significantly enhanced fluid circulation and heat and mass transfer, while for a great value of, the mentioned effect of starts to vanish. They also found that the orientation of the applied magnetics flux strongly affected heat and mass transfer. Kefayati [22] investigated the effect of a magnetic flux on free convection in a cavity with a linearly heated wall and filled with non-Newtonian power law fluids. Similarly, they reported the suppressing effect of the magnetic flux on heat transfer rate and the influence of the behavior index. According to the findings of Chtaibi et al. [23], an increase in the power index or Hartmann number has an adverse impact on both flow intensity and heat transfer. These outcomes were obtained through the application of the Boltzmann lattice Boltzmann method and involved a uniform magnetic field influencing a non-Newtonian fluid in a state of natural-convection within an inclined square cavity. In a similar vein, Nouri *et al.* [24] conducted a numerical assessment of the natural Rayleigh-Benard convection in a square cavity filled with a non-Newtonian fluid whose viscosity is temperature-dependent. Their investigation highlights that the initiation of convection is delayed with higher values of the power index, n, and Hartmann number. Moreover, it indicates a reduction in both flow intensity and heat transfer as the Hartmann number increases, a trend observed for both Newtonian and non-Newtonian fluids. Notably, at significantly elevated Hartmann number values, heat transfer is predominantly governed by the conduction regime.

The primary objective of this study is to investigate the heat transfer characteristics by free convection within a closed cavity charged with non-Newtonian fluids in the presence of a magnetic flux. Unlike previous research that predominantly considered Dirichlet thermal

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conditions at the cavity borders, this study specifically aims to explore the influence of Neumann-type thermal boundary conditions in conjunction with the magnetic flux. The existing literature has primarily focused on Dirichlet thermal conditions, leaving a notable gap regarding the consideration of Neumann-type thermal boundary conditions. By incorporating this aspect into the study, we aim to provide a more comprehensive understanding of the thermal behavior within closed cavities with non-Newtonian fluids. The inclusion of a magnetic flux parameter introduces a practical dimension the investigation. Understanding how magnetic flux interacts with non-Newtonian fluids in closed cavities can have implications in various fields. In summa-

ry, this research not only aims to fill a specific gap in the current literature but also strives to advance our understanding of heat transfer in complex systems, providing insights with practical applications and contributing to the broader theoretical framework in fluid dynamics.

Mathematical formulation

Figure 1 shows the studied configuration: a 2-D square cavity subjected to a horizontal magnetic field and heat flux on the vertical walls while the horizontal walls are insulated. The cavity is filled with non-Newtonian electically conductive fluids, whose rheological behavior can be characterized by the Ostwald-de Waele power-law model:



Figure 1. Schematic representation of the cavity with imposed thermal conditions and magnetic field

$$\mu_{a}' = k_{T} \left\{ 2 \left[\left(\frac{\partial U'}{\partial X'} \right)^{2} + \left(\frac{\partial V'}{\partial Y'} \right)^{2} \right] + \left(\frac{\partial U'}{\partial Y'} + \frac{\partial V'}{\partial X'} \right)^{2} \right\}^{\frac{n_{k}-1}{2}}$$
(1)

where n_k is the behavior of the flow and k_T – the consistency index and which are generally temperature dependent, However, the change of n as a function of temperature is negligible, $(n_k \approx \text{constant} = n)$ with respect to that of k_T , This may be calculated using the Frank-Kamenetskii exponential rule:

$$k_T = k e^{-b \left(T'^2 - T_r'^2\right)}$$

where b is the thermal dependence coefficient, is an exponent connected to the flow energy activation and the universal gas constant and k – the consistency index at the reference temperature, T_r .

For n = 1, the behavior is Newtonian, for 0 < n < 1, the apparent viscosity reduces with the shear rate and the behavior is pseudo plastic (or shears thinning), and for n > 1, the viscosity increases as the shear rate rises, and the behavior is dilatant (or shear-thickening).

Concerning the dimensionless variables, the characteristic scales:

$$H', \ \rho\left(\frac{\alpha^2}{{H'}^2}\right), \frac{{H'}^2}{\alpha}, \frac{\alpha}{H'}, \frac{q'H'}{\lambda}$$

corresponding to length, pressure, time, velocity, temperature and stream function, respectively, are used. As a result, the dimensionless governing equations are given:

- Continuity equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

– Momentum equation

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$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr\left[2\frac{\partial \mu_{a}}{\partial x}\frac{\partial U}{\partial X} + \frac{\partial \mu_{a}}{\partial Y}\left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right) + \mu_{a}\left(\frac{\partial^{2}U}{\partial Y^{2}} + \frac{\partial^{2}U}{\partial X^{2}}\right)\right]$$
(3)

