A NON-LINEAR CREEP MODEL CONSIDERING DISTURBANCE DAMAGE AT DIFFERENT DEPTHS

by

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The rock creep model is an important part of the study of the time-dependent mechanical behavior of rock. To ensure the safe excavation and long-term stable operation of deep engineering, a creep damage factor considering accelerating creep initiation time and creep failure time was proposed. By introducing the initial damage of excavation disturbances at different depths, a creep damage constitutive model for rocks at different depths considering disturbances was derived. This creep model has fewer parameters that are easy to obtain. It can better describe the accelerating creep characteristics of rock at different depths, especially for a more accurate description of the long-term creep mechanical behavior of deep rock.

Key words: *creep model, excavation disturbance, damage mechanics, accelerating creep*

Introduction

Major underground projects such as deep resource extraction, urban underground space utilization, and deep buried transportation tunnel construction not only need to ensure safe excavation, but also ensure long-term stability of surrounding rock during operation. Therefore, time parameters have become one of the important factors that must be considered [1].

Creep refers to the phenomenon where the strain of a material increases continuously over time under constant stress conditions. Creep is generally divided into three-stages: primary creep, steady-state creep, and accelerating creep. Establishing a suitable creep model by revealing the theoretical relationship between rock stress-strain-time is an important means of studying the mechanical properties of rock creep. Traditional empirical models are difficult to reflect the internal mechanisms and characteristics of rockcreep, and element combination models cannot describe the non-linear characteristics of rock creep. With the development of rock creep mechanics, some researchers have established creep models that can describe the

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non-linear accelerating creep stage of rock. At present, there are three main methods for establishing non-linear creep models:
– Replacing linear elements in element combination models with non-linear elements or adding new non-linear elements [2].
– Setting the parameters in element combination models to non-stationary values [3].
– Introducing new theories (fracture damage mechanics [4], endochronic theory [5]) to establish non-linear creep model.

However, there is a significant correlation between the time-dependent mechanical properties of deep rock masses and excavation disturbances. Regardless of the excavation method, the surrounding rock of deep engineering is affected by high stress concentration and stress redistribution, resulting in excavation damage zones [6]. At the same time, there are significant differences in the physical properties and mechanical behavior of rock at different depths [7]. Therefore, this paper defines the rock disturbance damage coefficient, proposes a new non-linear creep damage constitutive model considering excavation disturbance at different depths, and conducts graded loading creep tests of disturbed marble at different depths, which verifies the accuracy and rationality of theproposed creep model.

Disturbance damage characteristics of marble at different depths

During the disturbance process of local axial stress concentration or radial stress unloading, rocks will generate a certain number of initial cracks internally. During the subsequent loading process, secondary cracks will initiate and connect, forming internal damage to the rock mass [8]. However, as the burial depth increases, the amount of radial unloading of the surrounding rock will also significantly increase, leading to a further increase in the scope and degree of damage inside the rock mass [6]. To simulate the excavation unloading and damage evolution process of actual rock, the disturbance stress environment evolution equation of surrounding rock of deep cavern proposed by Zha *et al.* [9] was used to carry out stress loading and unloading pre-experiments of marble at different depths (400 m, 1200 m, 1800 m, 2400 m). Further details of the method can be found in Zha *et al.*'s [9] original paper.

The damage definition proposed by Lemaitre [10] is that the damage value inside the material, which can be measured by using changes in Young's modulus, *i.e.*:

$$D = 1 - \frac{\tilde{E}}{E} \tag{1}$$

where D is the damage variable, $\tilde{E}$ – the Young's modulus of the damaged material, and E – the Young's modulus of the material in the non-destructive state. It is difficult to obtain the unloading modulus in excavation disturbance simulation experiments, but the secant modulus is easier to obtain. Therefore, taking the instantaneous secant modulu E_s as the equivalent deformation modulus in the damaged state, we have:

$$E_s = \frac{(\sigma_1 + 2\sigma_3)(\sigma_1 - \sigma_3)}{\sigma_3(\varepsilon_1 - 2\varepsilon_3) + \sigma_1\varepsilon_1} \tag{2}$$

