NANO- AND MICRO-POLAR MAGNETOHYDRODYNAMIC FLUID-FLOW AND HEAT TRANSFER IN INCLINED CHANNEL

by

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Magnetohydrodynamic fluid-flows attract a lot of attention in the extrusion of polymers, in the theory of nanofluids, as well as in the consideration of biological fluids. The considered problem in the paper is the flow and heat transfer of nano- and micro-polar fluid in inclined channel. Fluid-flow is steady, while nano- and micro-polar fluids are incompressible, immiscible, and electrically conductive. The upper and lower channel plates are electrically insulated and maintained at constant and different temperatures. External applied magnetic field is perpendicular to the fluid-flow and considered problem is in induction-less approximation. The equations of the considered problem are reduced to ODE, which are analytically solved in closed form. The influence of characteristics parameters of nano- and micro-polar fluids on velocity, micro-rotation and temperature fields are graphically shown and discussed. The general conclusions given through the analysis of graphs can be used for better understanding of the flow and heat transfer of nano- and micro-polar fluid, which have a great practical application. Fluids with nanoparticles innovated the modern era, due to their comprehensive applications in nanotechnology and manufacturing processes, while the theory of micro-polar fluids explains the flow of biological fluids and various types of liquid metals and crystals.

Key words: *magnetohydrodynamic flow, heat transfer, micro-polar fluid, nanofluid*

Introduction

Magnetohydrodynamic (MHD) flow of electroconductive fluids has been in the center of scientific research starting from the middle of last century to the present day. The main reason of this trend is due to increasing use of these flows for scientific purposes, as well as in technological development. Influence of external magnetic field on flow of electroconductive fluid occurs in nuclear reactors, then in magnetic flow meters, but also in MHD generators and pumps. Nano- and micro-polar fluids are the most represented fluids in scientific research in recent decades, and investigation of their flow in the presence of external magnetic field attracted a lot of attention.

Eringen [1] developed the theory of micro-polar fluids in order to explain the flow and heat transfer of fluids containing microstructures. The distribution of fluids with microstructure in nature is large, starting with polymers, through biological fluids and up to

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various types of liquid metals and crystals. All these fluids are characterized by the presence of suspended particles which, in addition to translational movement, have the possibility of rotation around their axis. What is characteristic of these types of fluids is that unlike Newtonian fluids, micro-polar fluids do not have a symmetrical stress tensor. The non-symmetric stress tensor is a consequence of the presence of micro-rotation of suspended particles, which is defined in the equations of the considered problem by one additional equation for the micro-rotation vector. An extensive review of the flow of micro-polar fluids is given by Chamkha *et al.* [2] and Bachok *et al.* [3]. The application of micro-polar fluid-flow in today's modern technology is quite widespread. These applications include liquid crystals [4], blood flow in lungs or in arteries [5], flow and thermal control of polymeric processing [6], *etc.* There are also papers dealing with the study of the flow of micro-polar fluids over a moving body, such as work of Uddin *et al.* [7].

On the other hand fluids with nanoparticles innovated the modern era, due to their comprehensive applications in nanotechnology and manufacturing processes, such as automotive cooling, higher thermal transportation in microchips, food processing, and nuclear reactors. The term nanofluid refers to fluids in which nanoscale particles are suspended in the base fluid [8]. This type of fluids have been used in various fields of developed nanotechnology and engineering. Chemically, the nanoparticles of various forms include oxides, nitrides, carbides, carbon nanotubes, and metals.

The research of MHD nanofluid-flow and heat transfer is very represented in present day, with numerous researchers tackling the issue and with comprehensive body of results being available. Heat transfer in nanofluids was investigated by Wang and Mujumdar [9], providing guidance for further research. Also, research by some authors, such as Gorla and Chamka [10], is directed towards research in the boundary-layer in the flow of nanofluids in a porous medium around a horizontal plate. Mixed convection heat transfer flow of power law nanofluid over starching plate is investigated by Ellahi *et al.* [11]. The purpose of that paper was to study the effects of nanoparticles on mixed convection flow of power law fluid. In the paper of Khalili *et al.* [12], a numerical analysis of the non-stationary MHD flow of nanofluid over a stretchable surface near the stagnation point is given. The importance of applying and considering the flow of nanofluids in porous media is presented in the work of Petrović *et al.* [13]. In the paper by Abdul Latiff *et al.* [14], the unsteady forced bioconvection boundary-layer flow of a viscous incompressible micro-polar nanofluid containing microorganisms over a stretching/shrinking sheet is studied numerically. The model of blood flow of MHD non-Newtonian nanofluid with heat transfer and slip effects was studied by Elelamy *et al.* [15]. This paper aims to investigate a mathematical model with numerical simulation for bacterial growth in the heart valve.

Due to importance of applications of micro-polar and nanofluids, very soon occurred need for analyze the flow of two micro-polar fluids, as well as analyze the flow of two nanofluids or micro-polar and nanofluids. Jangili and Murthy [16] considered a thermodynamic analysis of the flow of two micro-polar fluids between the parallel plates. Jangili *et al.* [17] considered the flow of two immiscible micro-polar fluids in the inclined channel. Umavathi *et al.* [18] analyzed the flow of two micro-polar fluids separated by a viscous fluid layer, with a goal to set a better model for describing human blood behavior. The MHD fluid-flow and heat transfer of immiscible viscous and micro-polar fluid between inclined plates was investigated by Kocić *et al.* [19]. Elmaboud [20] examined the flow and heat transfer of two immiscible fluids in a vertical semi-corrugated channel under the influence of a homogeneous horizontal magnetic field. One region of the channel was filled with a nanofluid and the other with a clear non-conductive viscous fluid.