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr\left[2\frac{\partial \mu_{a}}{\partial Y}\frac{\partial V}{\partial Y} + \frac{\partial \mu_{a}}{\partial X}\left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right) + \mu_{a}\left(\frac{\partial^{2} V}{\partial X^{2}} + \frac{\partial^{2} V}{\partial Y^{2}}\right) + \operatorname{Ra}T\right] - \operatorname{Ha}^{2}\operatorname{Pr}V \quad (4)$$

Energy equation

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}$$
(5)

with

$$\mu_{a} = \left\{ 2 \left[\left(\frac{\partial U}{\partial X} \right)^{2} + \left(\frac{\partial V}{\partial Y} \right)^{2} \right] + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^{2} \right\}^{\frac{n-1}{2}}$$
(6)

The stream function Ψ is used to study the flux structure:

$$U = \frac{\partial \Psi}{\partial X}, \quad V = -\frac{\partial \Psi}{\partial Y} \tag{7}$$

Dominant parameters

In addition the power-law behavior index, n, three other dimensionless parameters appear, namely Rayleigh, Prandtl, and Harman numbers:

$$\operatorname{Ra} = \frac{g\beta H'^{(2+2n)}q'}{\left(\frac{K_T}{\rho}\right)\alpha^n\lambda}, \ \operatorname{Pr} = \frac{\left(\frac{K_T}{\rho}\right)H'^{(2-2n)}}{\alpha^{2-n}}, \ \operatorname{Ha} = B'_0H'^n\left(\frac{\sigma}{k}\alpha^{1-n}\right)^{1/2}$$
(8)

We mention that Prandtl number is fixed at 100 and for the present case, the associated non-dimensional boundary conditions are:

$$U = V = \Psi = \frac{\partial T}{\partial x} + 1 = 0, \text{ for } x = 0 \text{ and } x = 1$$
(9)

and

$$U = V = \Psi = \frac{\partial T}{\partial x} = \frac{\partial T}{\partial y} = 0 \text{ for } y = 0 \text{ and } y = 1$$
 (10)

Heat transfer

The Nusselt number, measuring local heat transfer in the horizontal direction is:

$$\operatorname{Nu}(y) = \frac{1}{\Delta T(y)}$$

where $\Delta T(y) = T(0, y) - T(1, y)$ which represent the dimensionless local temperature difference between the two vertical walls x = 0 and x = 1.

For the average horizontal Nusselt number describing the overall horizontal heat transfer the equation is:

$$Nu = \int_{0}^{1} Nu(y) dY$$
(11)

Numerical solution

The preceding eqs. (2)-(5) associated with the boundary conditions eq. (10) can be expressed [12]:

$$\frac{\partial}{\partial X} \left(U \boldsymbol{\Phi} - r \frac{\partial \boldsymbol{\Phi}}{\partial X} \right) + \frac{\partial}{\partial Y} \left(V \boldsymbol{\Phi} - r \frac{\partial \boldsymbol{\Phi}}{\partial Y} \right) = S_{\boldsymbol{\Phi}}$$
(12)

where Φ is the variable which can be either *T*, *U*, or *V*. To find the momentum equation, *r* is replaced by $Pr\mu_a$, and for the energy equation it is set to 1 where S_{ϕ} is the source term. To obtain a numerical solution, eq. (12) must be converted into a linear system:

$$A_P \Phi_P = A_W \Phi_W + A_E \Phi_E + A_S \Phi_S + A_N \Phi_N + S_{\Phi}$$
(13)

where Φ_P are the variables U, V, and T on Point P and the eq. (13) is the discretized equation that connects the calculation point to its adjacent grid point.

The discretized system is composed of a set of linear algebraic equations that can be quickly resolved using the line-by-line method based on the Thomas algorithm, [25-31]. The SIMPLE technique [29] is used to solve the connection between velocity and pressure. As for the convergence of the solution:

$$\max\left(\frac{\left(\boldsymbol{\varPhi}\right)^{n+1} - \left(\boldsymbol{\varPhi}\right)^{n}}{\left(\boldsymbol{\varPhi}\right)^{n+1}}\right) \le 10^{-6} \tag{14}$$

Grid size

Table 1 in vestigates the variation of the results as a function of the number of grid points in order to identify the optimal mesh size that leads to a satisfactory balance between accuracy and computation time. The results obtained for $Ra = 10^4$, n = 0.6, and Ha = 10, show that the 120×120 grid is sufficient to accurately simulate the problem at hand.

