

ERROR ANALYSIS OF TRANSIENT METHOD FOR ROCK PERMEABILITY MEASUREMENT

by

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In this paper, the permeability of sandstone, mudstone, gangue and limestone under the condition of initial elastic deformation is obtained by transient method test. And mercury intrusion test is carried out on the four kinds of rock samples to obtain the initial permeability of the rock samples. The results show that the magnitude of permeability obtained by mercury intrusion test and transient method test is completely consistent. The measurement error is analyzed. The relative error of permeability in this paper is less than 6.0%. It can be showed that the use of transient method to measure the permeability of rock samples is reasonable and reliable.

Keywords: error analysis; permeability; transient method

Introduction

Because of the complexity of rock materials and engineering rock mass structure, if various properties including seepage law need to be described, it is difficult to directly establish a theoretical model in the absence of experimental data. The test law based on probability statistics is also an indispensable empirical criterion for practical engineering applications. However, the rationality of the test method and the reliability of the test data need to be analyzed.

There are two commonly used methods for measuring rock permeability parameters in the laboratory. One is the constant flow method, also known as the steady state method, and the other is the pulse attenuation method, also known as the transient method^{[1][2]}. The steady state method is to measure the steady value of the seepage velocity of the rock sample under a constant pressure gradient. The transient method is to measure the time series of the seepage pressure difference at both ends of the rock sample. These two methods have certain applicable conditions. For high permeability rocks, such as soft rock and fractured rock mass,

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the steady state method should be adopted; for low permeability and ultra-low permeability rocks, such as complete and dense fine sandstone and mudstone, the transient method should be used for permeability measurement. The advantages of the transient method are that a rock sample can be used to obtain the permeability characteristics under different strain states, the test cycle is short, and the cost is economic.

In this paper, the theoretical background and experimental error of the transient method for testing rock permeability are analyzed, which provides an experimental basis for the theory of rock seepage.

Permeation and Seepage

The concepts of permeation velocity and seepage velocity are very important in groundwater dynamics. The definitions of permeation and seepage in different literatures are not completely the same [4]. There are three main types:

(1) There is only the concept of permeation, without mentioning the concept of seepage. Permeation is defined as the movement of water in porous media.

(2) Permeation and seepage are regarded as the same concept and used in confusion. Both are defined as the movement of water in pores or fractured rock layers.

(3) Permeation and seepage are regarded as different concepts. The movement of water in rock pores is called permeation, which describes the real water movement, as shown in Figure 1(a); the hypothetical flow of water filling the whole aquifer space (including the space occupied by the solid skeleton and the pore space) is called seepage, as shown in Figure 1(b). The hypothetical flow of water in seepage has the same flow rate and seepage resistance as the real flow. This paper agrees with the third viewpoint, that is, permeation and seepage are completely different flows, and they should be strictly distinguished.

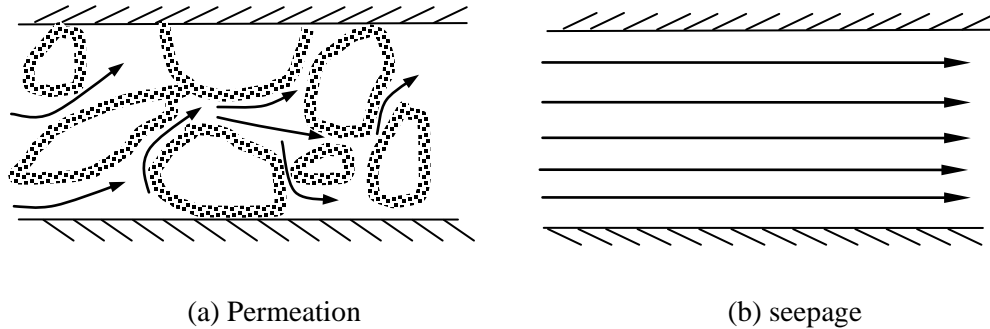


Figure 1. Rock permeation and seepage

With the concepts of penetration and seepage defined, the concepts of penetration velocity and seepage velocity are also defined. The penetration velocity is the actual velocity of fluid flow in the pore, while the seepage velocity is the hypothetical velocity of fluid flow when the pore is spread out over the whole space.

Suppose there is a water-crossing section ΔA_b in the rock, the sum of all pore areas in the section is ΔA_p , the area occupied by the solid skeleton is ΔA_s , and the flow through the water-flowing section per unit time is ΔQ .

The penetration velocity v can be written as

$$v = \frac{\Delta Q}{\Delta A_p} \quad (1)$$

The seepage velocity V can be written as

$$V = \frac{\Delta Q}{\Delta A_B} \quad (2)$$

The physical index to measure the pore of rock is porosity, which is defined as the ratio of the total volume of the pore space V_p in the rock to the total volume of the rock V_B .

