SIMULATION OF FRACTIONAL DIFFERENTIAL DIFFERENCE EQUATION VIA RESIDUAL POWER SERIES METHOD

by

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In the present article, the fractional order differential difference equation is solved by using the residual power series method. Residual power series method solutions for classical and fractional order are obtained in a series form showing good accuracy of the method. Illustrative models are considered to affirm the legitimacy of the technique. The accuracy of the chosen problems is represented by tables and plots which show good accuracy between the exact and assimilated solutions of the models.

Key words: *fractional differential difference equations, residual power series method, fractional PDE,*

Introduction

Fractional calculus is viewed as an integral asset for demonstrating physical occurrence. Recently, the analysts have indicated the best enthusiasm towards fractional calculus since its several applications in wide areas of sciences. Regardless of a complex history of fractional calculus, it seemed from an important question of L'Hospital. The dz/dx symbolize slope of a function what if we have $\mathrm{d}^{1/2}y/\mathrm{d}x^{1/2}$. To discover the appropriate response to this question, the mathematicians have figured out how to open another door of opportunities to improve the scientific demonstration of certifiable issues, which has brought forth numerous new questions and interesting outcomes. These recently settled outcomes have various execution in numerous regions of engineering [1, 2], fractional Caputo Fabrizio derivative for hepatitis B virus [3], fractional modelling for disease of chickenpox [4], fractional blood ethanol concentration model [5], fractional order pine wilt disease model [6], fractional order pine wilt disease model [7], *etc.* Currently, the attention of the scholars is to improve various analytical and numerical techniques for the solution of FDE. Consequently, various types of numerical and semi analytical methods have been settled and used for the simulation of FDE [8, 9].

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In the existing study we have examined the fractional view of some significant fractional differential difference equations (FDDE). These equations are basically the physical modelling of the nanotechnology problems *e.g.* electric current and flow in carbon nanotubes [10], electric lattices, molecular crystals and non-linear coupled optical waveguides and nanotechnology areas [11]. In this article we solved FDDE by using RPSM [12] which give the best evidence about the actual physical situation as compare to classical-order problems solution. Besides, the suggested method provided the solutions of the problems that have coincidence with the exact solutions. The procedure can be drawn towards other FPDE that are often happened in different field of real existence. The rest of the paper is sorted out as: in 2nd unit, we offered the fundamental definitions and hypothesis of the planned method, in 3rd unit we evaluated the numerical illustrations by means of the scheduled method and talked about the plots, and in 4th unit we in conclusion composed the end.

Methodology and convergence analysis

Definition 1. [13] Multiple fractional power series (MFPS) of $\Im(s, t)$ centered at $t = t_0$:

$$\sum_{j=0}^{\infty}\beta_j(s)(t-t_0)^{\mu j} = \beta_0(s)+\beta_1(s)(t-t_0)^{\mu}+\beta_2(s)(t-t_0)^{2\mu}+\beta_3(s)(t-t_0)^{3\mu}+... \tag{1}$$

Theorem 1. [13] Suppose that $\Im(s, t)$ has MFPS centered at $t = t_0$:

$$\Im(s,t)=\sum_{j=0}^{\infty}\beta_j(s)(t-t_0)^{\mu j},\ \ t_0\le t<t_0+\Re \tag{2}$$

If $\Im(s, t)$ is continuous on $[t_0, t_0 + \Re]$ and $D^{\mu j}\Im(s, t)$ is differentiable on $[t_0, t_0 + \Re]$ for $j = 0, 1, 2,...$ then:

$$\beta_j(s)=\frac{D^{\mu j}\Im(t_0)}{\Gamma(1+\mu j)}\ \text{ where }\ D^{\mu j}=\underbrace{D^{\mu}.D^{\mu}.D^{\mu}.D^{\mu}.......D^{\mu}}_{j\text{ times}}$$

Consider a coupled fractional differential difference equation:

$$\begin{gathered}D_t^{k^*\mu_i}E_i(s,t)+\chi\left[s,t,E_i(\upsilon,t),E_i(s-k,t),E_i(s+k,t)\right]+\\+\wp\left[s,t,E_i(s,t),E_i(s-k,t),E_i(s+k,t)\right],\ \text{ where }\ k,k^*\in\mathbb{N},\ \ 0<\mu\le 1,\ \ i=1,2\end{gathered} \tag{3}$$

where $D_t^{\mu_i}$ is the Caputo fractional derivative, χ – the linear, and $\wp$ – the non-linear terms:

$$E_i(s,t),E_i(s-k,t)\ \text{ and }\ E_i(s+k,t)$$

Subject to $E_i(s, 0) = \beta_i(s)$.