Research on the flow of nanofluids and micro-polar fluids that are still present in recent times, due to their applied applications primarily in nanotechnology but also in medicine, leaves an unequivocal need for the study of these types of flows. In the available literature there is not enough research of nano- and micro-polar fluids flow and heat transfer, with influence of externally applied magnetic field. Having in mind the previous statement and the practical application of MHD flow of nano- and micro-polar fluid-flow, in this paper the steady MHD flow and heat transfer of incompressible, immiscible, and electrically conducting nano- and micro-polar fluids, in inclined channel, is investigated.

Mathematic and physic model

The steady MHD flow of two immiscible fluids in inclined channel is considered. While considering this problem some assumptions are taken into account: first both nano- and micro-polar fluid, have constant and different electrical conductivity, second it is a 1-D problem and third the problem is considered in inductionless approximation.

The physical model of the problem is shown in fig. 1. There is a steady flow of nano- and micro-polar fluid in channel, where upper and lower plates are extending in the *x*- and *z*-direction and are inclined at an angle α towards the horizontal plane. Plates have been kept at constant and different temperatures, T_{w1} for upper plate and T_{w2} for lower plate, while $T_{w1} > T_{w2}$. Two mentioned fluids flow between the plates as consequence of constant pressure gradient in *x*-direction. Uniformly external magnetic field of strength *B* is perpendicular towards the fluid-flow (*y*-direction) and continuously acts along the entire *x*-axis. The flow space in the channel, between the upper and lower plates, is 2*h*, with both fluids occupying the same height of the channel *h*. The lower part of the channel is filled with micro-polar fluid, while the upper part of the channel contains nanofluid.

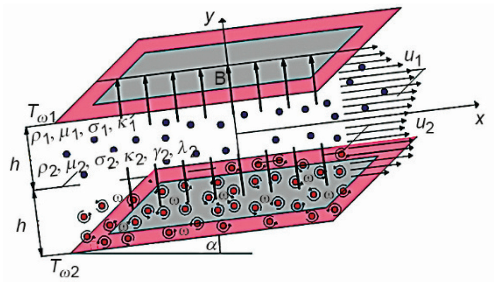

Figure 1. Physical model and co-ordinate system

The fluid velocity *v* and magnetic field **B** are:

$$v_i = u_i \vec{\mathrm{i}}, \quad i = 1, 2 \tag{1}$$

$$\mathbf{B} = B\vec{\mathrm{j}} \tag{2}$$

The general equations of considered problem are given in next form [12, 18, 21]:

– Nanofluid (upper one):

$$\mu_1 \frac{\mathrm{d}^2 u_1}{\mathrm{d}y^2} + \rho_1 \mathrm{g} \beta_1 (T_1 - T_{\mathrm{w}2}) \sin\alpha - \sigma_1 B^2 u_1 - \frac{\mathrm{d}p}{\mathrm{d}x} = 0 \tag{3}$$

$$k_1 \frac{\mathrm{d}^2 T_1}{\mathrm{d}y^2} + \mu_1 \left(\frac{\mathrm{d}u_1}{\mathrm{d}y} \right)^2 + \sigma_1 B^2 u_1^2 = 0 \tag{4}$$

– Micro-polar fluid (lower one):

$$(\mu_2+\lambda)\frac{\mathrm{d}^2u_2}{\mathrm{d}y^2}+\lambda\frac{\mathrm{d}\omega}{\mathrm{d}y}+\rho_2 g\beta_2(T_2-T_{w2})\sin\alpha-\sigma_2B^2u_2-\frac{\mathrm{d}p}{\mathrm{d}x}=0 \tag{5}$$

$$\gamma\frac{\mathrm{d}^2\omega}{\mathrm{d}y^2}-\lambda\frac{\mathrm{d}u_2}{\mathrm{d}y}-2\lambda\omega=0 \tag{6}$$

$$k_2\frac{\mathrm{d}^2T_2}{\mathrm{d}y^2}+(\mu_2+\lambda)\left(\frac{\mathrm{d}u_2}{\mathrm{d}y}\right)^2+\sigma_2B^2u_2^2=0 \tag{7}$$

and the boundary conditions are:

$$\begin{aligned}&u_1=0,\quad T_1=T_{w1}\quad \text{for}\quad y=h\\&u_2=0,\quad T_2=T_{w2},\quad \omega=0\quad \text{for}\quad y=-h\\&\left.\begin{aligned}&u_1=u_2,\quad T_1=T_2,\quad \omega=0\\&\mu_1\frac{\mathrm{d}u_1}{\mathrm{d}y}=(\mu_2+\lambda)\frac{\mathrm{d}u_2}{\mathrm{d}y}+\lambda\omega\\&k_1\frac{\mathrm{d}T_1}{\mathrm{d}y}=k_2\frac{\mathrm{d}T_2}{\mathrm{d}y}\end{aligned}\right\}\quad \text{for}\quad y=0\end{aligned} \tag{8}$$