The porosity ϕ of the rock is given as

$$\phi = \frac{V_p}{V_B} \times 100\% = \frac{V_B - V_s}{V_B} \times 100\% \quad (3)$$

where V_s is the volume of the solid skeleton.

Combining eqs (1), (2) and (3), the relationship between the permeation velocity and the seepage velocity can be given as

$$v = \frac{V}{\phi} \quad (4)$$

The permeability of the Darcy flow is calculated as follows:

$$k = - \frac{Q\mu\Delta L}{A\Delta p} \quad (5)$$

where L is the length of the specimen, and μ called the viscosity coefficient, dynamic viscosity coefficient or internal friction coefficient.

Combining eqs (2) and (5), we can obtain

$$V = - \frac{1}{\mu} k \xi \quad (6)$$

where ξ is the pressure gradient.

The Compressibility of the Fluid

Fluid volume will change with the pressure, this characteristic is called the compressibility of the fluid. When the pressure increases, the volume of the fluid decreases; when the pressure decreases, the volume will return to its original state. Therefore, this characteristic of the fluid can also be called elasticity. The compressibility of a fluid is usually measured by its volume compressibility coefficient c_f . It is the ratio of the relative increase of

the fluid density $\Delta\rho/\rho$ to the increase of the fluid pressure $d\rho$. Therefore, there is

$$c_f = \frac{1}{\rho} \frac{d\rho}{d\rho} \quad (7)$$

The inverse of c_f is called the volume elasticity coefficient E_L of a fluid. Here,

$$E_L = \frac{1}{c_f} \quad (8)$$

The fluid can be compressed difficultly when the value of E_L is greater. The volume elasticity coefficient of water is generally taken to be $2.1 \times 10^9 \text{ N/m}^2$, and the volume compressibility coefficient of water is $0.476 \times 10^{-9} \text{ Pa}^{-1}$. When the pressure increases by one standard atmosphere, the density of water increases by

$$\Delta\rho/\rho = c_f \Delta p = 0.476 \times 10^{-9} \times 1.013 \times 10^5 = 4.82 \times 10^{-5}.$$

The volume of water decreases by only about 5/100,000.

Principle of the transient permeability method test

The principle of transient permeability method test is as shown in Figure 2.

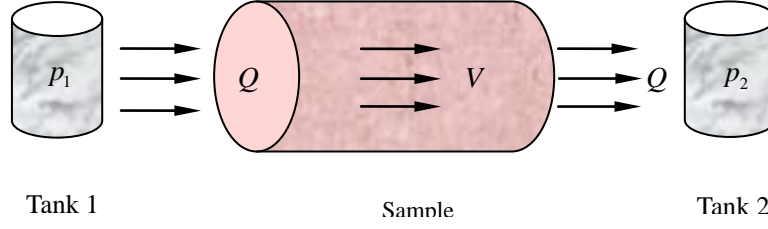


Figure 2. Principle of the transient permeability test method

In Figure 2, the volume of the two storage tanks is B , the pressure is p_1 and p_2 , the height and cross-sectional area of the rock sample are H and A , respectively. At the initial moment, the pressures at both ends of the rock sample are p_{10} and p_{20} ($p_{10} > p_{20}$), and the pressure gradient in the axial direction of the rock sample is $(p_{20} - p_{10})/H$. When the experiment begins, the liquid in storage tank 1 enters storage tank 2 through the rock sample, so that the pressure of storage tank 1 is constantly decreasing, the pressure of storage tank 2 is constantly increasing, and the pressure gradient is gradually decreasing until the pressures of the two storage tanks are equal and reach a balanced state, as shown in Figure 3.

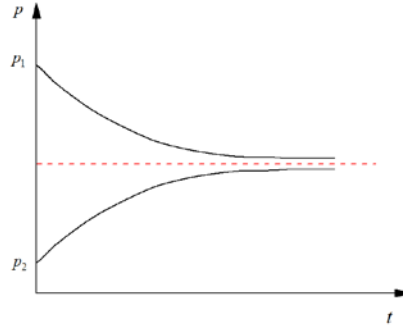


Figure 3 The variation curve of the storage tank pressure

Assuming that the mass flow of the liquid into the rock sample from storage tank 1 is Q , if the pore water of the rock sample is saturated, the mass flow of the liquid into storage tank 2 from the rock sample is also Q , and the seepage velocity in the rock sample is $V = Q/\rho A$. Combining with the relations

