DOES SHEAR VISCOSITY PLAY A KEY ROLE IN THE FLOW ACROSS A NORMAL SHOCK WAVE?

by

Huai-Chun ZHOU [a,b,*]

[a] School of Low-carbon Energy and Power Engineering, China University of Mining and Technology, Xuzhou 221116, China

[b] Jiangsu Smart Energy Technology and Equipment Research Center, China University of Mining and Technology, Xuzhou 221116, China

Once there is a velocity gradient in a viscous fluid flow, such as that across a shock wave, a viscous force and viscous energy loss exist inside the flow according to the Navier-Stokes equation (NSE), which may confuse the relative contribution of compressibility and viscosity. In this paper, a viscous shear vector is defined as the component of gradient vector of local velocity magnitude perpendicular to the velocity vector. Then, a local viscous energy flux vector is defined from the viscous shear vector after being multiplied by the viscosity and the velocity magnitude. The divergence of the viscous energy flux vector results in new expressions for viscous force and loss of viscous energy, in which all the square terms of derivative of velocity components correspond to irreversible energy loss. The rest part can be taken as a kind of mechanical energy transfer done by the viscous force, from which the viscous force components can be got based on the assumption that the viscous force vector is parallel to the velocity vector. The new equations are different from and more complex than those in the traditional NSE. By the new theory, it is shown that there is no shear viscous force and shear viscous energy loss in the flow across a normal shock wave without velocity gradient perpendicular to the flow direction.

Key words: *viscous shear vector, viscous energy flux vector, shock wave, compressible flow, energy dissipation*

Introduction

The role of turbulence can be seen in many different fields [1-4]. At present, turbulence can not be described by any direct method, and the statistical and structural characteristics of turbulence can also not be predicted by the Navier-Stokes equations (NSE) [5]. Long-range and short-range forces are the two types of force that affect fluid flow used in fluid mechanics [6]. The Euler equation for invisicd fluid has achieved great success [7]. The two types of forces are the basis to set up the famous NSE, and then the thermodynamic energy equation [6, 8-12].

In the past, some of the work of the author of this study was related to fluid flow, such as instability of incompressible flow [13], and laminar diffusion flame [14]. It can be said that the

[*] Corresponding author; e-mail: hczhou@cumt.edu.cn

author's research experience on the Radiation Transfer Equation (RTE) [15-16] provides an opportunity to think about the possible lack of rigor in NSE. In the previous study [17], the concept of a local viscous energy in a fluid was defined using the product of the local viscous force and the velocity, where the viscous force is got from the velocity gradient multiplied by the viscosity. For the flow across a shock wave, a viscous stress was given by the velocity difference divided by the thickness, also being a velocity gradient, of shock wave and multiplied by the viscosity, from which the thickness of shock wave was estimated [18]. The author of [18] also noted that, usually, viscosity plays a role when there is a velocity gradient perpendicular to the flow direction. What is the effect of viscosity when away from the boundary?

The purpose of this article is to explore the possibility that there may not be viscous stress in the direction across a normal shock wave, despite the presence of velocity gradient. At first, a concept of viscous shear vector is newly defined to clarify the relations between spatial velocity gradient and the effect of viscosity. Then, a local viscous energy flux vector in the fluid, which is extended from the local viscous energy proposed in [17], is defined from the viscous shear vector, and the divergence of the viscous energy flux results in new viscous force and loss of viscous energy, which are different from those in the traditional NSE.

Theoretical Derivation

Concept derivation in 2-D, steady, unidirectional, laminar flows

In order to quantitatively describe the dissipation process of mechanical energy in fluid motion by viscosity, we consider two cases in 2-D, steady, unidirectional, laminar flows as shown in Fig. 1(a) and (b). The velocity fields for the two flows are given as below:

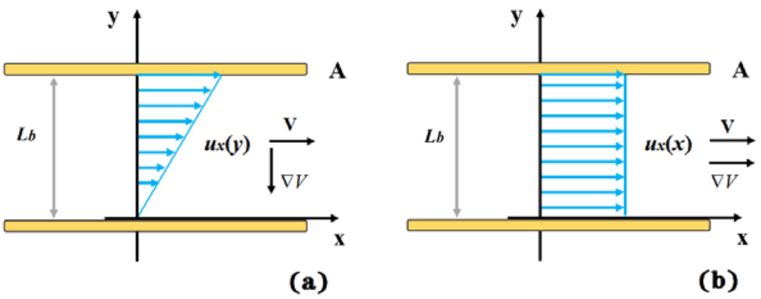

Figure 1. Steady, unidirectional, laminar flow of viscous fluid between two plates. (a) Case I, (b) Case II

$$\begin{cases} \text{Case} \quad \text{I}: \quad V = u_x = f(y), \quad u_y = 0, \quad u_z = 0 \\ \text{Case} \quad \text{II}: \quad V = u_x = g(x), \quad u_y = 0, \quad u_z = 0 \end{cases} \tag{1}$$

where u_x, u_y and u_z are the velocity components. Obviously, the flow in Case II in Fig. 1(b) must be compressible. Assume that the viscosity is constant in the flows for simplicity. According to the NSE [12], the viscous force in *x*-direction, $f_{v,x}$, in the two cases reads $\mu\partial^2 f/\partial y^2$ and $(4/3)\mu\partial^2 g/\partial x^2$, where μ is the viscosity. The viscous force component in *x*-direction for Case I is correct. But we can find that the viscous force component in *x*-direction for Case II is not zero, even the velocity does not have gradient in *y*-direction.

The viscous energy loss Φ for the two cases can be got as $\mu(\partial f/\partial y)^2$, and $(4\mu/3)(\partial g/\partial x)^2$, respectively, which indicates a fact that in the Case II, the viscous force exists, and causes a non-zero irreversible energy loss. One example of this flow is that across a shock wave [18].

Here we can figure out the velocity vector $\mathbf{V} = u_x\mathbf{i} + u_y\mathbf{j}$ and the gradient vector of velocity magnitude ∇V, where $V=\sqrt{u_x^2+u_y^2}$, as also shown in Fig. 1. The problem comes from the directional relationship between these two vectors. As shown in Fig. 1(a), these two vectors are perpendicular to each other, and the viscous force obtained from NSE is correct. But in Fig. 1(b), these two vectors are parallel to each other, and the viscous force obtained from NSE maybe not correct. In essence, the Newtonian friction law stands for the gradient vector of velocity magnitude perpendicular to the velocity vector in such as a boundary layer.