During the stress evolution of rock considering excavation disturbance [9], the rock at stress Point ①, see tab. 1, or stress Point ②, see tab. 1, is still in the elastic deformation stage, and the initial pores inside the rock are further compacted. It can be considered that the rock at stress Point ②, see tab. 1, is not damaged. Then, after the continuous strengthening of rock stress concentration and stress unloading to stress Point ④, see tab. 1, the surrounding rock forms an excavation damaged zone. To quantitatively describe the final damage of surrounding rock during the entire excavation simulation process, we defined the rock disturbance damage coefficient D_{dist} by:

$$D_{\text{dist}} = 1 - \frac{E_{s4}}{E_{s2}} \tag{3}$$

where E_{s2} is the secant modulus at stress point ②, see tab. 1, and E_{s4} – the secant modulus at stress point ④, see tab. 1. Table 1 shows the secant modulus and disturbance damage coefficient of each stress point in the simulated disturbance test. The results show that the damage coefficient of engineering disturbance to the rock mass at a depth of 400 m is 23.35%, and the impact on the stability of the surrounding rock cannot be ignored. The disturbance damage intensifies with the increase of the occurrence depth. The disturbance damage at a depth of 2400 m is about 1.5 times as much as that at a depth of 400 m, which may lead to more serious dynamic disasters.

Table 1. The disturbance damage coefficient of marble at different depths

Depth [m]	Secant modulus [GPa]			D_{dist}
	Point ②	Point ③	Point ④	
400	36.4	29.4	27.9	23.35%
1200	88.3	66.9	63.6	27.97%
1800	91.2	64.8	60.0	34.21%
2400	87.9	59.8	55.8	36.52%

Non-linear creep damage constitutive model

Firstly, the critical time point for the transition from steady-state creep to accelerating creep is defined as the accelerating creep initiation time (ACIT) t_A. Under a certain stress state, when the duration of rock creep does not reach the ACIT, the creep damage is 0. When the creep time reaches the ACIT, damage evolution occurs inside the rock, leading to a non-linear increase in creep deformation and creep rate, until the rock finally undergoes creep failure.At the same time, the creep failure time (CFT) t_F, as an important mechanical parameter of rocks, is of great significance for accurately predicting the failure time of rock masses. However, there are few accelerated creep models established based on the CFT of rocks. By introducing the ACIT and CFT, we define a non-linear creep damage factor D_c:

$$D_c = \begin{cases} 0\,, & t < t_A \\ 1 - \dfrac{1}{1 + \dfrac{2B(t - t_A)t_F}{(t_F - t)^3}}\,, & t > t_A \end{cases} \tag{4}$$

When $t \to t_F$, $D_c \to 1$, which corresponds to complete rock failure. Further combining the disturbance damage coefficient, D_{dist}, and depth effect [9], a creep damage evolution equation considering excavation disturbance at different depths is given:

$$\dot{\varepsilon}_{ac} = \left[\frac{\left(\dfrac{\sigma_1}{(1 - D_{\text{dist}})(1 - D_c)} - \left(\dfrac{9}{32}\lambda - \dfrac{1}{32} \right)\gamma H \right)}{F} \right]^m \tag{5}$$

where $\dot{\varepsilon}_{ac}$ is the axial creep rate during the accelerating creep, σ_1 – the maximum principal stress, F and m are material parameters, γ – the unit weight of rock strata, and λ – the horizontal and vertical ground stress ratio. Although the primary creep stage has a shorter duration compared to the steady-state creep stage, the creep rate is very high, so this part should be considered when describing creep behavior. The characterization of primary creep and steady-state creep of rocks at different depths can be described using the following empirical model [11], *i.e.*:

$$\varepsilon_t = A\left(1-\mathrm{e}^{-at}\right)+\dot{\varepsilon}_{sc}t \tag{6}$$

where ε_t is the sum of primary creep and steady-state creep, $\dot{\varepsilon}_{sc}$ – the steady-state creep rate, and A and a are the material parameters. Due to the total axial creep deformation $\varepsilon_1 = \varepsilon_t + \varepsilon_{ac}$, the creep constitutive model at different depths considering excavation disturbances is expressed:

$$\begin{gathered}\varepsilon_1 = A\left(1-e^{-at}\right)+\dot{\varepsilon}_{sc}t,\ \ t<t_A \\ \varepsilon_1 = \int_{t_A}^{t_F}\left[\frac{\left(\dfrac{\sigma_1}{\left(1-D_{\text{dist}}\right)\left(1-D_c\right)}-\left(\dfrac{9}{32}\lambda-\dfrac{1}{32}\right)\gamma H\right)}{F}\right]^m \mathrm{d}t + A\left(1-\mathrm{e}^{-at_A}\right)+\dot{\varepsilon}_{sc}t_A,\ \ t>t_A\end{gathered} \tag{7}$$

Model verification

After excavation disturbancepre-experiments, the multi-stage creep loading tests at different depths of marble were further conducted. The 70% of the estimated strength value of marble is used as the first level stress level, followed by an increase of 5% for each level. The loading time for each level is 120 hours until the sample is damaged. The parameters of the proposed rock creep accelerating stage model include five parameters: t_F, t_A, D_{dist}, F, and m. Among them, t_F and t_A, can be directly obtained from the rock creep curve; D_{dist} can be obtained from the

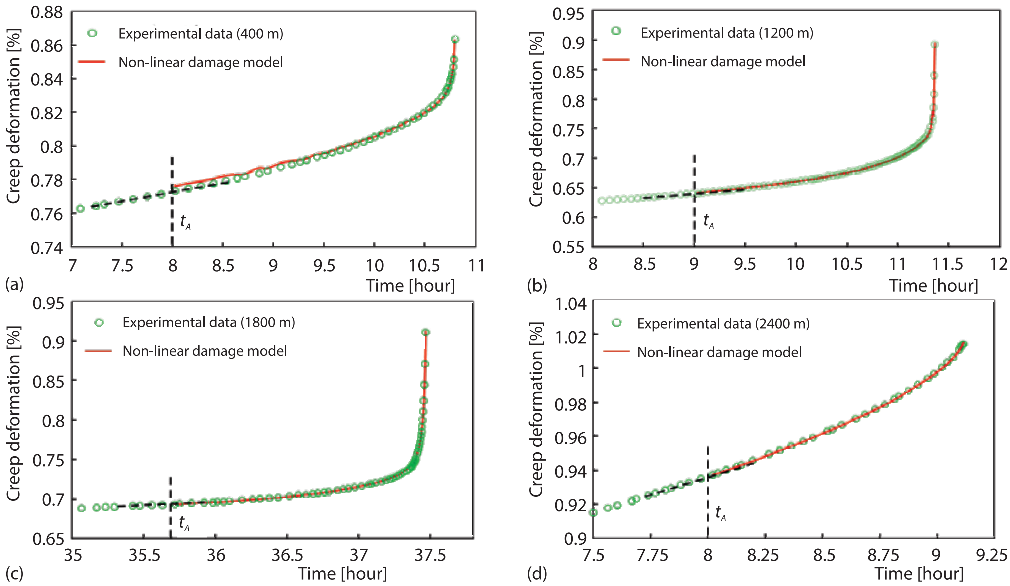

Figure 1. Comparison of experimental data and creep model for accelerating creep stage of marble at different depths; (a) 400 m, (b) 1200 m, (c) 1800 m, and (d) 2400 m

stress-strain curve under different disturbance stress paths, and the remaining parameters can be solved using the Levenberg Marquardt algorithm.The creep deformation results of the final stress level of marble at different depths are used for model validation. As shown in fig. 1, the model fitting curves of the marble atdifferent depthare in good agreement with the test data.The creep model parameters are shown in tab. 2.