In eqs. (3)-(8), the following notations were used: u_i, T_i – fluid velocity and fluid temperatures (*i* indicates upper *i* = 1 and lower fluid *i* = 2),

– For nanofluid (upper one): $\rho_1, \beta_1, \mu_1, \sigma_1, k_1$ are density, volumetric temperature expansion coefficient, dynamic viscosity, electrical conductivity and thermal conductivity of nanofluid and they are defined by appropriate expressions [22]:

$$\begin{gathered}\rho_1=\rho_f(1-\phi)+\phi\rho_s,\quad (\rho\beta)_1=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s\\\mu_1=\frac{\mu_f}{\varphi_1},\quad k_1=\varphi_2k_f,\quad \sigma_1=\varphi_3\sigma_f\end{gathered} \tag{9}$$

where

$$\begin{gathered}\varphi_1=(1-\phi)^{2.5},\\\varphi_2=[2k_f-2\phi(k_f-k_s)+k_s][2k_f+\phi(k_f-k_s)+k_s]^{-1},\\\varphi_3=1+3\phi\left(\frac{\sigma_s}{\sigma_f}-1\right)\left[\frac{\sigma_s}{\sigma_f}+2-\phi\left(\frac{\sigma_s}{\sigma_f}-1\right)\right]^{-1}\end{gathered} \tag{10}$$

where ϕ is the volume fraction of nanoparticles, μ, σ, k are dynamic viscosity, electrical conductivity, and thermal conductivity, respectively, while subscripts *f* refers to fluid and *s* refers to the nanoparticles.

– For micro-polar fluid (lower one): ω is vector of microrotation, $\rho_2, \beta_2, \mu_2, \sigma_2, k_2$ are density, volumetric temperature expansion coefficient, dynamic viscosity, electrical conductivity, and thermal conductivity of micro-polar fluid, respectively, λ – the vortex viscosity, and γ – the gyro viscosity.

After some mathematical transformations, described laminar MHD flow and heat transfer is mathematically presented with following equations in non-dimensional form:

– upper, nanofluid:

$$\frac{\mathrm{d}^2 u_1^*}{\mathrm{d}y^{*2}} + \frac{\mathrm{Gr}_1}{\mathrm{Re}_1}\sin\alpha - \mathrm{Ha}_1^2 u_1^* + \mathrm{Re}_1 P = 0 \tag{11}$$

$$\frac{\mathrm{d}^2\theta_1}{\mathrm{d}y^{*2}} + \mathrm{Pr}_1\,\mathrm{Ec}_1\left(\frac{\mathrm{d}u_1^*}{\mathrm{d}y^*}\right)^2 + \mathrm{Ha}_1^2\,\mathrm{Pr}_1\,\mathrm{Ec}_1 u_1^{*2} = 0 \tag{12}$$

– lower, micro-polar fluid:

$$(1+K)\frac{\mathrm{d}^2 u_2^*}{\mathrm{d}y^{*2}} + K\frac{\mathrm{d}\omega^*}{\mathrm{d}y^*} + \frac{\mathrm{Gr}_2}{\mathrm{Re}_2}\sin\alpha - \mathrm{Ha}_2^2 u_2^* + \mathrm{Re}_2\,\rho^* P = 0 \tag{13}$$

$$\Gamma\frac{\mathrm{d}^2\omega^*}{\mathrm{d}y^{*2}} - K\frac{\mathrm{d}u_2^*}{\mathrm{d}y^*} - 2K\omega^* = 0 \tag{14}$$

$$\frac{\mathrm{d}^2\theta_2}{\mathrm{d}y^{*2}} + (1+K)\mathrm{Pr}_2\,\mathrm{Ec}_2\left(\frac{\mathrm{d}u_2^*}{\mathrm{d}y^*}\right)^2 + \mathrm{Ha}_2^2\mathrm{Ec}_2\,\mathrm{Pr}_2\,u_2^{*2} = 0 \tag{15}$$

In previous general equations used symbols are common for the theory of MHD flows [16] and the following transformations have been used to transform previous equations to non-dimensional form:

