A PREDICTIVE MODEL FOR SANDSTONE PERMEABILITY BASED ON THE COULOMB-MOHR CRITERION

by

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In this paper, a permeability parameter calculation model with six influence coefficients is constructed by taking volume strain and equivalent strain under Coulomb-Mohr criterion as independent variables. The optimal estimates of permeability parameter influence coefficients are applied to the prediction of permeability parameters under a complex loading path, and the prediction results verify the applicability of the model.

Key words: *plastic flow, permeability parameter, influence coefficients, volume strain*

Introduction

The pore structure and micro-cracks of rock undergo changes following deformation, leading to alterations in permeability parameters as well. In the elastic deformation state, there exists a direct correspondence between stress components and strain components, as well as between permeability parameters and stress components [1-3]. However, once the plastic deformation state is reached, the one-to-one relationship between stress components and strain components in rock no longer holds true, similarly breaking the one-to-one correspondence between permeability parameters and stress components [4]. After yielding occurs, the plastic strain component of rock surpasses that of elastic strain component significantly, thus, resulting in a more pronounced alteration in rock permeability parameter [5]. Furthermore, the post-yielding strain component depends on the loading path adopted, hence making changes to permeability parameters much more intricate compared to those observed during elastic states.

In mining engineering, the evolution law of permeability in surrounding rock serves as the foundation for predicting water inflow in coal mines. The existence of different plastic zones within the surrounding rock necessitates studying changes in rock permeability parameters under plastic flow after yielding, which is particularly crucial for preventing and controlling water inrush disasters in coal mines [6]. However, few scholars have integrated plastic flow theory (incremental theory) with seepage mechanics to investigate post-peak rock permeability. The permeability of rock under plastic flow not only depends on their current stress (strain) state but also closely relates to their loading path history. Moreover, the increment of

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permeability under plastic flow is influenced by factors such as the yield criterion, plastic potential function, and intrinsic quantities reflecting loading history.

To address these issues, this paper constructs a calculation model for sandstone permeability based on Coulomb-Mohr yield criterion and verifies its applicability to provide a theoretical and experimental basis for predicting rock permeability parameters.

Calculation model of permeability parameters under the plastic deformation

The deformation of rock under stress leads to changes in the pore structure, resulting in corresponding variations in porosity and permeability. Numerous research findings have demonstrated that the alterations in porosity and permeability during the transition from elastic state to failure state are associated with different stages of rock deformation. During the initial stage of elastic compression, pores and micro-cracks within the rock close, reducing effective seepage channels and decreasing permeability. As axial strain increases, primary pores begin to expand while new cracks emerge, leading to an increase in effective seepage channels and a gradual rise in permeability. When axial strain reaches its peak value, there is a sharp increase in rock porosity as well as interconnected macroscopic cracks, resulting in a tenfold increase in rock permeability. Therefore, the evolution of porosity and permeability parameters can be linked to the strain state during rock deformation processes.

For steady Darcy flow conditions, only one parameter is required for describing permeability. For non-steady Darcy flow conditions, two parameters are permeability and non-Darcy flow β factor. For steady non-Darcy flow conditions, two parameters are permeability and acceleration coefficient. For unsteady non-Darcy flow situations, three parameters must be considered: permeability, non-Darcy flow β factor, and acceleration coefficient. The relationship between non-Darcy flow permeability and rock porosity can be described using a power exponent function:

$$k = k_0 \left(\frac{\phi}{\phi_0}\right)^{m_k}, \ \beta = \beta_0 \left(\frac{\phi}{\phi_0}\right)^{m_\beta}, \ c_a = c_{a0} \left(\frac{\phi}{\phi_0}\right)^{m_c} \tag{1}$$

where k_0 is the permeability, β_0 – the non-Darcy flow β factor, and c_{a0} – the acceleration coefficient under specific reference conditions. Power exponents and denote coefficients that reflect the rate of change of permeability, non-Darcy flow β factor, and acceleration coefficient with respect to porosity ϕ represents porosity.

Assuming negligible variation in rock mass density during deformation, and volumetric strain $\varepsilon_V \ll 1$ we have:

$$\Delta k = \frac{m_k}{\phi} k \Delta\varepsilon_V \tag{2}$$

Similarly, we can obtain:

$$\Delta\beta = \frac{m_\beta}{\phi} \beta \Delta\varepsilon_V \tag{3}$$

$$\Delta c_a = \frac{m_c}{\phi} c_a \Delta\varepsilon_V \tag{4}$$

where eqs. (2)-(4) represent the mathematical expressions that describe the influence of volumetric strain on permeability parameters.