The GRPSM assumes the solution of eq. (3) in multiple fractional power series form centered at $t_0 = 0$:

$$\begin{aligned}E_i(s,t)&=\sum_{j=0}^{\infty}\frac{\beta_{i,j}(s)}{\Gamma(\mu_i j+1)}(t)^{j\mu_i}\\E_i(s-k,t)&=\sum_{j=0}^{\infty}\frac{\beta_{i,j}(s-k)}{\Gamma(\mu_i j+1)}(t)^{j\mu_i}\\E_i(s+k,t)&=\sum_{j=0}^{\infty}\frac{\beta_{i,j}(s+k)}{\Gamma(\mu_i j+1)}(t)^{j\mu_i}\end{aligned} \tag{4}$$

where m^{th} truncated series of eq. (4) takes the form:

$$E_i(s,t)=\sum_{j=0}^{m}\frac{\beta_{i,j}(s)}{\Gamma(\mu_i j+1)}(t)^{j\mu_i}$$
$$E_i(s-k,t)=\sum_{j=0}^{m}\frac{\beta_{i,j}(s-k)}{\Gamma(\mu_i j+1)}(t)^{j\mu_i} \tag{5}$$
$$E_i(s+k,t)=\sum_{j=0}^{m}\frac{\beta_{i,j}(s+k)}{\Gamma(\mu_i j+1)}(t)^{j\mu_i}$$

If we take $m = 0$ then by eq. (5) we have zero order RPS truncated solutions:

$$E_{i,0}(s,t)=E_i(s,0)=\beta_{i,0}=\beta_i(s)$$

therefore, eq. (5) implies

$$E_i^m(s,t)=\beta_{i,0}(s)+\sum_{j=1}^{m}\frac{\beta_{i,j}(s)}{\Gamma(\mu_i j+1)}(t)^{j\mu_i}$$
$$E_i^m(s-k,t)=\beta_{i,0}(s-k)+\sum_{j=1}^{m}\frac{\beta_{i,j}(s-k)}{\Gamma(\mu_i j+1)}(t)^{j\mu_i}$$
$$E_i^m(s+k,t)=\beta_{i,0}(s+k)+\sum_{j=1}^{m}\frac{\beta_{i,j}(s+k)}{\Gamma(\mu_i j+1)}(t)^{j\mu_i}$$

By the representation of $E_i^m(s, t)$ the m^{th} RPS approximate solution will be obtained when β_{ij} are available for $j = 1,..., m$.

The residual function for eq. (3):

$$\operatorname{Re}s\left[E_i(s,t,s-k,s+k)\right]=D_t^{k^*\mu_i}E_i(s,t)+\chi\left[s,t,E_i(s,t),E_i(s-k,t),E_i(s+k,t)\right]+$$
$$+\wp\left[s,t,E_i(s,t),E_i(s-k,t),E_i(s+k,t)\right]$$

More over the m^{th} residual functions:

$$\operatorname{Re}s\left[E_i^m(s,t,s-k,s+k)\right]=D_t^{k^*\mu_i}E_i^m(s,t)+\chi\left[s,t,E_i^m(s,t),E_i^m(s-k,t),E_i^m(s+k,t)\right]+$$
$$+\wp\left[s,t,E_i^m(s,t),E_i^m(s-k,t),E_i^m(s+k,t)\right]$$

We have some useful facts which are essential for RPSM [14]:

$$\lim_{m\to\infty}\left[E_i^m(s,t)\right]=\left[E_i(s,t)\right]$$
$$\operatorname{Re}s\left[E_i(s,t)\right]=0,\ i=1,2$$
$$\lim_{m\to\infty}\operatorname{Re}s\left[E_i^m(s,t)\right]=\operatorname{Re}s\left[E_i(s,t)\right],\ s\in I\subseteq\mathbb{R},\ |t|\le\Re$$