$$\begin{gathered}
u_i^* = \frac{u_i}{U},\quad \omega^* = \frac{\omega}{U}h,\quad y^* = \frac{y}{h},\quad \rho^* = \frac{\rho_1}{\rho_2},\quad U = \frac{\mu_1}{\rho_1 P h},\quad P = -\frac{\mathrm{d}p}{\mathrm{d}x} = \text{constant}\\
\theta_1 = \frac{T_1 - T_{\mathrm{w}2}}{T_{\mathrm{w}1} - T_{\mathrm{w}2}},\quad \theta_2 = \frac{T_2 - T_{\mathrm{w}2}}{T_{\mathrm{w}1} - T_{\mathrm{w}2}},\quad \mathrm{Ha}_1 = Bh\sqrt{\frac{\sigma_1}{\mu_1}},\quad \mathrm{Ha}_2 = Bh\sqrt{\frac{\sigma_2}{\mu_2}},\quad K = \frac{\lambda}{\mu_2},\quad \Gamma = \frac{\gamma}{\mu_2 h^2}\\
\mathrm{Pr}_1 = \frac{\mu_1 c_{p,1}}{k_1},\quad \mathrm{Pr}_2 = \frac{\mu_2 c_{p,2}}{k_2},\quad \mathrm{Ec}_1 = \frac{U^2}{c_{p,1}(T_{\mathrm{w}1} - T_{\mathrm{w}2})},\quad \mathrm{Ec}_2 = \frac{U^2}{c_{p,2}(T_{\mathrm{w}1} - T_{\mathrm{w}2})}\\
\mathrm{Gr}_1 = \frac{\mathrm{g}\beta_1 h^3 (T_1 - T_{\mathrm{w}2})}{\nu_1^2},\quad \mathrm{Gr}_2 = \frac{\mathrm{g}\beta_2 h^3 (T_2 - T_{\mathrm{w}2})}{\nu_2^2},\quad \mathrm{Re}_1 = \frac{U_0 h}{\nu_1},\quad \mathrm{Re}_2 = \frac{U_0 h}{\nu_2}
\end{gathered} \tag{16}$$

We now introduce the appropriate boundary conditions for the velocity, micro-rotation and temperature of the plates and the dividing surface, as well as the boundary conditions for the shear stress and the heat flux on the boundary surface between the viscous and the micro-polar fluid. The boundary conditions in non-dimensional form are:

$$\begin{gathered}
u_1^* = 0,\quad \theta_1 = 1 \quad \text{for} \quad y^* = 1\\
u_2^* = 0,\quad \theta_2 = 0,\quad \omega^* = 0 \quad \text{for} \quad y^* = -1
\end{gathered} \tag{17}$$

and

$$\left.\begin{aligned}
&u_1^* = u_2^*,\quad \theta_1 = \theta_2,\quad \omega^* = 0\\
&\mu_1\frac{\mathrm{d}u_1^*}{\mathrm{d}y^*} = (\mu_2 + \lambda)\frac{\mathrm{d}u_2^*}{\mathrm{d}y^*} + \lambda\omega^*,\quad k_1\frac{\mathrm{d}\theta_1}{\mathrm{d}y^*} = k_2\frac{\mathrm{d}\theta_2}{\mathrm{d}y^*}
\end{aligned}\right\}\quad \text{for} \quad y^* = 0 \tag{18}$$

By solving eqs. (11) and (12), we obtain solutions for the dimensionless velocity and temperature for the nanofluid in the following form:

$$u_1^* = \mathrm{A}\exp(\lambda_1 y^*) + \mathrm{B}\exp(\lambda_1 y^*) + \frac{B}{A} \tag{19}$$

$$\begin{aligned}\theta_1 = \mathrm{M}_1 \exp(2\lambda_1 y^*) + \mathrm{M}_3 \exp(2\lambda_2 y^*) + \mathrm{M}_4 \exp(\lambda_1 y^*) + \\ + \mathrm{M}_5 \exp(\lambda_2 y^*) - \left(\frac{N+J}{2}\right) y^{*2} + C_1^* y^* + C_2^*\end{aligned} \tag{20}$$

While solving the system of eqs. (13) to (15), from eqs. (13) and (14), after basic mathematical transformations, the equation for velocity is:

$$\frac{\mathrm{d}^4 u_2^*}{\mathrm{d}y^{*4}} - a\frac{\mathrm{d}^2 u_2^*}{\mathrm{d}y^{*2}} + b u_2^* - d = 0 \tag{21}$$

where:

$$a = B^* + E^* - A^* D^*, \quad b = B^* E^*, \quad d = C^* E^*$$

$$A^* = \frac{K}{1+K}, \quad B^* = \frac{\mathrm{Ha}_2^2}{1+K}, \quad C^* = \frac{1}{1+K}\left(\frac{\mathrm{Gr}_2}{\mathrm{Re}_2}\sin\alpha + \mathrm{Re}_2 P\rho^*\right), \quad D^* = \frac{K}{\Gamma}, \quad E^* = \frac{2K}{\Gamma} \tag{22}$$

The solution of eq. (21), depending on the roots of characteristic equation, are giving three possible cases:

$$u_2^* = C_1 \exp(\delta_1 y^*) + C_2 \exp(\delta_2 y^*) + C_3 \exp(\delta_3 y^*) + C_4 \exp(\delta_4 y^*) + \frac{d}{b} \tag{23}$$

$$u_2^* = (C_5 + C_6 y^*)\exp(\xi_1 y^*) + (C_7 + C_8 y^*)\exp(\xi_2 y^*) + \frac{d}{b} \tag{24}$$

$$\begin{aligned}u_2^* = [C_9 \cos(\beta_1 y^*) + C_{10}\sin(\beta_1 y^*)]\exp(\alpha_1 y^*) + \\ + [C_{11}\cos(\beta_1 y^*) + C_{12}\sin(\beta_1 y^*)]\exp(-\alpha_1 y^*) + \frac{d}{b}\end{aligned} \tag{25}$$