Grids	Nu	$U_{\rm max}$	$V_{\rm max}$	$ \Psi_{max} $	Deviation [%]
80×80	2.335	11.038	11.206	2.669	_
120 × 120	2.334	11.053	11.242	2.668	0.32%
200×200	2.335	11.054	11.246	2.665	0.112%
300 × 300	2.334	11.055	11.248	2.669	0.149%

Table 1. Maximum stream function $|\Psi_{max}|$ and \overline{Nu} inside the enclosure for various mesh sizes

Numerical code validation

We compared our numerical results to previously published ones in order to confirm the numerical code. Table 2 presents the results in terms of Nu and $|\Psi_{max}|$. As can be seen, the agreement is reasonable, with the deviation not exceeding 4.2%, indicating the precision of the adotep numerical code.

			Present study		[29]		[30]		[31]		
Ra	Pr	п	На	$ \Psi_{max} $	Nu	$ \Psi_{max} $	Nu	$ \Psi_{max} $	$\overline{\mathrm{Nu}}$	$ \Psi_{max} $	\overline{Nu}
106	0.7	1	0	17.07	8.98	17.00	8.90	_	_	16.75	8.80
			50	10.79	6.39	10.51	6.39	-	_	_	_
			150	3.91	2.57	3.77	2.64	-	_	_	-
105	10 ²	1	0	9.67	4.51	9.75	4.62	_	4.70	_	_
		0.6		28.49	14.99	_	-	-	15	_	_
		1.8		3.19	1.57	-	_	_	1.55	_	_

 Table 2. Validation of current numerical results with previously published research for different n, Prandtl, Hartmann, and Rayleigh numbers values

Results and discussions

Effects of the Hartmann number on the dynamic and thermal structure of the flow

Figures 2 and 3 depict the streamlines for various values of the Rayleigh number, n, and Hartmann number with the magnetic flux applied in the *x*-direction. This analysis aims to examine the flow structure within the square cavities. It is notable that the flow structure undergoes changes in the presence of the magnetic flux (Ha = 60), where the streamlines elongate in the vertical direction perpendicular to the magnetic flux. This effect is more pronounced in the central region of the cavity, giving rise to the appearance of two small cells. The observed change becomes more evident as the n decreases and the Rayleigh number increases. This alteration can be attributed to the impact of the magnetic force, acting perpendicular to the magnetic flux, on the fluid-flow. Additionally, it is worth mentioning that the streamlines become



Figure 2. Streamlines for $Ra = 10^4$ and various values of the power-low index *n* and Hartmann number Ha = 0 (a) and Ha = 60 (b)

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more densely packed and nearly parallel to the adiabatic walls as both Hartmann and Rayleigh numbrs increase.



Figure 3. Streamlines for $Ra = 10^6$ and various values of the behavior index *n* and Hartmann number Ha = 0 (a) and Ha = 60 (b)

The observed changes can be interpreted as a result of the combined effects of magnetic attraction, fluid properties, and buoyancy forces. At high Hartmann numbers, the magnetic field intensifies, promoting the deformation of the flow structure under the Lorentz force with an increase in Rayleigh number or a decrease in the n. The heightened flow intensity further amplifies the impact of the Lorentz force, leading to the elongation of the central cells and the formation of convective cells near the adiabatic horizontal walls.

These phenomena can be explained by the synergistic effect of the magnetic force and Archimedes' thrust. At sufficiently high Hartmann numbers values, the flow is predominantly influenced by the magnetic force, which acts perpendicular to the direction of the magnetic field. For instance, when the magnetic field is horizontal in the *x*-direction, the magnetic force acts solely in the *y*-direction. Considering symmetry, the velocity u dominates along the vertical axis due to v = 0, indicating that the total velocity is nearly parallel to the magnetic field. Consequently, the magnetic force approaches zero since it can counteract the buoyancy effect. As a result, conductive fluids are stretched closer to the horizontal axis.

The corresponding isotherms are presented in figs. 4 and 5, and they exhibit a tendency to align parallel to the active normal walls as Hartmann number and n increase or Rayleigh number decreases. These findings suggest that convective heat transfer weakens while heat conduction intensifies. Consequently, the Lorentz force is enhanced by the magnetic flux, mitigating the convective regime by counteracting the buoyancy force responsible for natural-convection, which shows Nevaux that the magnetic field suppresses convection.



Figure 5. Isotherms for $Ra = 10^6$ and various values of the behavior index *n* and Hartmann number Ha = 0 (a) and Ha = 60 (b)

Effects of Hartmann number on flow and heat transfer

Figure 6 displays the profiles of the normal velocity, v, in the cavity center for various Rayleigh and Hartmann numbers, and n. The velocity maximum strongly decreases as

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(b)

Hartmann numbers increases confirming that the magnetic field slows down fluid circulation. The fluid-flow intensifies for increasing Rayleigh number or decreasing as natural-convection intensifies where the observed diminishing effect of Hartmann number on velocity profiles strengthens. We also notice that the velocity maximum value shifts toward the vertical active walls as Hartmann numbers and Rayleigh number increases and *n* decreases where the heart region of the enclosure becomes stagnant.



