PROMISES AND CHALLENGES OF FRACTAL THERMODYNAMICS

By

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Original scientific paper
https://doi.org/10.2298/TSCI233A735Z

The fractal thermodynamics deals with thermodynamical phenomena to find nature laws in a fractal space. Here is introduced a new concept of fractal entropy, and the traditional principle of entropy increase should be revised as the fractal entropy equilibrium. Additionally, the law of brachistofractality, or the minimal fractional dimensions law, is also introduced, it implies that an object remains its fractal dimensions unchanged without an external action, and it minimizes its fractal dimensions under an external perturbation. An example is given to measure the fractal dimensions of a fabric, so that a swimming vest with minimal friction can be optimally designed. This article offers a new angle for observing a thermodynamical phenomenon.

Key words: *fractal entropy, minimal energy consumption, fractal harmonic law, fractal diffusion, minimal fractal dimensions change*

Introduction

When travelling through a stark desert, you might be astonished with its aesthetic landscape. The dune moves chaotically every second under a wind, and your car might be covered with sands when you wake up in the morning, but the whole dune seems to be unchanged. What is the nature law that creates such an unconvincing phenomenon? If we observe the phenomenon in a fractal frame, the value of the dune's two-scale fractal dimensions [1] keeps almost unchanged. The wind is an external force, and it changes the tune's surface morphology, while the tune motion obeys the law of *brachistofractality* (the law of minimal fractal dimensions change) [2]. An object remains its fractal dimensions under a constant external force, and when subjected to an external perturbation, it will be deformed by minimizing its fractal dimensions.

In the traditional thermodynamics, air is assumed to be a 3-D continuum. However, when we measure the air on a molecule's scale, it becomes a porous medium, or a fractal space. When water molecules are evaporated from boiling water into air, their motion obeys physical laws in the fractal space, additionally, the air around the boiling water tries to minimize its fractal dimensions change, and the water molecules will gradually be distributed uniformly among the air. The boiling temperature of water in Tibet, the highest plateau in the

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Earth, is much less than 100 °C, this is because of the lower air density, resulting in a lower value of the fractal dimensions. Assume that the air were 3 dimensions on any scales, water would not boil at any high temperature.

Now we look through an experimental phenomenon arising in the nanoscale hydrodynamics, Majumder *et al.* [3] found an unprecedentedly rapid transport of water molecules through a carbon nanotube, which cannot be explained the traditional continuum mechanics, the conventional fluid-flow theory leads to an extremely enormous error of 1000000% to 10000000%, see the report published in Nature in 2005 [3]. Another example is the most famous Einstein's mass-energy equation, $E = mc^2$, which leads to about 4.5% accuracy to predict the energy in the cosmos, that means 95.5% energy is missing [4], this is because Einstein's equation is obtained under 4-D spacetime, however the fractal dimensions of our cosmos is 4.236 [5], the fractal spacetime leads to El-Naschie's mass-energy equation: $E = mc^2/22$ [4]. To give a heuristic explanation of these magic phenomena, we just think that a 2-D theory can never predict any phenomena arising in the 3rd dimension, and we cannot predict a 3-D truth from a 2-D projection. Similarly the continuum mechanics leads to a tremendous error or an opposite conclusion when applied to a discontinuous problem, human beings cannot image any phenomena arising in a four dimensional space.

Fractal thermodynamics

The terminology of *fractal thermodynamics* was also appeared in [6], here the fractal thermodynamics, or more specially the two-scale fractal thermodynamics, is to study any thermodynamical phenomena in a fractal space, following the natural laws and He-Liu's formulation for calculating the two-scale fractal dimensions [7]:

$$\alpha = 3(1-\varepsilon) \tag{1}$$

where α is the two-scale fractal dimensions and ε – the porosity. The two-scale fractal theory is now widely used to deal with problems arising in porous media or unsmooth boundaries, see for examples [8-12].