While the solutions for temperature are, respectively:

$$\begin{aligned}\theta_2 = -\mathrm{Pr}_2\,\mathrm{Ec}_2 \Bigg\{ \frac{1}{4\delta_1^2}\,{}^1C\exp(2\delta_1 y^*) + \frac{1}{4\delta_2^2}\,{}^2C\exp(2\delta_2 y^*) + \frac{1}{4\delta_3^2}\,{}^3C\exp(2\delta_3 y^*) + \\ + \frac{1}{4\delta_4^2}\,{}^4C\exp(2\delta_4 y^*) + \frac{1}{2}\,{}^1Dy^{*2} + \frac{{}^2D}{(\delta_1+\delta_3)^2}\exp[(\delta_1+\delta_3)y^*] + \\ + \frac{{}^3D}{(\delta_1+\delta_4)^2}\exp[(\delta_1+\delta_4)y^*] + \frac{{}^1E}{(\delta_2+\delta_3)^2}\exp[(\delta_2+\delta_3)y^*] + \\ + \frac{{}^2E}{(\delta_2+\delta_4)^2}\exp[(\delta_2+\delta_4)y^*] + \frac{1}{2}\,{}^3Ey^{*2} + \frac{1}{\delta_1^2}\,{}^1F\exp(\delta_1 y^*) + \\ + \frac{1}{\delta_2^2}\,{}^2F\exp(\delta_2 y^*) + \frac{1}{\delta_3^2}\,{}^3F\exp(\delta_3 y^*) + \frac{1}{\delta_4^2}\,{}^4F\exp(\delta_4 y^*) + \frac{1}{2}\,{}^5Fy^{*2} + + {}^1H_1 y^* + {}^1H_2 \Bigg\}\end{aligned} \tag{26}$$

$$\theta_2 = -\Pr \text{Ec}[(\Omega_{28}+\Omega_{29}y^*+\Omega_{30}y^{*2})\exp(2\xi_1 y^*)+(\Omega_{31}+\Omega_{32}y^*+\Omega_{33}y^{*2})\exp(2\xi_2 y^*)+ \\ +(\Omega_{34}+\Omega_{35}y^*)\exp(\xi_1 y^*)+(\Omega_{36}+\Omega_{37}y^*)\exp(\xi_2 y^*)+ \\ +\Omega_{38}y^{*2}+\Omega_{39}y^{*3}+\Omega_{40}y^{*4}+{}^2H_1 y^*+{}^2H_2] \tag{27}$$

$$\theta_2 = -\Pr_2 \text{Ec}_2\left\{\left[\frac{\Omega_{45}}{2\alpha_1}+\frac{(\chi_1\Omega_{47}-\chi_2\Omega_{49})}{2}\cos(2\beta_1 y^*)+\frac{(\chi_2\Omega_{47}+\chi_1\Omega_{49})}{2}\sin(2\beta_1 y^*)\right]\cdot\right. \\ \cdot\exp(2\alpha_1 y^*)+\left[\frac{\Omega_{46}}{2\alpha_1}-\frac{(\chi_1\Omega_{48}+\chi_2\Omega_{50})}{2}\cos(2\beta_1 y^*)+\frac{(\chi_2\Omega_{48}-\chi_1\Omega_{50})}{2}\sin(2\beta_1 y^*)\right]\cdot \\ \cdot\exp(-2\alpha_1 y^*)-\frac{\Omega_{51}}{2\beta_1}\sin(2\beta_1 y^*)-\frac{\Omega_{52}}{2\beta_1}\cos(2\beta_1 y^*)+[(\Omega_{53}\chi_1-\Omega_{55}\chi_2)\cos(\beta_1 y^*)+ \\ +(\Omega_{53}\chi_2+\Omega_{55}\chi_1)\sin(\beta_1 y^*)]\exp(\alpha_1 y^*)+[(\Omega_{54}\chi_1-\Omega_{56}\chi_2)\cos(\beta_1 y^*)- \\ \left.-(\Omega_{54}\chi_2+\Omega_{56}\chi_1)\sin(\beta_1 y^*)]\exp(-\alpha_1 y^*)+\frac{1}{2}\Omega_{57}y^{*2}+{}^3H_1 y^*+{}^3H_2\right\} \tag{28}$$

and for microrotation:

$$\omega^* = C_1\mathfrak{D}_1\exp(\delta_1 y^*)+C_2\mathfrak{D}_2\exp(\delta_2 y^*)+C_3\mathfrak{D}_3\exp(\delta_3 y^*)+C_4\mathfrak{D}_4\exp(\delta_4 y^*) \tag{29}$$

$$\omega^* = (E_{13}+E_{14}y^*)\exp(\xi_1 y^*)+(E_{15}+E_{16}y^*)\exp(\xi_2 y^*) \tag{30}$$

$$\omega^* = [P_3^*\cos(\beta_1 y^*)+P_4^*\sin(\beta_1 y^*)]\exp(\alpha_1 y^*)+ \\ +[P_5^*\cos(\beta_1 y^*)+P_6^*\sin(\beta_1 y^*)]\exp(-\alpha_1 y^*) \tag{31}$$

where in eqs. (19)-(31):

$$A;B;N;J;M_i,\quad i=1,..5;C_1^*;C_2^*; \\ C_i,\quad i=1,...12;\quad D_i,\quad i=1,2,3,4;\quad E_i,\quad i=13,...16;\quad P_i,\quad i=3,...6, \\ {}^iC,\quad i=1,...4;\quad {}^iD,\quad i=1,2,3;\quad {}^iE,\quad i=1,2,3;\quad {}^iF,\quad i=1,...5; \\ \Omega_i,\quad i=28,...57;\quad {}^iH_j,\quad i=1,2,3,\quad j=1,2 \tag{32}$$

represent constants, while:

$$\lambda_1 \text{ and } \lambda_2;\quad \delta_i, i=1,2,3,4;\quad \xi_i, i=1,2;\quad \alpha_1 \text{ and } \beta_1 \tag{33}$$

are roots of characteristic eqs. (21).