It should be noted that not only does volumetric strain affect rock permeability parameters, but shear strain also has an impact on them. Particularly in the post-peak state, the influence of shear strain gradually increases with plastic flow. The change in permeability parameters is primarily caused by the relative displacement of shear yield surfaces. This relative movement between shear yield surfaces can be decomposed into normal displacement and tangential displacement. The relative displacement in the normal direction leads to changes in volumetric strain and subsequently affects permeability parameters, while tangential displacement induces changes in shear strain. Although it is commonly believed that tangential displacement does not cause alterations in permeability parameters if the shear surface is smooth, it should be noted that actual shear yield surfaces are not smooth and thus tangential displacement can indeed result in changes both to shear strain and volumetric strain. In triaxial compression tests, it is possible to measure the angle between the normal of a shearing surface and principal stress; however, determining the direction of tangential displacement remains challenging. Therefore, an equivalent strain γ_{eq} corresponding to yield conditions is adopted to describe how shear deformation influences permeability characteristics, leading to modification of calculation formulas for permeability parameters. This implies that:

$$\begin{aligned}
\Delta k &= \frac{k}{\phi}\left(\lambda_1^k \Delta\varepsilon_{\mathrm{V}} + \lambda_2^k \Delta\gamma_{\mathrm{eq}}\right) \\
\Delta\beta &= \frac{\beta}{\phi}\left(\lambda_1^\beta \Delta\varepsilon_{\mathrm{V}} + \lambda_2^\beta \Delta\gamma_{\mathrm{eq}}\right) \\
\Delta c_a &= \frac{c_a}{\phi}\left(\lambda_1^c \Delta\varepsilon_{\mathrm{V}} + \lambda_2^c \Delta\gamma_{\mathrm{eq}}\right)
\end{aligned} \tag{5}$$

where λ_1^k, λ_2^k, λ_1^β, λ_2^β, λ_1^c, λ_2^c are, respectively the influences of volumetric strain and equivalent strain on permeability parameters. Here, we call these six parameters as permeability parameter influence coefficients.

Different yield rules correspond to different equivalent strains γ_{eq}. For the Coulomb-Mohr yield criterion, the equivalent strain can be given [6]:

$$\gamma_{\mathrm{CM}} = \frac{2}{1-2\nu}\left(1-\nu\sec^2\alpha\right)\varepsilon_{(1)} + \frac{2}{1-2\nu}\left(2\nu-\tan^2\alpha\right)\varepsilon_{(3)} \tag{6}$$

It is assumed in the experiment that the rock sample has N stress states:

$$A_i\left(\sigma_1^{(i)}, \sigma_2^{(i)}, \sigma_3^{(i)}\right)$$

corresponding to N permeability parameter values $k^{(i)}$, $\beta^{(i)}$, and $C_a^{(i)}$, $i = 1, 2,..., N$, where A_i is the peak stress state and $\sigma_1^{(i)}$, $\sigma_2^{(i)}$, $\sigma_3^{(i)}$ are the stress.

Let A_M be the peak stress state. The A_{M+1}, A_{M+2},... A_N are the plastic deformation state. In the plastic flow of $A_i \rightarrow A_{i+1}$, $i = M + 1, M + 2,... N - 1$, permeability parameters can be predicted:

$$k^{(M+1)} = k_0\left(\frac{\phi^{(M+1)}}{\phi_0}\right)^{m_k},\ \tilde{k}^{(i)} = k^{(i-1)} + \frac{k^{(i-1)}}{\phi^{(i-1)}}\left(\lambda_1^k \Delta\varepsilon_{\mathrm{V}}^{(i)} + \lambda_2^k \Delta\gamma_{\mathrm{CM}}^{(i)}\right),\ i = M+2, M+3, \cdots, N \tag{7}$$

$$\beta^{(M+1)} = \beta_0\left(\frac{\phi^{(M+1)}}{\phi_0}\right)^{m_\beta},\ \tilde{\beta}^{(i)} = \beta^{(i-1)} + \frac{\beta^{(i-1)}}{\phi^{(i-1)}}\left(\lambda_1^\beta \Delta\varepsilon_{\mathrm{V}}^{(i)} + \lambda_2^\beta \Delta\gamma_{\mathrm{CM}}^{(i)}\right),\ i = M+2, M+3, \cdots, N \tag{8}$$

$$c_a^{(M+1)} = c_{a0}\left(\frac{\phi^{(M+1)}}{\phi_0}\right)^{m_c}, \quad \tilde{c}_a^{(i)} = c_a^{(i-1)} + \frac{c_a^{(i-1)}}{\phi^{(i-1)}}\left(\lambda_1^c \Delta\varepsilon_V^{(i)} + \lambda_2^c \Delta\gamma_{CM}^{(i)}\right), \; i = M+2, M+3, \cdots, N \tag{9}$$

where $\tilde{k}^{(i)}$, $\tilde{\beta}^{(i)}$, and $c_a^{(i)}$ are the predicted value of permeability parameter under stress state A_i.