To understand the basic idea of fractal thermodynamics, we give an example. The air permeability through a porous medium obeys the mass conservation law and the energy conservation law in a fractal space, and a fractal-fractional differential model can be established as that discussed in [13, 14]. In this issue, Li [15] from Xi'an University of Architecture and Technology also suggested, for the first time ever, a fractal-fractional model for a nanofluid, and Sun [16] from Jiaozuo Teacher's College re-studied the most famous Schrodinger equation in a fractal spacetime and obtained a fractal variational principle. The fractal variational theory is to study the energy transformation between the kinetic energy and the potential energy in a fractal space [17-20].

Now we re-consider the entropy in a fractal space. We begin with the original definition:

$$\mathrm{d}S = \frac{\mathrm{d}Q}{T} \tag{2}$$

where S is the entropy, Q – the heat flux, and T – the temperature.

Equation (2) is established in a continuous space, and we now re-define it in a fractal space:

$$\frac{\mathrm{d}S}{\mathrm{d}x^{\alpha_i}} = \frac{1}{T}\frac{\mathrm{d}Q}{\mathrm{d}x^{\alpha_j}}, \quad (i, j = 1, 2, i \neq j) \tag{3}$$

where $\mathrm{d}S/\mathrm{d}x^{\alpha}$ is the two-scale fractal derivative [1], $\mathrm{d}x^{\alpha}$ can be explained as the average distance between two adjacent molecules, as shown in fig. 1, and α is the two-scale fractal dimensions.

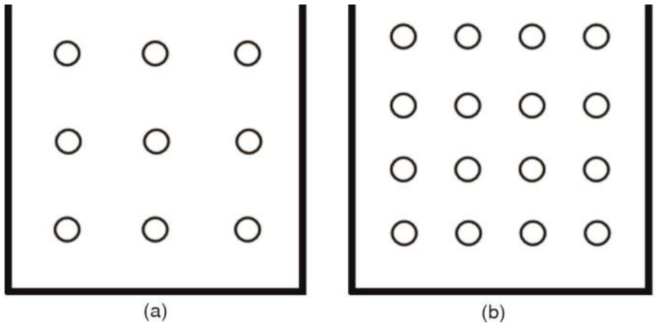

Figure 1. A heuristic explanation of $\mathrm{d}x^{\alpha}$, the average distance between two adjacent molecules, the cycles represent air molecules; (a) high temperature with an average distance $\mathrm{d}x^{\alpha_1}$ and (b) low temperature with an average distance $\mathrm{d}x^{\alpha_2}$

Now we consider a heat flux, Q, which is transferred from a hot body, fig. 1(a), to a cold one, fig. 1(b). The fractal entropy of the hot body decreases:

$$\mathrm{d}S_1 = \frac{\mathrm{d}Q_1}{T_1}\frac{\mathrm{d}x^{\alpha_1}}{\mathrm{d}x^{\alpha_2}} \tag{4}$$

While the entropy of a cold body increases:

$$\mathrm{d}S_2 = \frac{\mathrm{d}Q_2}{T_2}\frac{\mathrm{d}x^{\alpha_2}}{\mathrm{d}x^{\alpha_1}} \tag{5}$$

The fractal entropy equilibrium requires:

$$\mathrm{d}S_2 - \mathrm{d}S_1 = \frac{\mathrm{d}Q_2}{T_2}\frac{\mathrm{d}x^{\alpha_2}}{\mathrm{d}x^{\alpha_1}} - \frac{\mathrm{d}Q_1}{T_1}\frac{\mathrm{d}x^{\alpha_1}}{\mathrm{d}x^{\alpha_2}} = 0 \tag{6}$$

As shown in fig. 1, the average distance in a high body is larger than that in a cold body, that is:

$$\mathrm{d}x^{\alpha_1} > \mathrm{d}x^{\alpha_2} \tag{7}$$

From eq. (6), we have:

$$\frac{\dfrac{\mathrm{d}Q_2}{T_2}}{\dfrac{\mathrm{d}Q_1}{T_1}} = \left(\frac{\mathrm{d}x^{\alpha_1}}{\mathrm{d}x^{\alpha_2}}\right)^2 > 1 \tag{8}$$

Equation (8) implies:

$$\frac{\mathrm{d}Q_2}{T_2} - \frac{\mathrm{d}Q_1}{T_1} > 0 \tag{9}$$

This is the traditional principle of entropy increase. Please note the concept of our fractal entropy is different from the fractional entropy [21]. In case, $\mathrm{d}x^{\alpha_1} < \mathrm{d}x^{\alpha_2}$ we predict a negative entropy increase, such case arises in the black hole thermodynamics [22] and quantum mechanics [23].