Results and discussion

In this part of paper, obtained results from previous section are used to draw figures and discuss influence of characteristic parameters on velocity, microrotation, and temperature of nano- and micro-polar fluid-flow. All results presented in next figures are considered for inclination angle $\alpha = 0$, except figs. 8 and 9. First two figures represents influence of external magnetic field of strength, *B,* and volume fraction of nanoparticles, ϕ, on velocity profile of nano- and micro-polar fluid-flow.

From fig. 2 it can be noticed that the influence of the external magnetic field, has the same tendencies on both the nano- and the micro-polar fluid. In both cases, the increase of the strength of external magnetic field leads to a reduction of the velocity with the tendency to uniform the velocity profile over the entire height between the plates. This is the consequence of Lorentz force exposed opposite to fluid-flow direction, which appears when magnetic field influence electroconductive fluid-flow [23].

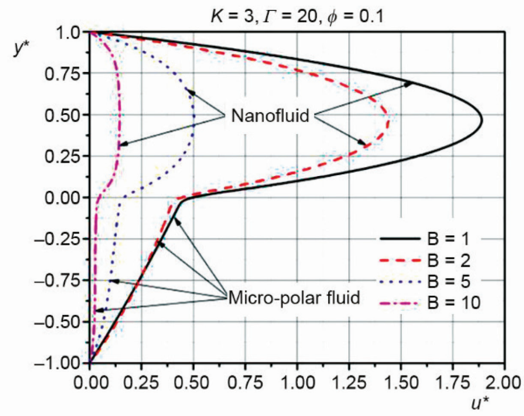

Figure 2. Velocity profiles for different values of external magnetic field B

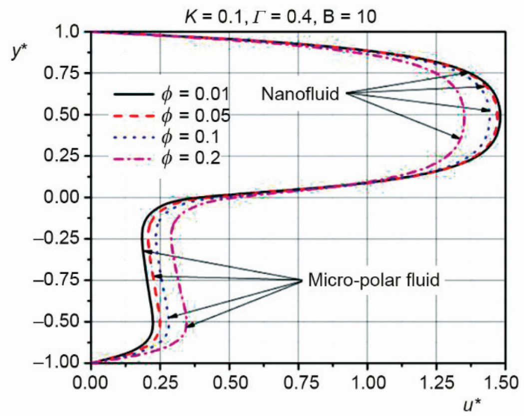

Figure 3. Influence of volume fraction of nanoparticles ϕ on velocity profile

Increase of the volume fraction of nanoparticles leads to decrease of velocity of nanofluid in the upper part of the channel, which is presented in fig. 3. The increase in volume fraction of nanoparticles ϕ increases nanofluid density and Lorentz force intensity, which consequently leads to a decrease in the flow rate of the nanofluid because the pressure drop along the flow direction is constant.

Figure 4 shows influence of coupling parameter, K, on velocity profile. While additional viscosity, λ, increases, the coupling parameter, K, increases and the total viscosity of fluid, as well. Fluids of higher viscosity create greater resistance to the flow, and as a result, for a constant drop in pressure along the flow, there is a decrease in the intensity of velocity in the case of a micro-polar fluid [24]. Also, it can be noted that there is reduction of fluid velocity in the upper part of the channel as consequence of merging effect between two fluids.