For Case I shown in Fig. 1(a), the flow is driven by the upper plate, and the mechanical energy is input by the external force $\mathbf{F}_0$ in the fluid. The work done by the force $\mathbf{F}_0$ acting on a body with velocity $\mathbf{V}$ will be $\mathbf{F}_0\cdot\mathbf{V}$. It will be transferred into the fluid, and then partially or totally dissipated into heat. Thus, viscous energy flux was introduced as [17]:

$$E_{v,y}=F_{v,x}u_x=\mu u_x\frac{du_x}{dy} \tag{2}$$

where $F_{v,x}$ is a Newtonian internal friction stress, and represents the viscous force in the *x*-direction caused by the gradient of velocity in the *x*-direction. Obviously, the dimension of $E_{v,y}$ is [N / m^2 · (m / s)] = [J / m^2 /s] = [W / m^2], and it is a kind of energy flux, and also a vector. Similarly, concept of mechanical energy in fluid motion was adopted in literature (for the example, see [19]).

If we take the derivative of eq. (2), we get [17]

$$e_{v,y}=\frac{dE_{v,y}}{dy}=\frac{d}{dy}\left(\mu u_x\frac{du_x}{dy}\right)=\mu\left(\frac{du_x}{dy}\right)^2+\mu u_x\frac{d^2u_x}{dy^2} \tag{3}$$

with the assumption of constant viscosity for simplicity, which represents the change of viscous energy flux in the fluid along the vertical direction, as the change of an internal energy. Under the plate laminar flow condition as shown in Fig. 1(a), the dissipation rate of viscous energy per unit volume of fluid caused by the viscous effect is exactly equal to $\mu(du_x/dy)^2$ [17]. As analyzing a steady, straight flow in a horizontal pipe in literature (e.g., see p181 of ref.[6]), the energy dissipation rate of viscous fluid per unit mass in a horizontal pipe is obtained as $\Phi=(\mu/\rho)(du/dr)^2$.

Under unsteady flow conditions, the second term on the right-hand side of eq. (3), $\mu u_x(d^2u_x/dy^2)$, may appear, which can be positive or negative, representing the mutual transformation with mechanical energy in the process. From eq. (3), $\mu(d^2u_x/dy^2)$, the remaining part of $\mu u_x(d^2u_x/dy^2)$ after removal of the speed term u_x, is just a viscous, body force, $f_{v,x}$, which would occur in the momentum equation as an acting force and cause acceleration of fluid [17].

Therefore, eq. (3) satisfies the first law of thermodynamics: the left-hand side of the equation represents the change in internal energy, the first term on the right-hand side is irreversible viscous heat loss, which indicates the effect of the second law of thermodynamics, and the second term is the conversion part of mechanical energy.

For Case II shown in Fig. 1(b), the velocity vector $\mathbf{V}$, and the gradient vector of velocity magnitude ∇V, are parallel to each other, and the viscous force obtained from NSE maybe not correct. In essence, the Newtonian friction law stands for the gradient vector of velocity magnitude perpendicular to the velocity vector in such as a boundary layer. So, we need to define a viscous shear vector at first, and then to introduce a viscous energy flux vector, in order to obtain general equations for the viscous fluid flow.

Introduction of a viscous shear vector

For the velocity components (u_x,u_y,u_z) of a point in a flow field, the gradient of fluid velocity magnitude is [17]

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{i} + \frac{\partial V}{\partial y}\mathbf{j} + \frac{\partial V}{\partial z}\mathbf{k}\,, \tag{4}$$

which is a kind of vector, where the partial derivatives of the velocity magnitude in the *x*-, *y*- and *z*-directions are given by [17]:

$$\begin{cases} \dfrac{\partial V}{\partial x} = \dfrac{1}{V}\left(u_x \dfrac{\partial u_x}{\partial x} + u_y \dfrac{\partial u_y}{\partial x} + u_z \dfrac{\partial u_z}{\partial x}\right) \\ \dfrac{\partial V}{\partial y} = \dfrac{1}{V}\left(u_x \dfrac{\partial u_x}{\partial y} + u_y \dfrac{\partial u_y}{\partial y} + u_z \dfrac{\partial u_z}{\partial y}\right) \\ \dfrac{\partial V}{\partial z} = \dfrac{1}{V}\left(u_x \dfrac{\partial u_x}{\partial z} + u_y \dfrac{\partial u_y}{\partial z} + u_z \dfrac{\partial u_z}{\partial z}\right) \end{cases} \tag{5}$$

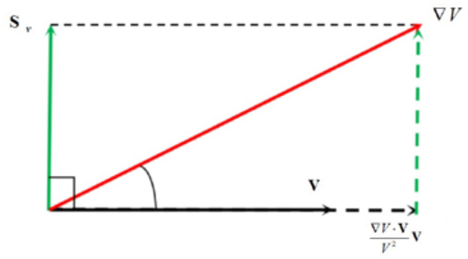

Figure 2. The gradient vector of velocity magnitude, ∇V, the local viscous shear vector, $\mathbf{S}_v$, and the velocity vector, $\mathbf{V}$, on the same plane.

As shown in Fig. 2, ∇V, the local velocity magnitude gradient vector, may not be always perpendicular to the velocity vector, $\mathbf{V}$. We define a viscous shear vector, $\mathbf{S}_v$, that should be always perpendicular to the velocity vector, $\mathbf{V}$, on the same plane with ∇V and $\mathbf{V}$, as follows

$$\mathbf{S}_v = \nabla V - \frac{\nabla V \cdot \mathbf{V}}{V^2}\mathbf{V}\,, \tag{6}$$

where $\left[(\nabla V \cdot \mathbf{V})/V^2\right]\mathbf{V}$ is just the component of the gradient vector of velocity magnitude, ∇V, parallel to the velocity vector, $\mathbf{V}$, as shown in Fig. 2.

In fact, eq.(6) is expressed as follows

$$\begin{aligned} \mathbf{S}_v = \frac{1}{V}&\left[\begin{aligned}&\left(1-\frac{u_x^2}{V^2}\right)\left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_y}{\partial x} + u_z \frac{\partial u_z}{\partial x}\right) - \frac{u_x u_y}{V^2}\left(u_x \frac{\partial u_x}{\partial y} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_z}{\partial y}\right) \\ &- \frac{u_x u_z}{V^2}\left(u_x \frac{\partial u_x}{\partial z} + u_y \frac{\partial u_y}{\partial z} + u_z \frac{\partial u_z}{\partial z}\right)\end{aligned}\right]\mathbf{i} \\ + \frac{1}{V}&\left[\begin{aligned}&-\frac{u_x u_y}{V^2}\left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_y}{\partial x} + u_z \frac{\partial u_z}{\partial x}\right) + \left(1-\frac{u_y^2}{V^2}\right)\left(u_x \frac{\partial u_x}{\partial y} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_z}{\partial y}\right) \\ &- \frac{u_y u_z}{V^2}\left(u_x \frac{\partial u_x}{\partial z} + u_y \frac{\partial u_y}{\partial z} + u_z \frac{\partial u_z}{\partial z}\right)\end{aligned}\right]\mathbf{j} \\ + \frac{1}{V}&\left[\begin{aligned}&-\frac{u_x u_z}{V^2}\left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_y}{\partial x} + u_z \frac{\partial u_z}{\partial x}\right) - \frac{u_y u_z}{V^2}\left(u_x \frac{\partial u_x}{\partial y} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_z}{\partial y}\right) \\ &+ \left(1-\frac{u_z^2}{V^2}\right)\left(u_x \frac{\partial u_x}{\partial z} + u_y \frac{\partial u_y}{\partial z} + u_z \frac{\partial u_z}{\partial z}\right)\end{aligned}\right]\mathbf{k} \end{aligned} \tag{7}$$

We can verify that the following equation holds

$$\mathbf{S}_v \cdot \mathbf{V} = 0\,, \tag{8}$$

that means that the viscous shear vector is perpendicular to the velocity vector.