$$d\rho = -Qdt/B, \quad Q = \rho AV$$

and eq. (7), we can get

$$\frac{dp_1}{dt} = -\frac{AV}{c_f B} \quad (9)$$

Similarly, we have

$$\frac{dp_2}{dt} = \frac{AV}{c_f B} \quad (10)$$

Combining eqs. (9) and (10), it can be obtained by

$$V = \frac{c_f B}{2A} \frac{d(p_1 - p_2)}{dt} \quad (11)$$

or alternately,

$$V = \frac{c_f BH}{2A} \frac{d\xi}{dt} \quad (12)$$

Combining eqs. (6) and (11), we have

$$\frac{d(p_1 - p_2)}{p_1 - p_2} = -2 \frac{Ak}{c_f BH\mu} dt \quad (13)$$

Darcy flow permeability of the rock sample can be gotten after the integration of the Equation (13). Here,

$$k = \frac{c_f BH\mu}{2t_f A} \ln \frac{p_{10} - p_{20}}{p_{1f} - p_{2f}} \quad (14)$$

where p_{1f} and p_{2f} are the storage tank pressures at the end of sampling t_f .

The Equation (14) represents the mathematical expression for determining the permeability of rock samples during the transient permeability test, encompassing the entire stress-strain process of rock samples [5-6].

Experimental system

The rock sample is placed in the permeameter, as shown in Figure 4. The upper steel plate is designed as two parts in contact with the spherical surface (respectively called upper steel plate-concave side and upper steel plate-convex side), which can rotate relatively to adjust the parallelism between the rock sample, the porous disc, the steel plate and the piston. The porous disc above and below the rock sample is responsible for apportioning the liquid pressure and uniformizing the seepage velocity. The transient permeability test system of rock is formed by adding a permeability loop in the triaxial compression test system.

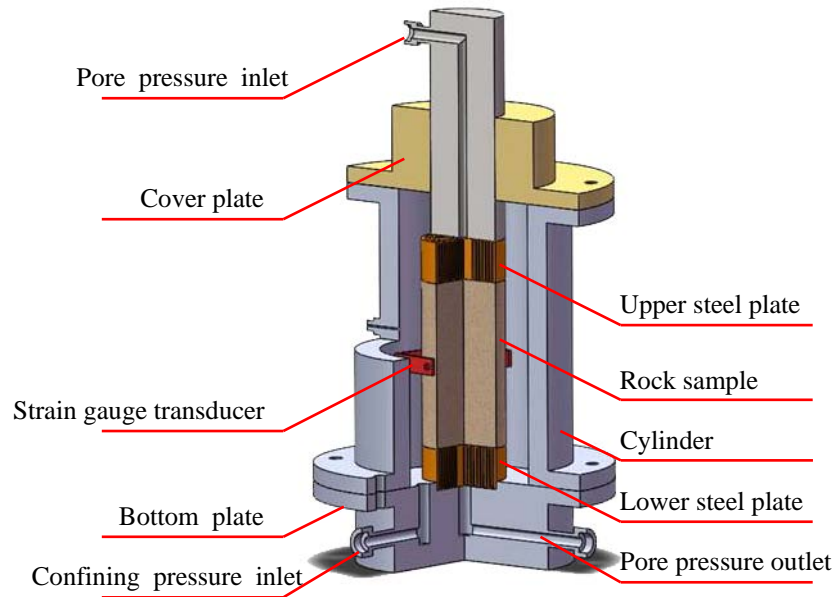


Figure 4 The permeameter

Measurement of water flow in rock sample

In the transient permeability test, the liquid mass in storage tank 1 and storage tank 2 varies with pressure. According to the relations $V = Q/\rho A$ and equation (10), the flow of the cross-section of the rock sample is

$$Q = \frac{c_f B}{2} \frac{d(p_1 - p_2)}{dt}.$$

The compressibility of water is $c_f = 0.459 \times 10^{-9} \text{ (Pa}^{-1}\text{)}$, and the volume of each tank is 1 L. When the pressure of storage tank 1 decreases by $\Delta p_1^* = 1 \text{ MPa}$, the volume of water flowing into the rock sample is given as

$$\Delta B_1^* = -c_f V_1 \Delta p_1^* = -(0.459 \times 10^{-9}) \times (1 \times 10^{-3}) \times (-1 \times 10^6) = 0.459 \text{ (mm}^3\text{)}$$

Similarly, when the pressure of storage tank 2 increases by $\Delta p_2^* = 1 \text{ MPa}$, the volume of water flowing into storage tank 2 from the rock sample is $\Delta B_2^* = 0.459 \text{ (mm}^3\text{)}$. Assuming that the initial pressures of storage tank 1 and 2 are 4 MPa and 2 MPa respectively, and the initial pressure difference is 2 MPa, the pressure difference gradually decreases with the increase of time, and the pressure change of each tank is not more than 2 MPa, so the volume of water flowing into the inner part of the rock sample from storage tank 1 and the volume of water flowing into tank 2 from the rock sample are both not more than 1.112 mm^3 . Therefore, the flow rate is extremely small and cannot be directly measured, but can only be calculated through the change of pressure.