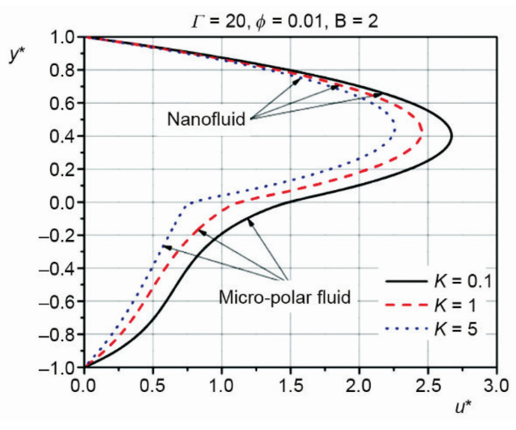

Figure 4. Influence of coupling parameter *K* on velocity profile

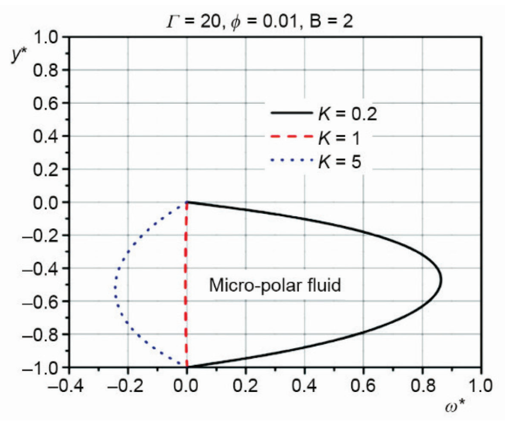

Figure 5. The microrotation in function of coupling parameter *K*

The same tendency additional viscosity of micro-polar fluid, λ, has on intensity of vector of microrotation. The coupling parameter, K, is characteristic of a coupling between linear and rotational motion, resulting from micro motion of molecules or particles embedded in the fluid. With increase of coupling parameter, K, there is decrease in intensity and change of direction of the microrotation in micro-polar fluid, which is shown in fig. 5. The vector of microrotation, defined by eq. (6) is one of the most important characteristics of micro-polar fluids, and in this case appears just in the lower part of the channel, where the micro-polar fluid is.

Figures 6 and 7 represent influence of spin gradient viscosity parameter, Γ, on velocity and microrotation.

From fig. 7 it can be noted that with increase of spin gradient viscosity parameter, Γ, the microrotation changes it direction and decreases, which brings us to the conclusion that the increase in additional angular viscosity γ aggravates the microrotation and lessens the generation of vortex in the fluid. The influence of spin gradient viscosity parameter Γ on velocity profile is negligible, fig. 6, and it is consequence of change of vector of microrotation, as impulse eq. (5) and angular momentum eq. (6) are the coupled equations.

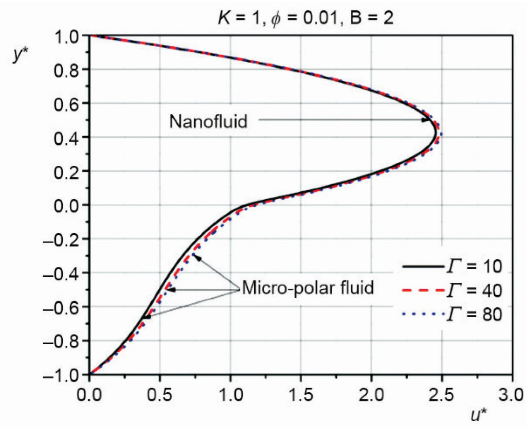

Figure 6. Velocity profiles for different values of spin gradient viscosity parameter Γ

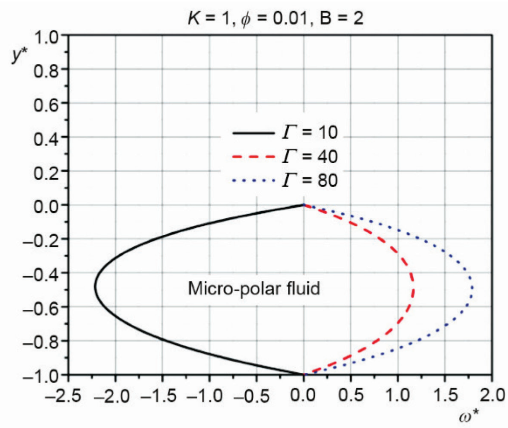

Figure 7. Microrotation for different values of spin gradient viscosity parameter Γ

As the considered problem is MHD fluid-flow in inclined channel, it is interesting to give influence of inclination angle α on velocity profile. Figures 8 and 9 show that with increase of inclination angle α, the intensity of velocity along the entire height of the channel increases, as well as the intensity of microrotation in the lower part of the channel where the micro-polar fluid-flows. This behavior of nano- and micro-polar fluid is expected and is a direct consequence of the second term of eq. (3), as far as nanofluid is concerned, and the third term of eq. (5), when it comes to micro-polar fluid, which is a consequence of the buoyancy effect. It should also be noted that in both cases, as in figs. 8 and 9 Grashof numbers have a positive value.

Heat transfer and temperature distribution in channel, are very important part of considered problem. It is already mentioned that fluids with nanoparticles have comprehensive applications in automotive cooling, higher thermal transportation in microchips, and heath transfer in nuclear reactors. Due to that, last two figures represents influence of external magnetic field of strength, **B,** and volume fraction of nanoparticles ϕ on non-dimensional temperature of nano- and micro-polar fluid in channel.