For 2-D flows, the viscous shear vector, $\mathbf{S}_v$, becomes

$$\mathbf{S}_v=\frac{1}{V}\left[\left(1-\frac{u_x^2}{V^2}\right)\left(u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_y}{\partial x}\right)-\frac{u_xu_y}{V^2}\left(u_x\frac{\partial u_x}{\partial y}+u_y\frac{\partial u_y}{\partial y}\right)\right]\mathbf{i}$$
$$+\frac{1}{V}\left[-\frac{u_xu_y}{V^2}\left(u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_y}{\partial x}\right)+\left(1-\frac{u_y^2}{V^2}\right)\left(u_x\frac{\partial u_x}{\partial y}+u_y\frac{\partial u_y}{\partial y}\right)\right]\mathbf{j} \quad (9)$$

From eq. (9), it is obvious that there exists a viscous shear in y-direction in Case I in Fig. 1(a), $s_{v,x}=0$, $s_{v,y}=\partial u_x/\partial y=\partial f/\partial y$. But there does not exist a viscous shear in Case II in Fig. 1(b), since $u_y=0$, $s_{v,x}=0$, and $s_{v,y}=0$.

Definition of viscous energy flux vector in 2-D flows and its divergence

Based on the viscous shear vector, $\mathbf{s}_v$, a local viscous energy flux vector, $\mathbf{E}_v$, is defined as below:

$$\mathbf{E}_v=\mu V\mathbf{S}_v=\mu V\left(\nabla V-\frac{\nabla V\cdot\mathbf{V}}{V^2}\mathbf{V}\right)=\mu V\nabla V-\mu\frac{\nabla V\cdot\mathbf{V}}{V}\mathbf{V} \quad (10)$$

Then the divergence of the viscous energy flux can be calculated as

$$e_v=\nabla\cdot\mathbf{E}_v=\nabla\cdot\left(\mu V\nabla V\right)-\nabla\cdot\left(\mu\frac{\nabla V\cdot\mathbf{V}}{V}\mathbf{V}\right) \quad (11)$$

In 2-D flows, it becomes

$$e_v=\frac{\partial}{\partial x}\left[\mu\left(1-\frac{u_x^2}{V^2}\right)\left(u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_y}{\partial x}\right)-\frac{\mu u_xu_y}{V^2}\left(u_x\frac{\partial u_x}{\partial y}+u_y\frac{\partial u_y}{\partial y}\right)\right]$$
$$+\frac{\partial}{\partial y}\left[-\frac{\mu u_xu_y}{V^2}\left(u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_y}{\partial x}\right)+\mu\left(1-\frac{u_y^2}{V^2}\right)\left(u_x\frac{\partial u_x}{\partial y}+u_y\frac{\partial u_y}{\partial y}\right)\right] \quad (12)$$

It is stated that the divergence of the viscous energy flux can be divided into two parts. The first part includes all the square terms of derivative of velocity components,

$$\Phi_v=\mu\left[1+\frac{u_x^2}{V^2}\left(\frac{2u_x^2}{V^2}-3\right)\right]\left(\frac{\partial u_x}{\partial x}\right)^2+\mu\left[1+\frac{u_x^2}{V^2}\left(\frac{2u_y^2}{V^2}-1\right)\right]\left(\frac{\partial u_y}{\partial x}\right)^2$$
$$+\mu\left[1+\frac{u_y^2}{V^2}\left(\frac{2u_x^2}{V^2}-1\right)\right]\left(\frac{\partial u_x}{\partial y}\right)^2+\mu\left[1+\frac{u_y^2}{V^2}\left(\frac{2u_y^2}{V^2}-3\right)\right]\left(\frac{\partial u_y}{\partial y}\right)^2 \quad (13)$$

They are always non-negative as an irreversible loss, corresponds to the energy dissipated by the viscosity of the fluid and is converted into thermal energy in the fluid (i.e., raising the fluid temperature) [17].

For Case I in Fig. 1(a), the viscous energy loss is suggested as $\Phi_v=\mu(\partial f/\partial y)^2$. For Case II in Fig. 1(b), the viscous energy loss is given as $\Phi_v=\mu[1+1\cdot(2-3)](\partial g/\partial x)^2=0$. So, in this case, there is no viscous energy loss.

The rest part on the right-hand side of eq. (12) contains at least one velocity component, and as a kind of energy transfer, they are related to the mechanical energy done by the viscous force, being equal to the product of the velocity component and the viscous force component. Then we have the divergence of the viscous energy flux in the following form for 2-D flows

$$e_v = \nabla \cdot \mathbf{E}_v = f_{v,x} \cdot u_x + f_{v,y} \cdot u_y + \Phi_v \tag{14}$$

As we can see in the Fig. 1(a) that the viscous force is parallel to the velocity vector, it is suggested that in 3-D flows, the viscous force is always parallel to the fluid velocity vector inside the flows. The suggestion is based on the principle that the viscous forces hinder the movement of fluid, thus opposing the direction of flow. Then, for the viscous force components, $\mathbf{F}_v \times \mathbf{V} = 0$, that means $f_{v,x} \cdot u_y - f_{v,y} \cdot u_x = 0$. For a term c on right-hand side of eq. (12), from eq. (14) we have $f_{v,x} \cdot u_x + f_{v,y} \cdot u_y = c$. Then we have

$$f_{v,x} = \frac{u_x}{V^2} c, \qquad f_{v,y} = \frac{u_y}{V^2} c. \tag{15}$$

The numbers of terms are 20 in 2-D flows, and the viscous force components in two directions for 2-D flows can be given as

$$f_{v,x} = \frac{u_x}{V^2} \sum_{i=1}^{20} C_i P_i, \qquad f_{v,y} = \frac{u_y}{V^2} \sum_{i=1}^{20} C_i P_i\ , \tag{16}$$

and all the terms P_i and their coefficients C_i are given in Table 1.