We assume:

$$D_t^{(m-k^*)\mu_i}\operatorname{Re}s\left[E_i^m(s,0)\right]=0,\ D_t^{(m-k^*)\mu_i}\operatorname{Re}s\left[E_i^m(s-k,0)\right]=0$$
$$\text{and }D_t^{(m-k^*)\mu_i}\operatorname{Re}s\left[E_i^m(s+k,0)\right]=0$$

Since

$$D_t^{(m-k^*)\mu_i}\operatorname{Re}s\left[E_i^m(s,0)\right]=\beta_{i,j}+D_t^{(m-k^*)\mu_i}\left.\left\{\begin{array}{l}\chi\left[s,t,E_i^m(s,t),E_i^m(s-k,t),E_i^m(s+k,t)\right]\\+\wp\left[s,t,E_i^m(s,t),E_i^m(s-k,t),E_i^m(s+k,t)\right]\end{array}\right\}\right|_{t=0}$$

this relation is essential rule in GPSM so fractional power series solution of eq. (3) is considered:

$$E_i(s,t)=\beta_{i,0}(s)+\sum_{j=1}^{\infty}\frac{\beta_{i,j}(s)}{\Gamma(j\mu_i+1)}t^{j\mu_i}$$

$$E_i(s-k,t)=\beta_{i,0}(s-k)+\sum_{j=1}^{\infty}\frac{\beta_{i,j}(s-k)}{\Gamma(j\mu_i+1)}t^{j\mu_i}$$

$$E_i(s+k,t)=\beta_{i,0}(s+k)+\sum_{j=1}^{\infty}\frac{\beta_{i,j}(s+k)}{\Gamma(j\mu_i+1)}t^{j\mu_i}$$

where

$$\beta_{i,j}=-D_t^{(m-k^*)\mu_i}\left\{\begin{array}{l}\chi\left[s,t,E_i^m(s,t),E_i^m(s-k,t),E_i^m(s+k,t)\right]+\\+\wp\left[s,t,E_i^m(s,t),E_i^m(s-k,t),E_i^m(s+k,t)\right]\end{array}\right\}_{t=0} \tag{6}$$

where $m=k^*,k^*+1,\ k^*+2,...$

Applications and discussion

Model 1. Fractional MKDV Lattice equation [15]:

$$D_t^{\mu}E(s,t)=\left\{1-\left[E(s,t)\right]^2\right\}\left[E(s+1,t)-E(s-1,t)\right],\ 0<\mu\le 1 \tag{7}$$

Subject to IC $E(s, 0) = b\tanh[s]$, where $b = \tanh[k]$ and k is any constant.
For $\mu = 1$ exact solution of eq. (7) implies $E(s, t) = b\tanh[ks + 2bt]$.
Using the process of RPSM we can write solution of eq. (7):

$$E(s,t)=\sum_{m=0}^{\infty}\beta_m(s)\frac{t^{m\mu}}{\Gamma(m\mu+1)}=\beta_0(s)+\beta_1(s)\frac{t^{\mu}}{\Gamma(\mu+1)}+\beta_2(s)\frac{t^{2\mu}}{\Gamma(2\mu+1)}+... \tag{8}$$

Equation (8) further takes the form:

$$E(s,t)=\beta_0(s)+\sum_{m=1}^{k}\beta_m(s)\frac{t^{m\mu}}{\Gamma(m\mu+1)},\ k=1,2,3...$$

where the initial approximation

$$\beta_0(s)=b\tanh[s] \tag{9}$$

Using algorithm, we write our 3rd order approximation:

$$E(s,t)=\beta_0(s)+\beta_1(s)\frac{t^{\mu}}{\Gamma(\mu+1)}+\beta_2(s)\frac{t^{2\mu}}{\Gamma(2\mu+1)}+\beta_3(s)\frac{t^{3\mu}}{\Gamma(3\mu+1)} \tag{10}$$