Brachistofractality

The terminology *brachistofractality* comes from Greek root brachisto- and Latin fractits [2] as that of *brachistochrone problem*, it is also called the law of minimal fractal dimensions change. The *brachistofractality* implies a minimal change of the fractal dimensions under an external perturbation, which includes direct perturbation (*e.g.* a force) and indirect perturbation (*e.g.* temperature). Mathematically it can be written in the form:

$$\alpha - \alpha_0 \to \min \tag{10}$$

where α and α_0 the fractal dimensions without and with an external perturbation. To understand *brachistofractality* clearly, we consider a bubble's formation. Under an external air injection, a water surface can form a bubble by minimizing its surface, and a spherical shape is obtained, because the spherical surface has the minimal fractal dimensions among all curved surfaces.

The fractal harmonic law [2] implies the friction between two moving objects reaches minimum when the surfaces' fractal dimensions are same. As an example, we consider the fabric surface of the swimming vest, the two-scale fractal dimensions, α_{fabric}, can be calculated as:

$$\alpha_{\text{fabric}} = \frac{2(ij + hj - hi)}{jk} \tag{11}$$

where *i*, *j*, *h*, and *k* are the length of the micro-cell as illustrated in fig. 2, the area of the micro-cell is *jk*. To design a swimming vest with minimal friction, according to the fractal harmonic law, it requires:

$$\alpha_{\text{fabric}} - \alpha_{\text{water}} \to \min \tag{12}$$

where α_{water} is the fractal dimensions of water surface acting on the fabric's surface on a molecule scale.

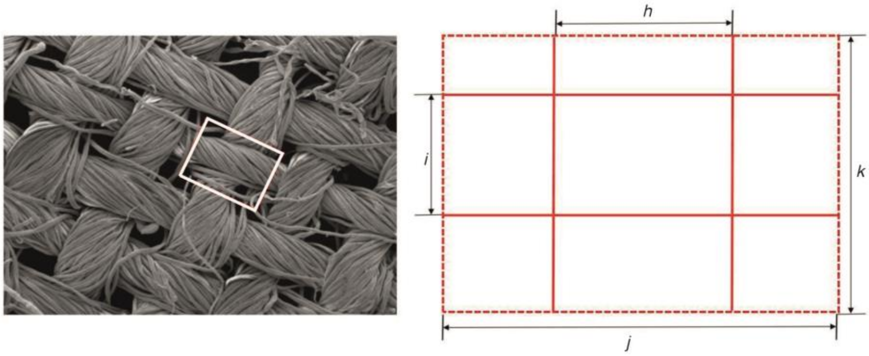

Figure 2. Fabric structure

The fractal harmonic law can explain the lotus effect [24], the fractal dimensions of the lotus surface are almost equal to that of the water. *Brachistofractality* and the fractal harmonic law were used to study hemicyanine fluorescent dye [25] and bone repair [26], and can be extended to shell deformation [2, 27, 28].

Conclusion remarks

This short note elucidates the originality and promising of the fractal thermodynamics, which offers a totally new angle for studying thermodynamical problems, and this issue provides with good reference for future research.

Acknowledgment

The paper is supported by Zhejiang Provincial Natural Science Foundation of China under Grant No. LQ21E030016, China Postdoctoral Science Foundation under Grant No. 2021M692866, Jiangsu Higher Vocational College Teachers' Professional Leaders' High and Training Team Visit Project(Grant number: 2022TDFX008), and Brand Major Construction Project of International Talent Training in Colleges and Universities-Modern Textile Technology Major(Grant number: Jiangsu Foreign Cooperation Exchange Education 2022. No 8).

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Paper submitted: December 1, 2022
Paper revised: March 15, 2023
Paper accepted: March 15, 2023