Experimental result

In this paper, sandstone, mudstone, gangue and limestone were selected for analysis. Firstly, the initial permeability of various rock samples was measured by PoreMaster 33 mercury intrusion meter. The rock samples were made into cylinders with nominal diameter of 50 mm and nominal height of 100 mm, and the permeability of sandstone, mudstone, gangue and limestone was measured by the test system. The confining pressure was set as 1 MPa, and the axial strain was set as 0.3. The permeability under the condition of initial elastic deformation was measured. The initial pressures of storage tank 1 and tank 2 were set as 4 MPa and 2 MPa respectively, and the initial pressure difference was 2 MPa. The results of mercury intrusion test and transient method test are shown in Table 1.

Table 1 Permeability of rock samples

Rock samples	permeability (m^2)	
	mercury intrusion test	transient method test
sandstone	1.23×10^{-17}	6.85×10^{-17}
mudstone	1.66×10^{-18}	6.22×10^{-18}
gangue	5.32×10^{-18}	3.82×10^{-18}
limestone	3.83×10^{-17}	7.36×10^{-17}

As can be seen from Table 1, the magnitude of the mercury intrusion test results and the transient method test results of the four rock permeability are the same, but the value is different, because the mercury intrusion test measures the permeability of the rock sample in the initial state, while the transient method test must be carried out under a certain stress state, that is to say, the rock sample has undergone a certain deformation. Because the pore distribution inside the engineering rock mass itself has a great discreteness, as long as the results obtained by the two methods are the same magnitude, it is enough to verify the accuracy of the test.

Error analysis of permeability measurement

The calculation formula of Darcy flow permeability expressed in Equation (14) has an intuitive relationship with pressure p_1 and p_2 , so Equation (14) is used to analyze the measurement error of the test.

Assuming that the measurement range of the pressure transmitter in the test system is 0~16MPa, and the measurement accuracy is 0.1% of the full scale, the measurement error of p_{10} , p_{20} , p_{1f} and p_{2f} is $16 \times 0.1 \times 10^{-2} = 1.6 \times 10^{-2}$ MPa.

The relative error of permeability is [6]

$$\frac{\Delta k}{k} = \frac{\frac{\Delta p_{1f} + \Delta p_{2f}}{p_{1f} - p_{2f}}}{\ln \frac{p_{10} - p_{20}}{p_{1f} - p_{2f}}} \quad (15)$$

In this study, we take $p_{10} - p_{20} = 2.0(\text{MPa})$ and $p_{1f} - p_{2f} \geq 0.3(\text{MPa})$.

Therefore, we have

$$\ln \frac{p_{10} - p_{20}}{p_{1f} - p_{2f}} \geq 1.90 \quad \text{and} \quad \frac{\Delta p_{1f} + \Delta p_{2f}}{p_{1f} - p_{2f}} \leq 0.11$$

Put the above calculation results into eq. (18), the relative error of permeability is given as

$$\frac{\Delta k}{k} \leq \frac{0.11}{1.90} = 5.80 \times 10^{-2}$$

It can be seen from the above results that the relative error of permeability in this study is less than 6.0%.

Conclusions

Transient permeability test is an important method for measuring the permeability of rocks. It is necessary to analyze the rationality of the test method and the reliability of the test data. This paper calculates the water flow in the rock sample. In the permeability test, the water flow of the rock sample is not more than 1.112 mm^3 . The flow is too small to be measured directly, and can only be calculated by the change of pressure. The initial permeability of the rock sample is obtained by mercury intrusion test of sandstone, mudstone, gangue and limestone. The permeability of the four rock samples is measured by the test system containing the permeameter, and the permeability under the condition of initial elastic deformation is obtained. The test results show that the magnitude of the permeability obtained by mercury intrusion test and transient method test is completely consistent. The measurement error is analyzed. The relative error of the permeability in the test is less than 6.0%. It is shown that the transient method for measuring the permeability of rock samples is reasonable, and the test results are reliable.

Nomenclature

k - permeability, [m^2]

μ - viscosity coefficient, [$\text{Pa} \cdot \text{s}$]

v - penetration velocity, [$\text{m} \cdot \text{s}^{-1}$]

V - seepage velocity, [$\text{m} \cdot \text{s}^{-1}$]

p - water pressure, [MPa]

c_f - volume compressibility coefficient, [Pa^{-1}]

ϕ - porosity, [1]

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