Using eq. (6) we compute:

$$\beta_1(s)=-\beta_0(s-1)+\beta_0(s-1)\left[\beta_0(s)\right]^2+\beta_0(s+1)-\left[\beta_0(s)\right]^2_0(s+1)$$

$$\beta_2(s)=-\beta_1(s-1)+\left[\beta_0(s)\right]^2\beta_1(s-1)+2\beta_0(s-1)\beta_0(s)\beta_1(s)-$$
$$-2\beta_0(s)\beta_0(s+1)\beta_1(s)+\beta_1(s+1)-\left[\beta_0(s)\right]^2\beta_1(s+1)$$

and

$$\beta_3(s)=-\beta_2(s-1)+\left[\beta_0(s)\right]^2\beta_2(s-1)+2\beta_0(s-1)\beta_0(s)\beta_2(s)-$$
$$-2\beta_0(s)\beta_0(s+1)\beta_2(s)+\beta_2(s+1)-\left[\beta_0(s)\right]^2\beta_2(s+1)$$

By putting all these values in eq. (10) we have:

$$E(s,t)=\beta_0(s)+\left\{-\beta_0(s-1)+\beta_0(s-1)\left[\beta_0(s)\right]^2+\beta_0(s+1)-\left[\beta_0(s)\right]^2\beta_0(s+1)\right\}\frac{t^{\mu}}{\Gamma(\mu+1)}+$$

$$+\begin{pmatrix}-\beta_1(s-1)+\left[\beta_0(s)\right]^2\beta_1(s-1)+2\beta_0(s-1)\beta_0(s)\beta_1(s)-\\ -2\beta_0(s)\beta_0(s+1)\beta_1(s)+\beta_1(s+1)-\left[\beta_0(s)\right]^2\beta_1(s+1)\end{pmatrix}\frac{t^{2\mu}}{\Gamma(2\mu+1)}+$$

$$+\begin{pmatrix}-\beta_2(s-1)+\left[\beta_0(s)\right]^2\beta_2(s-1)+2\beta_0(s-1)\beta_0(s)\beta_2(s)\\ -2\beta_0(s)\beta_0(s+1)\beta_2(s)+\beta_2(s+1)-\left[\beta_0(s)\right]^2\beta_2(s+1)\end{pmatrix}\frac{t^{3\mu}}{\Gamma(3\mu+1)}$$

Table 1 display the comparison of RPSM and MLHM solution for classical MKDV lattice equation. Figure 1 display the 2-D plot of lattice equation using different values of μ it is experienced from the figure that space fractional order varies towards space integer order graph. Figure 2(a) and 2(b) represent exact and approximate surface of classical lattice equation.

Table 1. Comparison of absolute error of MLHM and RPSM at t = 0.4 and k = 0.2

s	Abs error of 2nd order MLHM [16]	Abs error of 2nd order RPSM	Abs error of 4th order MLHM [16]	Abs error of 3rd order RPSM
–10	$3.50 \cdot 10^{-5}$	$3.50081 \cdot 10^{-5}$	$3.43 \cdot 10^{-5}$	$2.47792 \cdot 10^{-6}$
–6	$8.61 \cdot 10^{-5}$	$8.60799 \cdot 10^{-5}$	$7.96 \cdot 10^{-5}$	$2.72077 \cdot 10^{-6}$
–2	$1.461 \cdot 10^{-4}$	$1.46099 \cdot 10^{-4}$	$1.446 \cdot 10^{-4}$	$2.26698 \cdot 10^{-5}$
2	$1.048 \cdot 10^{-4}$	$1.04839 \cdot 10^{-4}$	$1.086 \cdot 10^{-4}$	$2.85904 \cdot 10^{-5}$
6	$8.44 \cdot 10^{-5}$	$8.44154 \cdot 10^{-5}$	$7.94 \cdot 10^{-5}$	$1.05628 \cdot 10^{-6}$
10	$3.06 \cdot 10^{-5}$	$3.06169 \cdot 10^{-5}$	$3.01 \cdot 10^{-5}$	$1.91326 \cdot 10^{-6}$