Table 1. Terms and coefficients in new viscous force in 2-D flows.

i	P_i	C_i	i	P_i	C_i	i	P_i	C_i
1	$\frac{\partial \mu}{\partial x}\frac{\partial u_x}{\partial x}$	$u_x\left(1-\frac{u_x^2}{V^2}\right)$	8	$\frac{\partial \mu}{\partial y}\frac{\partial u_y}{\partial x}$	$-\frac{u_x u_y^2}{V^2}$	15	$\frac{\partial u_x}{\partial x}\frac{\partial u_x}{\partial y}$	$\frac{4\mu u_x u_y}{V^2}\left(\frac{u_x^2}{V^2}-1\right)$
2	$\frac{\partial \mu}{\partial y}\frac{\partial u_y}{\partial y}$	$u_y\left(1-\frac{u_y^2}{V^2}\right)$	9	$\frac{\partial^2 u_x}{\partial x^2}$	$\mu u_x\left(1-\frac{u_x^2}{V^2}\right)$	16	$\frac{\partial u_y}{\partial x}\frac{\partial u_y}{\partial y}$	$\frac{4\mu u_x u_y}{V^2}\left(\frac{u_y^2}{V^2}-1\right)$
3	$\frac{\partial \mu}{\partial y}\frac{\partial u_x}{\partial y}$	$u_x\left(1-\frac{u_y^2}{V^2}\right)$	10	$\frac{\partial^2 u_y}{\partial y^2}$	$\mu u_y\left(1-\frac{u_y^2}{V^2}\right)$	17	$\frac{\partial u_x}{\partial x}\frac{\partial u_y}{\partial x}$	$\frac{2\mu u_x u_y}{V^2}\left(\frac{2u_x^2}{V^2}-1\right)$
4	$\frac{\partial \mu}{\partial x}\frac{\partial u_y}{\partial x}$	$u_y\left(1-\frac{u_x^2}{V^2}\right)$	11	$\frac{\partial^2 u_y}{\partial x^2}$	$\mu u_y\left(1-\frac{u_x^2}{V^2}\right)$	18	$\frac{\partial u_x}{\partial y}\frac{\partial u_y}{\partial y}$	$\frac{2\mu u_x u_y}{V^2}\left(\frac{2u_y^2}{V^2}-1\right)$
5	$\frac{\partial \mu}{\partial x}\frac{\partial u_x}{\partial y}$	$-\frac{u_x^2 u_y}{V^2}$	12	$\frac{\partial^2 u_x}{\partial y^2}$	$\mu u_x\left(1-\frac{u_y^2}{V^2}\right)$	19	$\frac{\partial u_x}{\partial x}\frac{\partial u_y}{\partial y}$	$\frac{\mu}{V^2}\left(\frac{4u_x^2 u_y^2}{V^2}-u_x^2-u_y^2\right)$
6	$\frac{\partial \mu}{\partial y}\frac{\partial u_x}{\partial x}$	$-\frac{u_x^2 u_y}{V^2}$	13	$\frac{\partial^2 u_x}{\partial x \partial y}$	$-\frac{2\mu u_x^2 u_y}{V^2}$	20	$\frac{\partial u_x}{\partial y}\frac{\partial u_y}{\partial x}$	$\frac{\mu}{V^2}\left(\frac{4u_x^2 u_y^2}{V^2}-u_x^2-u_y^2\right)$
7	$\frac{\partial \mu}{\partial x}\frac{\partial u_y}{\partial y}$	$-\frac{u_x u_y^2}{V^2}$	14	$\frac{\partial^2 u_y}{\partial x \partial y}$	$-\frac{2\mu u_x u_y^2}{V^2}$			

For Case I in Fig. 1(a), the viscous force is given as $f_{v,x} = \mu \partial^2 f / \partial y^2$, $f_{v,y} = 0$. For Case II in Fig. 1(b), $f_{v,x} = \mu(1-1)\partial^2 g / \partial x^2 = 0$, $f_{v,y} = 0$, and the fluid is not affected by the viscous force.

Viscous energy flux and its transfer in 3-D flows

The divergence of the viscous energy flux in 3-D flows

$$e_v = \nabla \cdot \mathbf{E}_v = f_{v,x} \cdot u_x + f_{v,y} \cdot u_y + f_{v,z} \cdot u_z + \Phi_v \tag{17}$$

detailed derivation is not given here, and in the result,

$$\Phi_v = \mu\left[1\text{-}\frac{u_x^2}{V^2}\left(3\text{-}\frac{2u_x^2}{V^2}\right)\right]\left(\frac{\partial u_x}{\partial x}\right)^2 + \mu\left[1-\frac{u_x^2}{V^2}\left(1\text{-}\frac{2u_y^2}{V^2}\right)\right]\left(\frac{\partial u_y}{\partial x}\right)^2 + \mu\left[1-\frac{u_x^2}{V^2}\left(1\text{-}\frac{2u_z^2}{V^2}\right)\right]\left(\frac{\partial u_z}{\partial x}\right)^2$$

$$+\mu\left[1-\frac{u_y^2}{V^2}\left(1-\frac{2u_x^2}{V^2}\right)\right]\left(\frac{\partial u_x}{\partial y}\right)^2+\mu\left[1-\frac{3u_y^2}{V^2}\left(3-\frac{2u_y^2}{V^2}\right)\right]\left(\frac{\partial u_y}{\partial y}\right)^2+\mu\left[1-\frac{u_y^2}{V^2}\left(1-\frac{2u_z^2}{V^2}\right)\right]\left(\frac{\partial u_z}{\partial y}\right)^2$$
$$+\mu\left[1-\frac{u_z^2}{V^2}\left(1-\frac{2u_x^2}{V^2}\right)\right]\left(\frac{\partial u_x}{\partial z}\right)^2+\mu\left[1-\frac{u_z^2}{V^2}\left(1+\frac{2u_y^2}{V^2}\right)\right]\left(\frac{\partial u_y}{\partial z}\right)^2+\mu\left[1-\frac{u_z^2}{V^2}\left(3-\frac{2u_z^2}{V^2}\right)\right]\left(\frac{\partial u_z}{\partial z}\right)^2 \quad (18)$$

For the viscous force in 3-D flows, $\mathbf{F}_v \times \mathbf{V} = 0$ leads to

$$f_{v,x}\cdot u_y - f_{v,y}\cdot u_x = 0,\ f_{v,x}\cdot u_z - f_{v,z}\cdot u_x = 0,\ f_{v,y}\cdot u_z - f_{v,z}\cdot u_y = 0\,. \quad (19)$$

For a term c ,we have

$$f_{v,x}\cdot u_x + f_{v,y}\cdot u_y + f_{v,z}\cdot u_z = c\,. \quad (20)$$

Together with eq. (19), we have

$$f_{v,x}=\frac{u_x}{V^2}c,\quad f_{v,y}=\frac{u_y}{V^2}c,\quad f_{v,z}=\frac{u_z}{V^2}c\,. \quad (21)$$

The viscous force components in three directions can be given as

$$f_{v,x}=\frac{u_x}{V^2}\sum_{i=1}^{81}C_iP_i,\quad f_{v,y}=\frac{u_y}{V^2}\sum_{i=1}^{81}C_iP_i,\quad f_{v,z}=\frac{u_z}{V^2}\sum_{i=1}^{81}C_iP_i\,, \quad (22)$$

and the total number of terms is 81 in 3-D flows. The details in the derivation are not given here. All the terms P_i and their coefficients C_i are given in Table 2.