Here:

$$\Delta\varepsilon_V^{(i)} = \varepsilon_V^{(i)} - \varepsilon_V^{(i-1)}$$

is the increment of volumetric strain in $A_i \rightarrow A_{i+1}$, $i = M + 1, M + 2,... N - 1$.

Here:

$$\Delta\gamma_{CM}^{(i)} = \gamma_{CM}^{(i)} - \gamma_{CM}^{(i-1)}$$

is the increment of equivalent strain in $A_i \rightarrow A_{i+1}$, $i = M + 1, M + 2,... N - 1$.

Verification of the predictive model

When $\lambda_1^{k*} = 2.55$, $\lambda_2^{k*} = 0.151$, $\lambda_1^{\beta*} = 2.50$, $\lambda_2^{\beta*} = 0.156$, $\lambda_1^{c*} = 2.45$, and $\lambda_2^{c*} = 0.153$ are taken, eqs. (7)-(9) are used to obtain the predicted values of permeability parameters under various stress states in the plastic flow of sandstone samples, fig. 1.

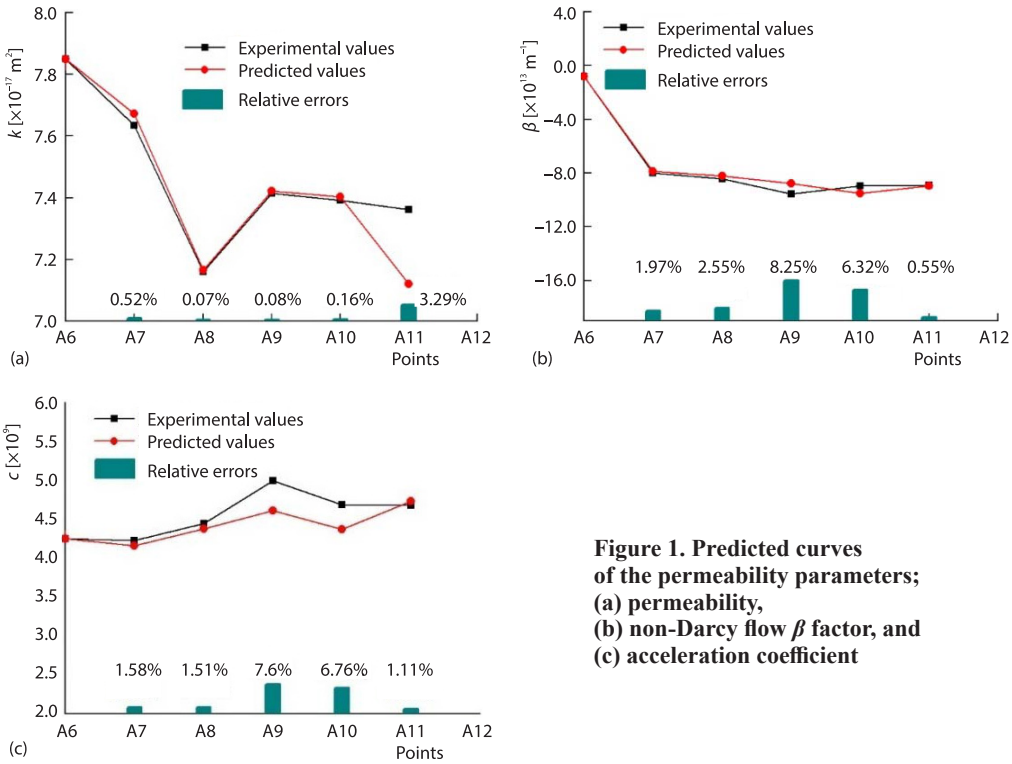

Figure 1. Predicted curves of the permeability parameters; (a) permeability, (b) non-Darcy flow β factor, and (c) acceleration coefficient

Conclusion

In this paper, a prediction model of permeability parameters based on the Coulomb-Mohr yield criterion is constructed. Through a simple loading path of the permeability test, the optimal estimate of the permeability parameter influence coefficient is determined. The optimal estimate of the permeability parameter influence coefficient is applied to the prediction

of the permeability parameters under a complex loading path, and the prediction model error is analyzed to verify the applicability of the model. The model is used to predict the permeability parameters of sandstone. The prediction errors of the permeability, non-Darcy flow factor and acceleration coefficient are less than 4.0%, 9.0%, and 8.0%, respectively, indicating that the model has good applicability.

Nomenclature

c_a – acceleration coefficient, [–]
k – permeability, [m^2]

Greek symbols

β – non-Darcy flow factor, [m^{-1}]
γ_{eq} – equivalent strain, [–]
ε_V – volumetric strain, [–]
ν – Poisson's ratio, [–]
ϕ – porosity, [%]

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