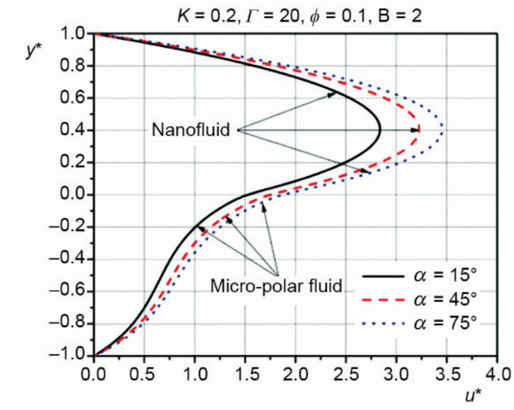

Figure 8. Influence of inclination angle, *α*, on velocity profile

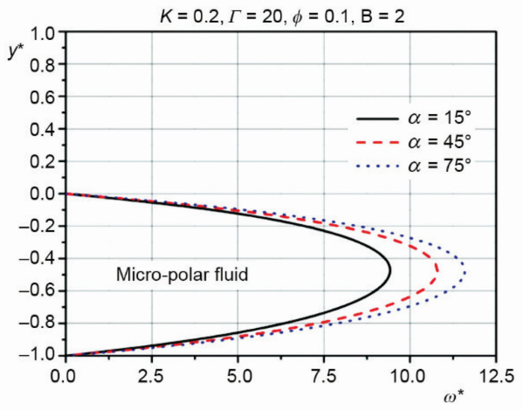

Figure 9. Influence of inclination angle, *α*, on microrotation

From figure 10 it is obvious that with increase of strength of external magnetic field, **B**, non-dimensional temperature in the middle of the channel increases. This is a consequence of Joule heating, because the Lorentz force causes a greater *braking* of the fluid in the middle of the channel. Last fig. 11 shows change in temperature distribution in entire height of the channel with change of volume fraction of nanoparticles, ϕ. It can be noted that with increase of volume fraction of nanoparticles, ϕ, non-dimensional temperature increases in the upper part of the channel, where is nanofluid. The increase in temperature is due to the viscous heating and more intense heat transfer because total viscosity and mass heat capacity of nanofluid increases with increase of volume fraction of nanoparticles, ϕ.

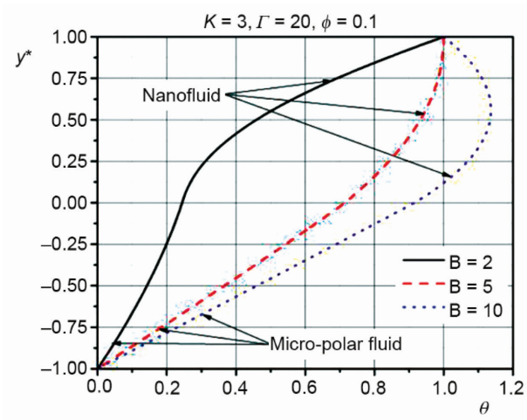

Figure 10. Influence of external magnetic field, B, on non-dimensional temperature, *θ*

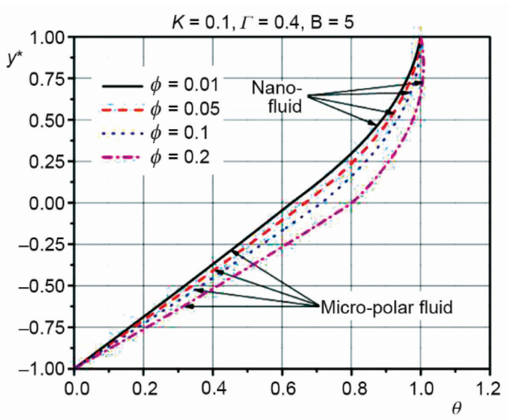

Figure 11. Non-dimensional temperature, *θ*, in fuction of volume fraction of nanoparticles, *ϕ*

Conclusions

The steady MHD flow and heat transfer of incompressible, immiscible, and electrically conducting nano- and micro-polar fluid, in inclined channel, is investigated in this paper. Considered problem solutions for velocity, microrotation, and temperature are obtained in closed form. The general conclusions given through the analysis of graphs can be used for better understanding of the flow and heat transfer of nano- and micro-polar fluid which have a great practical application in nanotechnology and manufacturing processes, such as automotive cooling, higher thermal transportation in microchips, food processing, and nuclear reactors. Some of them are:

- The increase of the strength of external magnetic field leads to a reduction of the velocity with the tendency to uniform the velocity profile over the entire height between the plates.
- Increase of the volume fraction of nanoparticles leads to decrease of velocity of nanofluid.
- With increase of coupling parameter, K, additional viscosity of micro-polar fluid, λ, there is decrease in intensity and change of direction of the microrotation in micro-polar fluid.
- Increase in additional angular viscosity, γ, aggravates the microrotation and lessens the generation of vortex in the fluid.
- With increase of volume fraction of nanoparticles, ϕ, the increase in temperature is due to the viscous heating and more intense heat transfer because total viscosity and mass heat capacity of nanofluid increases.

Future research may involve obtaining numerical solutions for similar problems like the one given in the manuscript and comparing the obtained results, as well as investigating the flow of nano- and micro-polar fluids with other extended boundary conditions.

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Nomenclature

B – magnetic field vector, [T]
c_p – specific heat capacity, [Jkg^{-1}K^{-1}]
Ec – Eckert number
Gr – Grashof number
h – height of channel, [m]
Ha – Hartmann number
k – thermal conductivity of fluid, [WK^{-1}m^{-1}]
Pr – Prandtl number
p – pressure, [Pa]
Re – Reynolds number
T – temperature, [K]
u – fluid velocity, [ms^{-1}]
x – longitudinal co-ordinate, [m]
y – transversal co-ordinate [m]

Greek symbols

ρ – density of fluid, [kgm^{-3}]
μ – dynamic viscosity, [kgm^{-1}s^{-1}]
σ – electrical conductivity, [Sm^{-1}]
ν – kinematic viscosity, [m^{2}s^{-1}]
γ – spin gradient viscosity, [kgms^{-1}]
λ – vortex viscosity, [kgm^{-1}s^{-1}]
ω – micro-rotation vector, [s^{-1}]
θ – dimensionless temperature

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