As shown in Fig. 3, according to the derivation in the literature such as [20,21], the energy balance equation in a volume element $dxdydz$ is given by [17]:

$$\begin{aligned}&\frac{\partial}{\partial x}(pu_x)+\frac{\partial}{\partial y}(pu_y)+\frac{\partial}{\partial z}(pu_z)\\&+\frac{\rho}{2}\frac{D}{Dt}(u_x^2+u_y^2+u_z^2)+\rho\frac{DE}{Dt}\\&=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\frac{\partial Q}{\partial t}\\&+\frac{\partial}{\partial x}(E_{v,x})+\frac{\partial}{\partial y}(E_{v,y})+\frac{\partial}{\partial z}(E_{v,z})\end{aligned}\,. \quad (23)$$

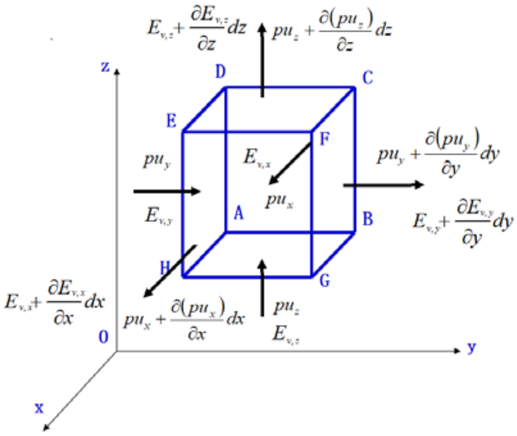

Figure 3. A micro parallelepiped element used to analyze transfer of pressure potential energy and viscous energy flux in viscous fluid movement.

where E is the internal energy, Q is the heat generation, and the last nine terms on the right side of eq. (23) represent the work done by the viscous force. In this paper, the rate of change of viscous energy flux, where $E_{v,x}$, $E_{v,y}$ and $E_{v,z}$ represent its components, is used to replace the work done by the viscous force in the traditional equation. Eq. (23) can directly give the reversible conversion of viscous energy, which contains the viscous force term that can be used in the momentum equation.

The pressure related term on the left side of eq. (23), dealing with transfer of the pressure potential energy, is derived as follows [17]:

$$
\begin{aligned}
&\frac{\partial}{\partial x}(pu_x)+\frac{\partial}{\partial y}(pu_y)+\frac{\partial}{\partial z}(pu_z)=p\cdot\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)+u_x\frac{\partial p}{\partial x}+u_y\frac{\partial p}{\partial y}+u_z\frac{\partial p}{\partial z} \\
&=p\cdot(\nabla\cdot\mathbf{V})+u_x\frac{\partial p}{\partial x}+u_y\frac{\partial p}{\partial y}+u_z\frac{\partial p}{\partial z}=-\frac{p}{\rho}\frac{D\rho}{Dt}+u_x\frac{\partial p}{\partial x}+u_y\frac{\partial p}{\partial y}+u_z\frac{\partial p}{\partial z}
\end{aligned}
. \quad (24)
$$

It can be seen from the above equation that the divergence of velocity,

$$
\nabla\cdot\mathbf{V}=\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z},
$$

is included in transfer of the pressure potential energy, and should not be related directly to the viscosity.

After derivation, eq. (23) leads to [17]:

$$
\begin{aligned}
&u_x\cdot\left[\rho a_x+\frac{\partial p}{\partial x}-f_{v,x}\right]+u_y\cdot\left[\rho a_y+\frac{\partial p}{\partial y}-f_{v,y}\right]+u_z\cdot\left[\rho a_z+\frac{\partial p}{\partial z}-f_{v,z}\right] \\
&-\frac{p}{\rho}\frac{D\rho}{Dt}+\rho\frac{DE}{Dt}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\frac{\partial Q}{\partial t}+\Phi_v
\end{aligned}
, \quad (25)
$$

where a_x, a_y, a_z are the acceleration components, and the irreversible heat loss term Φ_v is given in eq. (18). In eq. (25), the first, second and third terms in square brackets on the left-hand side are just momentum equations, which are balanced and the principle of conservation of mechanical energy is met. Then, the motion equation of viscous fluid can be obtained as follows [17]:

$$
\begin{cases}
\rho a_x=\rho\frac{Du_x}{Dt}=\rho\left(\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+f_{v,x} \\
\rho a_y=\rho\frac{Du_y}{Dt}=\rho\left(\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}\right)=-\frac{\partial p}{\partial y}+f_{v,y} \\
\rho a_z=\rho\frac{Du_z}{Dt}=\rho\left(\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+f_{v,z}
\end{cases}
, \quad (26)
$$

where the viscous forces are given in eq. (22) and all the terms P_i and their coefficients C_i are given in Table 2.

Here, eq. (25) becomes:

$$
\rho\frac{DE}{Dt}=\frac{p}{\rho}\frac{D\rho}{Dt}+\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\frac{\partial Q}{\partial t}+\Phi_v . \quad (27)
$$

For unit mass fluid, the expressions of irreversible loss, viscous force and viscous energy flux change rate can be obtained after being divided by density. The change of local fluid temperature comes mainly from: (1) the contribution of external mechanical work input through viscous energy dissipation; (2) the contribution of external heat input [11].

Discussion

The viscous force terms in the NSE are [12,20,21]:

$$
f_{x,NS}=\frac{\partial}{\partial x}\left(2\mu\frac{\partial u_x}{\partial x}-\frac{2}{3}\mu\nabla\cdot\mathbf{V}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u_x}{\partial y}+\mu\frac{\partial u_y}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial u_x}{\partial z}+\mu\frac{\partial u_z}{\partial x}\right) \quad (28)
$$

$$
f_{y,NS}=\frac{\partial}{\partial x}\left(\mu\frac{\partial u_x}{\partial y}+\mu\frac{\partial u_y}{\partial x}\right)+\frac{\partial}{\partial y}\left(2\mu\frac{\partial u_y}{\partial y}-\frac{2}{3}\mu\nabla\cdot\mathbf{V}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial u_y}{\partial z}+\mu\frac{\partial u_z}{\partial y}\right) \quad (29)
$$

$$f_{z,NS} = \frac{\partial}{\partial x}\left(\mu\frac{\partial u_x}{\partial z} + \mu\frac{\partial u_z}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u_y}{\partial z} + \mu\frac{\partial u_z}{\partial y}\right) + \frac{\partial}{\partial z}\left(2\mu\frac{\partial u_z}{\partial z} - \frac{2}{3}\mu\nabla\cdot\mathbf{V}\right) \tag{30}$$

