EXACT SOLITARY WAVE SOLUTIONS FOR NON-LINEAR OPTIC MODEL BY VARIATIONAL PERSPECTIVE

by

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A variational principle for the non-linear optic model is established by semi-inverse method. Two new exact solitary wave solutions are obtained by using the variational transform method. Numerical examples show the novel method is efficient and simple, and can be applied to find solitary wave solutions for different types of wave equations. The physical properties of solitary wave solutions are illustrated by some figures.

Key words: *variational principle, optic model, solitary wave solution*

Introduction

Consider the non-linear optic model [1]:

$$\frac{\partial^2 \vartheta}{\partial t^2} - \frac{\partial^2 \vartheta}{\partial x^2} + \vartheta - \vartheta^2 = 0 \tag{1}$$

where can be successfully used to demonstrate the non-linear optical phenomena. Many powerful algorithms have been proposed and developed to obtain the approximate analytical solution of the non-linear equation, such as tanh-function method (TFM), variational iteration algorithm (VIA) [2], Laplace transform method (LTM) [3, 4], Haar wave method (HWM) [5], Yang-Machado-Baleanu-Cattain wave method (YMBCWM) [6-8], Extended rational sine-cosine method [9], and so on [10-17].

In this paper, a direct and efficient method, called variational transform method, is proposed to find the exact solitary wave solution of the non-linear optic model. These obtained solitary wave solutions are expressed as hyperbolic function, and are completely new. Finally, these obtained solitary wave solutions are illustrated by drawing some corresponding 3-D and 2-D graphs.

Variational principle

Consider the non-linear optic model:

$$\frac{\partial^2 \vartheta}{\partial t^2} - \frac{\partial^2 \vartheta}{\partial x^2} + \vartheta - \vartheta^2 = 0 \tag{2}$$

By using the travelling wave transformation:

$$\vartheta(x,t) = \vartheta(\xi),\ \xi = x - ct \tag{3}$$

where c is the constant.

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Substitute eq. (3) into eq. (2), and have:

$$\left(c^2-1\right)\vartheta_{\xi\xi}+\vartheta-\vartheta^2=0 \tag{4}$$

where

$$\vartheta_{\xi\xi}=\frac{\partial^2\vartheta}{\partial\xi^2}$$

According to the semi-inverse method, the variational principle of eq. (4) can be arrived:

$$J\left(\vartheta\right)=\int_2^\infty\left\{-\frac{c^2-1}{2}\left(\vartheta_\xi\right)^2+\frac{\vartheta^3}{2}-\frac{\vartheta^3}{3}\right\}\mathrm{d}\xi \tag{5}$$

Solitary wave solutions

By the Ritz method, we assume the solitary wave solution of non-linear optic model is the following two forms.

Case 1:

$$\vartheta\left(\xi\right)=p\,\mathrm{sech}\left(\xi\right) \tag{6}$$

where p the constant and require be determined.

Substitute eq. (6) into eq. (5), and obtain:

$$\begin{gathered}J\left(\xi\right)=\int_0^\infty\left\{-\frac{\left(c^2-1\right)p^2}{2}\sec h^2\left(\xi\right)\tanh^2\left(\xi\right)+\frac{p^2}{2}\sec h^2\left(\xi\right)-\frac{p^3}{3}\sec h^3\left(\xi\right)\right\}\mathrm{d}\xi= \\ =-\frac{1}{12}p^3\pi+\frac{2}{3}p^2-\frac{1}{6}c^2p^2\end{gathered} \tag{7}$$

The parameter p is the can be obtained:

$$\frac{\mathrm{d}J}{\mathrm{d}p}=-\frac{1}{4}p^2\pi+\frac{4}{3}p-\frac{1}{3}c^2p=0 \tag{8}$$

By solving eq. (8), we get the parameter:

$$p=-\frac{4(c^2-4)}{3\pi} \tag{9}$$

Therefore, the solitary wave solution of eq. (2) is derived:

$$\vartheta\left(x,t\right)=-\frac{4\left(c^2-4\right)}{3\pi}\mathrm{sech}\left(x-ct\right) \tag{10}$$