Figure 6. Normal velocity profiles at the center of the cavity (y = 1/2) for various values of the Hartman number, *n*, and Raleigh number, Ra = 10⁴ (a) and Ra = 10⁶ (b)

Figure 7 depicts the evolutions of flow intensity with the regulating Hartmann number, behavior index, and Rayleigh number. As expected, the fluid-flow intensifies as Rayleigh number augments due to the enhanced contribution of buoyancy force. As for the Hartmann number, increasing it reduces Ψ_{max} confirming what we mentioned about the magnetic field slowing down fluid circulation. Furthermore, a decreasing power-law behavior index, *n*, strengthens fluid-flow as the fluid becomes less resistant to motion; however, the magnitude of the effect diminishes as the applied magnetic field further intensifies.

The variations of the average Nusselt number are illustrated in fig. 8 for different values of the Hartmann number, power-law index, and Rayleigh number. First, it is clear that increasing the Rayleigh number enhances the influence of decreasing the behavior index on the heat transfer rate. Further, raising Hartmann decreases the average Nusselt number with the introduction of the magnetic flux, significantly reducing the observed effect of the behavior index on the heat transfer rate Nusselt number especially for shear-thinning fluids, n < 1.



Figure 7. Variations of flow intensity Ψ_{max} for different values of Hartmann number, *n*, and Rayleigh number



Figure 8. Variations of average Nusselt number for different values of Hartmann number, *n*, and Rayleigh number

Figure 9 shows how the maximum temperature, T_{max} , varies as a function of Hartmann number, *n*, and Rayleigh number. First, increasing Rayleigh number decreases the maximum temperature due to the associated strong convective heat transfer. Second, raising the behavior index increases the maximum temperature due to increasing fluid apparent viscosity, which slows down fluid circulation resulting in lower heat transfer. Finally, strengthening the applied



Figure 9. Variations of T_{max} for different values of Hartman number, *n*, and Rayleigh number

magnetic field augments T_{max} , where an increase of Hartmann number from 0-60 results in 28% augmentation for n = 0.6, while only 8.21% augmentation is observed for n = 1.4. Thus, the fluid nature influences the observed magnetic force effect with shear thickening fluids, which are less sensitive to magnetic field presence compared to shear thinning fluids. This is due to the fact that increasing the *n* weakens convective fluid-flow and heat transfer.

Conclusions

The present numerical work implements the FVM to investigate free convection in a square cavity charged with non-Newtonian conducting fluids and subjected to constant heat flux on the vertical walls and a uniform horizontal external magnetic flux. The examination of governing parameters: Rayleigh number, behavior index, n, and Hartman number effects on fluid-flow and heat transfer characteristics lead to the following key findings.

- The absence of magnetic flux and the lowering of the behavior index increase the intensity of the flux and the heat transfer, and the maximum temperature decreases.
- The application of the magnetic flux affects the flow structure as the streamlines stretch in the vertical direction perpendicular to the direction of the applied magnetic flux while the isotherms become parallel to the active vertical walls.
- Applying the magnetic flux slows down fluid circulation and decreases the heat transfer rate as the magnitude of the applied magnetics flux, increases.
- The magnetic flux diminishes the enhancing role of the decreasing behavior index.

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Nomenclature

- B_0 magnetic field strength, [T]
- g gravitational acceleration, $[ms^{-2}]$ H' cavity dimension, [m]
- Ha Hartmann number, [–]
- k consistency index for a power-law fluid, [Pa·s]
- flow behavior index for a power-law fluid, [-] п
- Nu average nusselt number, [-]
- Р - dimensionless pressure, [-]
- generalised Prandtl number, [-] Pr
- constante heat flux, [Wm⁻²] q'
- Ra generalized Rayleigh number, [-]
- T dimensionless temperature, [–]
- U dimensionless normal velocities, [–]
- U'- normal velocities, [ms-1]
- V- Dimensionless horizontal velocities, [-]
- V'- horizontal velocities, [ms⁻¹]
- dimensionless normal co-ordinates, [-] Х
- X' normal co-ordinates, [m]

- dimensionless horizontal co-ordinates, [-]
- Y'- horizontal co-ordinates, [m]

Greek symbols

- α thermal diffusivity of fluid, [m²s⁻¹]
- thermal expansion coefficient of fluid, $[k^{-1}]$ ß
- thermal conductivity of fluid, [Wm⁻¹k⁻¹] λ
- μ'_{a} dimensionless effective viscosity, [–]
- density of fluid, [Kgm⁻³] ρ
- σ electrical conductivity of fluid, [sm⁻¹] Ψ dimensionless stream function, [–]

Superscript

- dimensional variables

Subscripts

- а - effective variable
- max Maximum value

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