Table 2. Terms and coefficients in new viscous force expressions

i	P_i	C_i	i	P_i	C_i	i	P_i	C_i	i	P_i	C_i
1	$\frac{\partial\mu}{\partial x}\frac{\partial u_x}{\partial x}$	$u_x\left(1-\frac{u_x^2}{V^2}\right)$	22	$\frac{\partial\mu}{\partial y}\frac{\partial u_x}{\partial z}$	$-\frac{u_x u_y u_z}{V^2}$	42	$\frac{\partial^2 u_z}{\partial y\partial z}$	$-\frac{2\mu u_y u_z^2}{V^2}$	62	$\frac{\partial u_x}{\partial z}\frac{\partial u_z}{\partial x}$	$\frac{\mu}{V^2}\left(\frac{4u_x^2u_z^2}{V^2}-u_x^2-u_z^2\right)$
2	$\frac{\partial\mu}{\partial y}\frac{\partial u_y}{\partial y}$	$u_y\left(1-\frac{u_y^2}{V^2}\right)$	23	$\frac{\partial\mu}{\partial z}\frac{\partial u_x}{\partial y}$	$-\frac{u_x u_y u_z}{V^2}$	43	$\frac{\partial^2 u_x}{\partial y\partial z}$	$-\frac{2\mu u_x u_y u_z}{V^2}$	63	$\frac{\partial u_z}{\partial y}\frac{\partial u_y}{\partial z}$	$\frac{\mu}{V^2}\left(\frac{4u_y^2u_z^2}{V^2}-u_y^2-u_z^2\right)$
3	$\frac{\partial\mu}{\partial z}\frac{\partial u_z}{\partial z}$	$u_z\left(1-\frac{u_z^2}{V^2}\right)$	24	$\frac{\partial\mu}{\partial z}\frac{\partial u_y}{\partial x}$	$-\frac{u_x u_y u_z}{V^2}$	44	$\frac{\partial^2 u_y}{\partial x\partial z}$	$-\frac{2\mu u_x u_y u_z}{V^2}$	64	$\frac{\partial u_x}{\partial y}\frac{\partial u_x}{\partial z}$	$\frac{2\mu u_y u_z}{V^2}\left(\frac{2u_x^2}{V^2}-1\right)$
4	$\frac{\partial\mu}{\partial y}\frac{\partial u_x}{\partial y}$	$u_x\left(1-\frac{u_y^2}{V^2}\right)$	25	$\frac{\partial\mu}{\partial x}\frac{\partial u_y}{\partial z}$	$-\frac{u_x u_y u_z}{V^2}$	45	$\frac{\partial^2 u_z}{\partial x\partial y}$	$-\frac{2\mu u_x u_y u_z}{V^2}$	65	$\frac{\partial u_y}{\partial x}\frac{\partial u_y}{\partial z}$	$\frac{2\mu u_x u_z}{V^2}\left(\frac{2u_y^2}{V^2}-1\right)$
5	$\frac{\partial\mu}{\partial z}\frac{\partial u_x}{\partial z}$	$u_x\left(1-\frac{u_z^2}{V^2}\right)$	26	$\frac{\partial\mu}{\partial x}\frac{\partial u_z}{\partial y}$	$-\frac{u_x u_y u_z}{V^2}$	46	$\frac{\partial u_x}{\partial x}\frac{\partial u_x}{\partial y}$	$\frac{4\mu u_x u_y}{V^2}\left(\frac{u_x^2}{V^2}-1\right)$	66	$\frac{\partial u_z}{\partial x}\frac{\partial u_z}{\partial y}$	$\frac{2\mu u_x u_y}{V^2}\left(\frac{2u_z^2}{V^2}-1\right)$
6	$\frac{\partial\mu}{\partial x}\frac{\partial u_y}{\partial x}$	$u_y\left(1-\frac{u_x^2}{V^2}\right)$	27	$\frac{\partial\mu}{\partial y}\frac{\partial u_z}{\partial x}$	$-\frac{u_x u_y u_z}{V^2}$	47	$\frac{\partial u_x}{\partial x}\frac{\partial u_x}{\partial z}$	$\frac{4\mu u_x u_z}{V^2}\left(\frac{u_x^2}{V^2}-1\right)$	67	$\frac{\partial u_y}{\partial x}\frac{\partial u_z}{\partial x}$	$\frac{4\mu u_x^2 u_y u_z}{V^4}$
7	$\frac{\partial\mu}{\partial z}\frac{\partial u_y}{\partial z}$	$u_y\left(1-\frac{u_z^2}{V^2}\right)$	28	$\frac{\partial^2 u_x}{\partial x^2}$	$\mu u_x\left(1-\frac{u_x^2}{V^2}\right)$	48	$\frac{\partial u_y}{\partial x}\frac{\partial u_y}{\partial y}$	$\frac{4\mu u_x u_y}{V^2}\left(\frac{u_y^2}{V^2}-1\right)$	68	$\frac{\partial u_z}{\partial y}\frac{\partial u_x}{\partial y}$	$\frac{4\mu u_x u_y^2 u_z}{V^4}$
8	$\frac{\partial\mu}{\partial x}\frac{\partial u_z}{\partial x}$	$u_z\left(1-\frac{u_x^2}{V^2}\right)$	29	$\frac{\partial^2 u_y}{\partial y^2}$	$\mu u_y\left(1-\frac{u_y^2}{V^2}\right)$	49	$\frac{\partial u_y}{\partial y}\frac{\partial u_y}{\partial z}$	$\frac{4\mu u_y u_z}{V^2}\left(\frac{u_y^2}{V^2}-1\right)$	69	$\frac{\partial u_x}{\partial z}\frac{\partial u_y}{\partial z}$	$\frac{4\mu u_x u_y u_z^2}{V^4}$
9	$\frac{\partial\mu}{\partial y}\frac{\partial u_z}{\partial y}$	$u_z\left(1-\frac{u_y^2}{V^2}\right)$	30	$\frac{\partial^2 u_z}{\partial z^2}$	$\mu u_z\left(1-\frac{u_z^2}{V^2}\right)$	50	$\frac{\partial u_z}{\partial z}\frac{\partial u_z}{\partial x}$	$\frac{4\mu u_x u_z}{V^2}\left(\frac{u_z^2}{V^2}-1\right)$	70	$\frac{\partial u_x}{\partial x}\frac{\partial u_z}{\partial y}$	$\frac{\mu u_y u_z}{V^2}\left(\frac{4u_x^2}{V^2}-1\right)$
10	$\frac{\partial\mu}{\partial x}\frac{\partial u_x}{\partial y}$	$-\frac{u_x^2 u_y}{V^2}$	31	$\frac{\partial^2 u_y}{\partial x^2}$	$\mu u_y\left(1-\frac{u_x^2}{V^2}\right)$	51	$\frac{\partial u_z}{\partial z}\frac{\partial u_z}{\partial y}$	$\frac{4\mu u_y u_z}{V^2}\left(\frac{u_z^2}{V^2}-1\right)$	71	$\frac{\partial u_x}{\partial x}\frac{\partial u_y}{\partial z}$	$\frac{\mu u_y u_z}{V^2}\left(\frac{4u_x^2}{V^2}-1\right)$