Table 2. Parameters of non-linear creep damage constitutive model

Depth [m]	t_A [hour]	t_F [hour]	B [hour]	F [MPa]	m
400	8	10.9	$4.0287 \cdot 10^{-7}$	292.91	149.30
1200	9	11.4	$3.6454 \cdot 10^{-5}$	2655.4	1.6923
1800	35.7	37.48	0.01168	4172221.7	0.49769
2400	8	9.116	0.54865	$4.6316 \cdot 10^{17}$	0.09037

The classical Burgers model is an element model composed of elastic body, viscoelasticity body and viscous body, and its expression is:

$$\varepsilon_t = \sigma_0 \left[\frac{1}{E_0} + \frac{t}{\eta_1} + \frac{1}{E_2} \left(1 - \mathrm{e}^{-\frac{E_2}{\eta_2} t} \right) \right] \tag{8}$$

where σ_0 is the eccentric stress load, E_0, η_1, E_2, and η_2 are the Burgers model parameters. Taking the 2400 m deep marble as an example, comparative verification was conducted based on the creep loading data of the last two stress levels before failure (105% stress level and 110% stress level), as shown in fig. 2. The creep parameters are shown in tab. 3. It can be seen that the Burgers model can also relatively well characterize the deformation behavior of primary creep and steady-state creep in fig. 2(a). However, the fitting of the creep process for the final stress level is poor in fig. 2(b), and the fitted parameter values do not match their physical significance, which are very unreasonable. This is because the rock sample at a depth of 2400 m experienced very severe initial disturbance damage. The rock with more severe initial disturbance damage will exhibit larger creep deformation and faster creep deformation rate at the final stress level, and have extremely strong non-linear damage deformation accumulation. Therefore, the creep curve shows a slight concave shape, resulting in poor fitting performance of the Burgers model. The model proposed in this paper can accurately describe the deformation throughout the entire creep process, especially for the strong non-linear mechanical behavior during the accelerating creep stage. This model has fewer parameters whichare easy to obtain from test data. Therefore, the creep damage model at different depths considering excavation disturbances has good applicability.

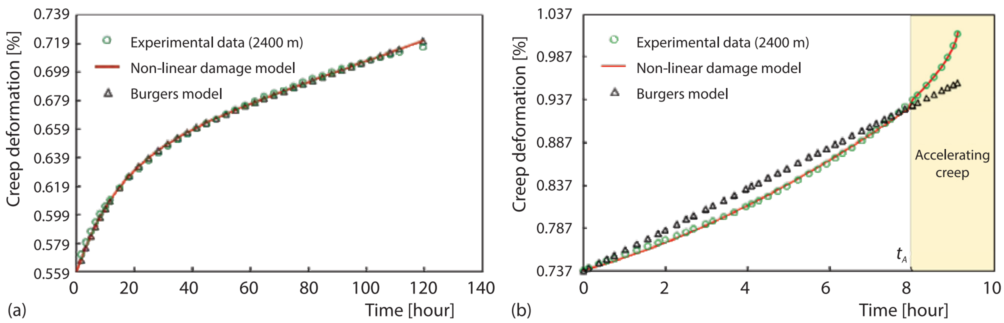

Figure 2. Comparison between non-linear damage model and Burgers model; (a) 105% stress level and (b) 110% stress level

Table 3. Parameters of non-linear damage model and Burgers model

Stress level	Non-linear damage model			Burgers model			
	$\dot{\varepsilon}_{sc}$ [hour^{-1}]	A	a [hour^{-1}]	E_0 [GPa]	E_1 [GPa]	η_1 [GPah]	η_2 [GPah]
105%	$7.2487 \cdot 10^{-6}$	$7.5419 \cdot 10^{-4}$	0.06919	47.36	349.94	36379	5002.9
110%	0.0014	–0.05822	0.0217	37.6	$1.1968 \cdot 10^{21}$	1146.2	0.654

Conclusion

In the present work, the disturbance damage coefficient of rocks has been proposed. The experimental results show that the initial disturbance damage intensifies with the increase of occurrence depth. It was found that the proposed model has a good description of the non-linear deformation characteristics of the accelerating creep stage of rock masses disturbed at different depths.

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Nomenclature

D_c – non-linear creep damage factor, [–]
D_{dist} – disturbance damage coefficient, [–]
E_s – instantaneous secant modulus, [GPa]
t_F – creep failure time, [hour]

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