Case 2:

$$\vartheta\left(\xi\right)=p\,\mathrm{sech}^2\left(\xi\right) \tag{11}$$

As the aforementioned method, we have:

$$\begin{gathered}J\left(\xi\right)=\int_0^\infty\left\{-2\left(c^2-1\right)p^2\sec h^4\left(\xi\right)\tanh^2\left(\xi\right)+\frac{p^2}{2}\sec h^4\left(\xi\right)-\frac{p^3}{3}\sec h^6\left(\xi\right)\right\}\mathrm{d}\xi= \\ =-\frac{4}{15}c^2p^2+\frac{3}{5}p^2-\frac{8}{45}p^3\end{gathered} \tag{12}$$

By eq. (12), we can get the result:

$$\frac{\mathrm{d}J}{\mathrm{d}p} = -\frac{8}{15}c^2 p + \frac{6}{5}p - \frac{8}{15}p^2 \tag{13}$$

Solve eq. (13), and the parameter p is presented:

$$p = \frac{4}{9} - c^2 \tag{14}$$

Consequently, the another solitary wave solution of eq. (2) is presented:

$$\vartheta(x,t) = \left(\frac{4}{9} - c^2\right)\operatorname{sech}^2(x - ct) \tag{15}$$

In fig. 1, we plot two different kinds of solitary wave solutions of eq. (2) with speed c = 1. Figures 2 and 3 show the two kinds of solitary wave solutions of eq. (2) at the different time t with c = 1. The two figures fully demonstrate that the solitary wave moves along the x-axis in a positive direction with increasing time, while the shape of the wave and amplitude remain unchanged.

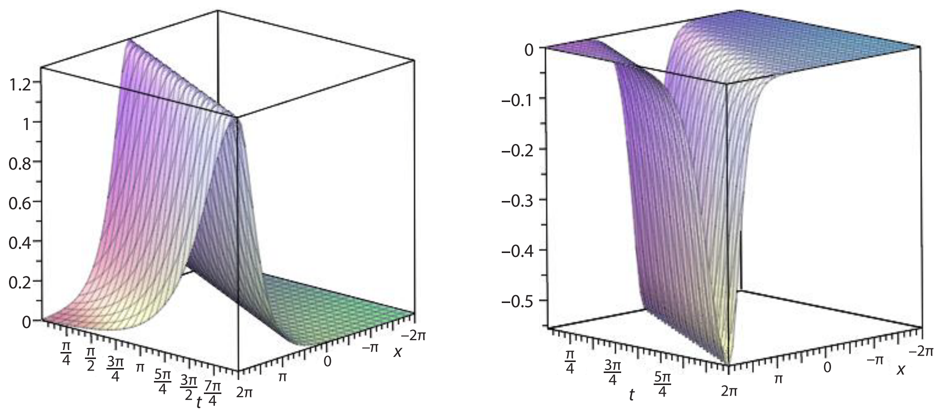

Figure 1. The 3-D plot of two kinds of solitary wave solutions of eq. (2) with c = 1

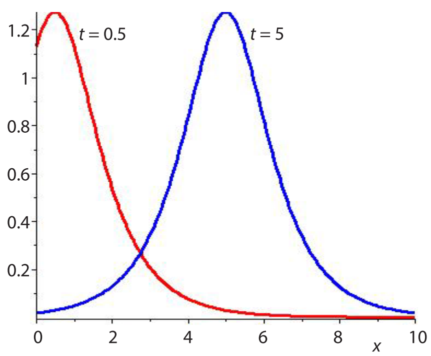

Figure 2. Solitary wave solution of eq. (2) with different t (Case 1)

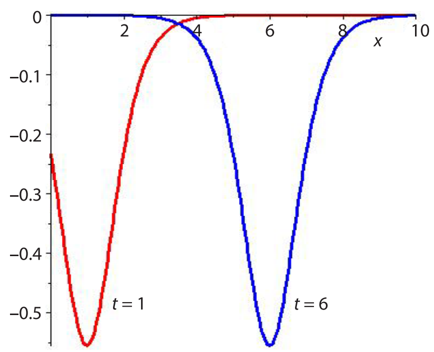

Figure 3. Solitary wave solution of eq. (2) with different t (Case 2)

Conclusion

The variational principle of non-linear optic model is established via semi-inverse method, and its two kinds of solitary wave solutions are obtained by using the variational transform method with very easy and convenient. Some 3-D and 2-D graphs are sketched by selecting proper parameters.These graphs are very helpful in further understanding of the physical characteristics of solitary waves. In future work, we will explore using the proposed method to study the structure of solutions to fractional order evolution equations.

Nomenclature

t – time, [second]

x – space co-ordinate, [m]

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