11	$\frac{\partial\mu}{\partial y}\frac{\partial u_x}{\partial x}$	$-\frac{u_x^2 u_y}{V^2}$	32	$\frac{\partial^2 u_z}{\partial x^2}$	$\mu u_z\left(1-\frac{u_x^2}{V^2}\right)$	52	$\frac{\partial u_x}{\partial x}\frac{\partial u_y}{\partial x}$	$\frac{2\mu u_x u_y}{V^2}\left(\frac{2u_x^2}{V^2}-1\right)$	72	$\frac{\partial u_x}{\partial z}\frac{\partial u_y}{\partial y}$	$\frac{\mu u_x u_z}{V^2}\left(\frac{4u_y^2}{V^2}-1\right)$
12	$\frac{\partial\mu}{\partial z}\frac{\partial u_x}{\partial x}$	$-\frac{u_x^2 u_z}{V^2}$	33	$\frac{\partial^2 u_x}{\partial y^2}$	$\mu u_x\left(1-\frac{u_y^2}{V^2}\right)$	53	$\frac{\partial u_x}{\partial x}\frac{\partial u_z}{\partial x}$	$\frac{2\mu u_x u_z}{V^2}\left(\frac{2u_x^2}{V^2}-1\right)$	73	$\frac{\partial u_y}{\partial y}\frac{\partial u_z}{\partial x}$	$\frac{\mu u_x u_z}{V^2}\left(\frac{4u_y^2}{V^2}-1\right)$
13	$\frac{\partial\mu}{\partial x}\frac{\partial u_x}{\partial z}$	$-\frac{u_x^2 u_z}{V^2}$	34	$\frac{\partial^2 u_z}{\partial y^2}$	$\mu u_z\left(1-\frac{u_y^2}{V^2}\right)$	54	$\frac{\partial u_x}{\partial y}\frac{\partial u_y}{\partial y}$	$\frac{2\mu u_x u_y}{V^2}\left(\frac{2u_y^2}{V^2}-1\right)$	74	$\frac{\partial u_x}{\partial y}\frac{\partial u_z}{\partial z}$	$\frac{\mu u_x u_y}{V^2}\left(\frac{4u_z^2}{V^2}-1\right)$
14	$\frac{\partial\mu}{\partial x}\frac{\partial u_y}{\partial y}$	$-\frac{u_x u_y^2}{V^2}$	35	$\frac{\partial^2 u_x}{\partial z^2}$	$\mu u_x\left(1-\frac{u_z^2}{V^2}\right)$	55	$\frac{\partial u_y}{\partial y}\frac{\partial u_z}{\partial y}$	$\frac{2\mu u_y u_z}{V^2}\left(\frac{2u_y^2}{V^2}-1\right)$	75	$\frac{\partial u_y}{\partial x}\frac{\partial u_z}{\partial z}$	$\frac{\mu u_x u_y}{V^2}\left(\frac{4u_z^2}{V^2}-1\right)$
15	$\frac{\partial\mu}{\partial y}\frac{\partial u_y}{\partial x}$	$-\frac{u_x u_y^2}{V^2}$	36	$\frac{\partial^2 u_y}{\partial z^2}$	$\mu u_y\left(1-\frac{u_z^2}{V^2}\right)$	56	$\frac{\partial u_x}{\partial z}\frac{\partial u_z}{\partial z}$	$\frac{2\mu u_x u_z}{V^2}\left(\frac{2u_z^2}{V^2}-1\right)$	76	$\frac{\partial u_x}{\partial y}\frac{\partial u_z}{\partial x}$	$\frac{\mu u_y u_z}{V^2}\left(\frac{4u_x^2}{V^2}-1\right)$
16	$\frac{\partial\mu}{\partial y}\frac{\partial u_y}{\partial z}$	$-\frac{u_y^2 u_z}{V^2}$	37	$\frac{\partial^2 u_x}{\partial x\partial y}$	$-\frac{2\mu u_x^2 u_y}{V^2}$	57	$\frac{\partial u_y}{\partial z}\frac{\partial u_z}{\partial z}$	$\frac{2\mu u_y u_z}{V^2}\left(\frac{2u_z^2}{V^2}-1\right)$	77	$\frac{\partial u_x}{\partial z}\frac{\partial u_y}{\partial x}$	$\frac{\mu u_y u_z}{V^2}\left(\frac{4u_x^2}{V^2}-1\right)$
17	$\frac{\partial\mu}{\partial z}\frac{\partial u_y}{\partial y}$	$-\frac{u_y^2 u_z}{V^2}$	38	$\frac{\partial^2 u_x}{\partial x\partial z}$	$-\frac{2\mu u_x^2 u_z}{V^2}$	58	$\frac{\partial u_x}{\partial x}\frac{\partial u_y}{\partial y}$	$\frac{\mu}{V^2}\left(\frac{4u_x^2u_y^2}{V^2}-u_x^2-u_y^2\right)$	78	$\frac{\partial u_y}{\partial x}\frac{\partial u_z}{\partial y}$	$\frac{\mu u_x u_z}{V^2}\left(\frac{4u_y^2}{V^2}-1\right)$
18	$\frac{\partial\mu}{\partial y}\frac{\partial u_z}{\partial z}$	$-\frac{u_y u_z^2}{V^2}$	39	$\frac{\partial^2 u_y}{\partial x\partial y}$	$-\frac{2\mu u_x u_y^2}{V^2}$	59	$\frac{\partial u_x}{\partial x}\frac{\partial u_z}{\partial z}$	$\frac{\mu}{V^2}\left(\frac{4u_x^2u_z^2}{V^2}-u_x^2-u_z^2\right)$	79	$\frac{\partial u_x}{\partial y}\frac{\partial u_y}{\partial z}$	$\frac{\mu u_x u_z}{V^2}\left(\frac{4u_y^2}{V^2}-1\right)$
19	$\frac{\partial\mu}{\partial z}\frac{\partial u_z}{\partial y}$	$-\frac{u_y u_z^2}{V^2}$	40	$\frac{\partial^2 u_y}{\partial y\partial z}$	$-\frac{2\mu u_y^2 u_z}{V^2}$	60	$\frac{\partial u_y}{\partial y}\frac{\partial u_z}{\partial z}$	$\frac{\mu}{V^2}\left(\frac{4u_y^2u_z^2}{V^2}-u_y^2-u_z^2\right)$	80	$\frac{\partial u_z}{\partial y}\frac{\partial u_x}{\partial z}$	$\frac{\mu u_x u_y}{V^2}\left(\frac{4u_z^2}{V^2}-1\right)$
20	$\frac{\partial\mu}{\partial z}\frac{\partial u_z}{\partial x}$	$-\frac{u_x u_z^2}{V^2}$	41	$\frac{\partial^2 u_z}{\partial x\partial z}$	$-\frac{2\mu u_x u_z^2}{V^2}$	61	$\frac{\partial u_x}{\partial y}\frac{\partial u_y}{\partial x}$	$\frac{\mu}{V^2}\left(\frac{4u_x^2u_y^2}{V^2}-u_x^2-u_y^2\right)$	81	$\frac{\partial u_y}{\partial z}\frac{\partial u_z}{\partial x}$	$\frac{\mu u_x u_y}{V^2}\left(\frac{4u_z^2}{V^2}-1\right)$
21	$\frac{\partial\mu}{\partial x}\frac{\partial u_z}{\partial z}$	$-\frac{u_x u_z^2}{V^2}$									