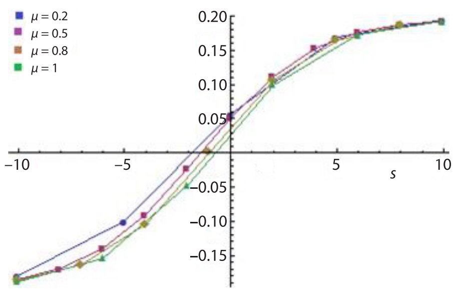

Figure 1. Numerical solution of fractional MKDV lattice equation at t = 0.4 and k = 0.2

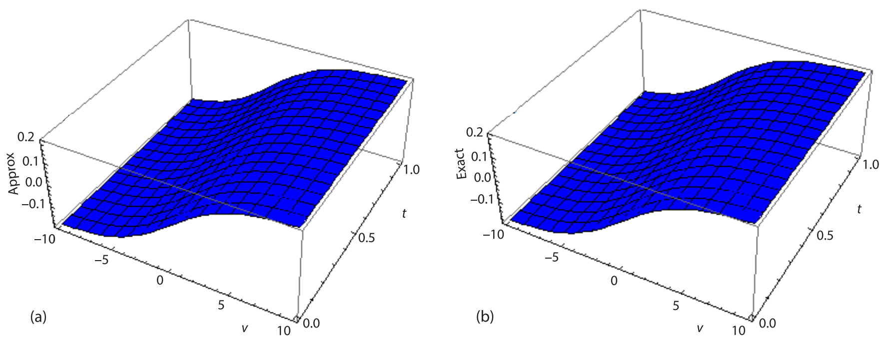

Figure 2. (a) Numerical solution of MKDV lattice equation at μ = 1 and (b) exact solution

Model 2. Time fractional Toda Lattice non-linear differential difference equation [16]:

$$\frac{\partial^{\mu} E_1}{\partial t^{\mu}} = E_1(s,t)\left[E_2(s,t) - E_2(s-1,t)\right]$$
$$\frac{\partial^{\lambda} E_2}{\partial t^{\lambda}} = E_2(s,t)\left[E_1(s+1,t) - E_1(s,t)\right] \tag{11}$$

With IC's

$$E_1(s,0) = -\alpha\coth(\vartheta) + \alpha\tanh(\vartheta s)$$
$$E_2(s,0) = -\alpha\coth(\vartheta) - \alpha\tanh(\vartheta s) \tag{12}$$

For $\mu = \lambda = 1$ exact solution for given system is:

$$E_1(s,t) = -\alpha\coth(\vartheta) + \alpha\tanh(\vartheta s + \alpha t + \delta)$$
$$E_2(s,t) = -\alpha\coth(\vartheta) - \alpha\tanh(\vartheta s + \alpha t + \delta)$$

where α, ϑ, and δ are constant.

Using the process of RPSM we can write solution of eq. (11):

$$E_1(s,t) = \sum_{j=0}^{\infty} \beta_{1,j}(s)\frac{t^{j\mu}}{\Gamma(j\mu+1)} = \beta_{1,0}(s) + \beta_{1,1}(s)\frac{t^{\mu}}{\Gamma(\mu+1)} + \beta_{1,2}(s)\frac{t^{2\mu}}{\Gamma(2\mu+1)} + ...$$
$$E_2(s,t) = \sum_{j=0}^{\infty} \beta_{1,j}(s)\frac{t^{j\lambda}}{\Gamma(j\lambda+1)} = \beta_{2,0}(s) + \beta_{2,1}(s)\frac{t^{\lambda}}{\Gamma(\lambda+1)} + \beta_{2,2}(s)\frac{t^{2\lambda}}{\Gamma(2\lambda+1)} + ... \tag{13}$$

Equation (13) can further be written:

$$E_1(s,t) = \beta_{1,0}(s) + \sum_{j=1}^{m} \beta_{1,j}(s)\frac{t^{j\mu}}{\Gamma(j\mu+1)},\ m = 1,2,3...$$
$$E_2(s,t) = \beta_{2,0}(s) + \sum_{j=1}^{m} \beta_{2,j}(s)\frac{t^{j\lambda}}{\Gamma(j\lambda+1)},\ m = 1,2,3...$$

Where the initial approximation:

$$\beta_{1,0}(s) = -\alpha\coth[\vartheta] + \alpha\tanh[\vartheta s]$$
$$\beta_{2,0}(s) = -\alpha\coth[\vartheta] - \alpha\tanh[\vartheta s] \tag{14}$$