Many differences for the viscous force exist between our new equations (Eq. 22) and the

NSE (eqs. (28-30)). Among them, the term $\nabla \cdot \mathbf{V}$, determined by compressibility and multiplied by the viscosity coefficient, is included in the momentum equation in NSE. So, the viscosity and compressibility are mixed in the NSE, which may be questionable. The term of divergence of velocity does not appear in the new momentum equation, indicating a decoupling between the compressibility and the viscosity. The result is different from the theory of Yang's scaling-law [22] and fractal power-law [23] flows. Much more work needs to verify the theory derived in this paper for compressible fluid flows in the future.

Acknowledgments

The present work is supported by the National Natural Science Foundation of China (under the grant of No. 51827808).

Nomenclature

a_x, a_y, a_z - acceleration components, [m·s^{-2}]
E -internal energy of fluid, [W]
E_v -viscous energy flux, [W·m^{-2}]
e_v -derivative of viscous energy flux, [W·m^{-3}]
F_0 -external force, [N·m^{-2}]
F_v -viscous force, [N·m^{-2}]
$\mathbf{F}_v$ -viscous force vector, [N·m^{-2}]
f_v -volume viscous force, [N·m^{-3}]
p -pressure of fluid, [J·kg^{-1}]
s_v -viscous shear vector, [s^{-1}]
T -fluid temperature, [K]
u_x, u_y, u_z -velocity components, [m·s^{-1}]
V -velocity magnitude, [m·s^{-1}]
μ -viscosity, [N·s·m^{-2}]
Φ_v -viscous energy dissipation, [W·m^{-3}·s^{-1}]
ρ -density of fluid, [kg·m^{-3}]

Subscripts

0 -initial or external
v -viscous
x, y, z -coordinate components

References

[1] Saini, G. L., Some Basic Results on Relativistic Fluid Mechanics and Shock Waves, *Journal of Mathematical Analysis and Applications,* 56(1976), 3, pp.711-717

[2] Falkovich, G., *et al*., Particles and Fields in Fluid Turbulence, *Reviews of Modern Physics,* 73(2001), 4, pp.913-75

[3] McKeown, R., *et al.*, Turbulence Generation through an Iterative Cascade of the Elliptical Instability, *Science Advances,* 6(2020) , 9, Article ID eaaz2717

[4] Xia, H., *et al.*, Upscale Energy Transfer in Thick Turbulent Fluid Layers, *Nature Physics*, 7(2011), pp.321-4

[5] Bailly, C., and Comtme-Bellot, G., *Turbulence,* Springer, 2015

[6] Batchelor, G. K., *An Introduction to Fluid Dynamics,* Cambridge University Press, 2000

[7] Prandtl, L., Oswatitsch, K., and Wieghardt, K., *Introduction to Hydrodynamics*, Science Press, Bejing, 1981*(in Chinese)*

[8] Toth, A., and Bobok, E., Chapter 2. Basic Equations of Fluid Mechanics and Thermodynamics. In: *Flow and Heat Transfer in Geothermal Systems*, Elsevier, 2017. p. 21-55

[9] Spurk, J. H., and Aksel, N., *Fluid Mechanics*, 3 ed, Springer, 2020

[10] Rieutord, M., *Fluid Dynamics: An Introduction,* Springer, New York, 2015

[11] Masui, T., and Inoue, M., *Viscous Fluid Mechanics*, Beijing, Ocean Press, China, 1984*(in Chinese)*

[12] Nakayama, Y., Chapter 6. Flow of Viscous Fluid. In: *Introduction to Fluid Mechanics*, Elsevier Ltd., 2018, pp. 99-133

[13] Xie , M. L., Lin, J. Z., Zhou, H. C., Temporal Stability of a Particle-laden Blasius Boundary Layer, *Modern Physics Letters B*, 23(2009), 2, pp.203-216

[14] Li, Y. Q., Cao, H. L., Zhou, H. C., Zhou, J. -J., Liao, X. -Y., Research on Dynamics of a Laminar Diffusion Flame with Bulk Flow Forcing, *Energy*, 141 (2017), pp.1300-1312

[15] Zhou, H. C., Chen, D. L., Cheng, Q., A New Way to Calculate Radiative Intensity and Solve Radiative Transfer Equation through Using the Monte Carlo Method, *Journal of Quantitative Spectroscopy and Radiative Transfer*, (83)(2004), 3-4, pp.459-481

[16] Wang, D. D., Zhou, H. C., Quantitative Evaluation of the Computational Accuracy for the Monte Carlo Calculation of Radiative Heat Transfer, *Journal of Quantitative Spectroscopy & Radiative Transfer*, 226 (2019), pp. 100-114

[17] Zhou, H. C., Yu, B., Chen, Z. S., et al., A New Theory on Transfer and Dissipation of Viscous Energy in Compressible, Viscous Fluid Flow, https://doi.org/10.21203/rs.3.rs-2402083/v1, Dec. 22, 2022

[18] Tsien, H-S., *Notes on Aerodynamics of Compressible Fluids*, Shanghai, Shanghai Jiaotong University Press, China, 2022 *(in Chinese)*

[19] Dou, H. -S., *Origin of Turbulence-Energy Gradient Theory*, Springer, 2022

[20] John, D., and Anderson, J., *Computational Fluid Dynamics: the Basics with Applications*, McGraw-Hill, Education, 1995

[21] Rao, S. S., Basic Equations of Fluid Mechanics. In: *The Finite Element Method in Engineering*, Elsevier, 2018, pp. 589-610

[22] Yang, X. J., The Scaling-Law Flows: An Attempt at Scaling-Law Vector Calculus, *Fractals*, 2023, 3, doi:10.1142/S0218348X23401266

[23] Yang, X. J.,. An Insight on the Fractal Power Law Flow: from a Hausdorff Vector Calculus Perspective, *Fractals*, 30(2022), 3, Article ID 2250054

Paper submitted: March 28, 2023
Paper revised: April 21, 2023
Paper accepted: December 13, 2023