Using algorithm we write our 3rd order approximation for $E_1(s, t)$ and $E_2(s, t)$ of eq. (11):

$$E_1(s,t) = \beta_{1,0}(s) + \beta_{1,1}(s)\frac{t^{\mu}}{\Gamma(\mu+1)} + \beta_{1,2}(s)\frac{t^{2\mu}}{\Gamma(2\mu+1)} + \beta_{1,3}(s)\frac{t^{3\mu}}{\Gamma(3\mu+1)}$$

and

$$E_2(s,t) = \beta_{2,0}(s) + \beta_{2,1}(s)\frac{t^{\lambda}}{\Gamma(\lambda+1)} + \beta_{2,2}(s)\frac{t^{2\lambda}}{\Gamma(2\lambda+1)} + \beta_{2,3}(s)\frac{t^{3\lambda}}{\Gamma(3\lambda+1)} \tag{15}$$

Using eq. (14) we compute:

$$\begin{gathered}\beta_{1,1}(s)=-\beta_{1,0}(s)\beta_{2,0}(-1+s)+\beta_{1,0}(s)\beta_{2,0}(s)\\ \beta_{2,1}(s)=-\beta_{1,0}(s)\beta_{2,0}(s)+\beta_{1,0}(1+s)\beta_{2,0}(s)\\ \beta_{1,2}(s)=-\beta_{1,1}(s)\beta_{2,0}(-1+s)+\beta_{1,1}(s)\beta_{2,0}(s)\\ \beta_{2,2}(s)=-\beta_{1,0}(s)\beta_{2,1}(s)+\beta_{1,0}(1+s)\beta_{2,1}(s)\\ \text{and}\\ \beta_{1,3}(s)=-\beta_{1,2}(s)\beta_{2,0}(-1+s)+\beta_{1,2}(s)\beta_{2,0}(s)\\ \beta_{2,3}(s)=-\beta_{1,0}(s)\beta_{2,2}(s)+\beta_{1,0}(1+s)\beta_{2,2}(s)\end{gathered} \tag{16}$$

By putting all these functions in eq. (15) we have:

$$\begin{gathered}E_1(s,t)=\beta_{1,0}(s)+\left[-\beta_{1,0}(s)\beta_{2,0}(-1+s)+\beta_{1,0}(s)\beta_{2,0}(s)\right]\frac{t^{\mu}}{\Gamma(\mu+1)}+\\ +\left[-\beta_{1,1}(s)\beta_{2,0}(-1+s)+\beta_{1,1}(s)\beta_{2,0}(s)\right]\cdot\\ \cdot\frac{t^{2\mu}}{\Gamma(2\mu+1)}+\left[-\beta_{1,2}(s)\beta_{2,0}(-1+s)+\beta_{1,2}(s)\beta_{2,0}(s)\right]\frac{t^{3\mu}}{\Gamma(3\mu+1)}\end{gathered}$$

and

$$\begin{gathered}E_2(s,t)=\beta_{2,0}(s)+\left[-\beta_{1,0}(s)\beta_{2,0}(s)+\beta_{1,0}(1+s)\beta_{2,0}(s)\right]\frac{t^{\lambda}}{\Gamma(\lambda+1)}+\\ +\left[-\beta_{1,0}(s)\beta_{2,1}(s)+\beta_{1,0}(1+s)\beta_{2,1}(s)\right]\cdot\\ \cdot\frac{t^{2\lambda}}{\Gamma(2\lambda+1)}+\left[-\beta_{1,0}(s)\beta_{2,2}(s)+\beta_{1,0}(1+s)\beta_{2,2}(s)\right]\frac{t^{3\lambda}}{\Gamma(3\lambda+1)}\end{gathered}$$

Table 2 demonstrate the result for $E_1(s, t)$ part of fractional Toda non-linear lattice equation using different values of μ. Figures 3(a) and 3(b) shows the space graph for $E_1(s, t)$ and $E_2(s, t)$ parts of coupled system. It is observed from the graphs that space fractional order converges towards space integer order graph. Figure 4 display the time graph for $E_1(s, t)$ and $E_2(s, t)$ parts of coupled system. It is observed that absolute error of time fractional order converges to zero as we move from fractional to classical order.

Table 2. Solution of RPSM for $E_1(s, t)$ part of fractional Toda Lattice equation at $t = 1$

s	3rd order RPSM at $\mu = 1/3$	3rd order RPSM at $\mu = 2/3$	3rd order RPSM at $\mu = 1$	Abs error
–40	–1.1032490	–1.1032492	–1.1032494	$2.3896923 \cdot 10^{-7}$
–20	–1.0989432	–1.0989516	–1.0989649	$1.0048038 \cdot 10^{-5}$
–20	–1.0748055	–1.0748520	–1.0749348	$2.6070591 \cdot 10^{-5}$
10	–0.9224896	–0.9225354	–0.9226189	$6.6221259 \cdot 10^{-4}$
20	–0.9061378	–0.9061462	–0.9061595	$1.2641990 \cdot 10^{-4}$
40	–0.9033832	–0.9033833	–0.9033836	$2.4500022 \cdot 10^{-6}$

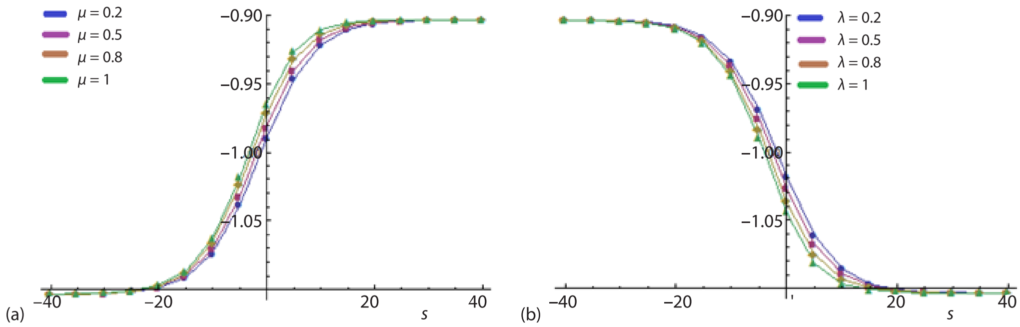

Figure 3. Numerical solution for $E_1(s, t)$ (a) and (b) $E_2(s, t)$ parts of fractional Toda Lattice equation at $t = 4$

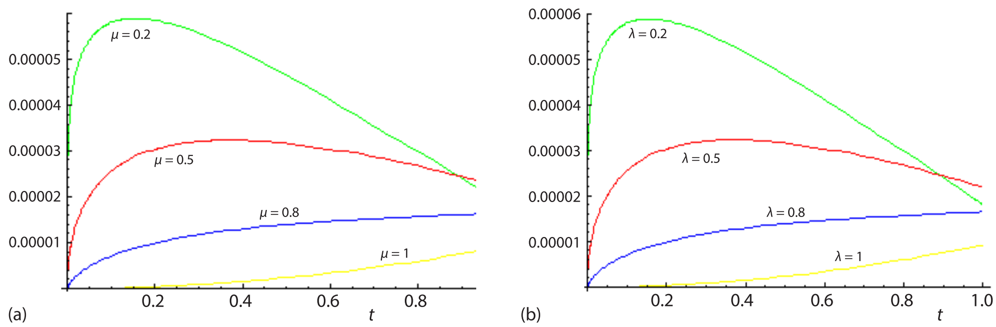

Figure 4. (a), (b) display absolute error graph for $E_1(s, t)$ (a) and (b) $E_2(s, t)$ part of eq. (11) at $s = 40$ seconds

Conclusion

In the current article, we presented some FDDE, arising in modern sciences. A novel and classy technique, which is identified as RPSM is applied for both fractional and classical problems. For pertinence and unwavering quality of the proposed method, some illustrative models are solved. It has been explored through graphical and tabulated results that the current method gives a precise and meriting investigation about the physical occurring of the problems. Also, the current method is favored when contrasted with other technique in light of its better pace of convergence. This course rouses the scientists towards the execution of the present method for other non-linear FDE.

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