

# New Method for Calculating Heat Transfer in Unsteady MHD Mixed Boundary Layers with Radiative and Generation Heat over a Cylinder

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**Abstract:** *This paper is devoted to the analysis of an unsteady two-dimensional MHD dynamic and thermal boundary layer over a horizontal cylinder in mixed convection, in the presence of suction/injection, heat source/sink, and heat radiation. Fluid electrical conductivity is constant. The system of MHD equations of dynamic and temperature boundary layer, which describe complex non-auto model problems, has been solved by a new approach. New variables and sets of parameters were introduced and transformed equations were obtained, in which the influence of the magnitude  $Z$  was explicitly retained. In order to close the system of equations, to the equations of the boundary layers momentum equation was added. The solution of the obtained system of nonlinear differential equations was performed numerically using the finite difference method, with the simultaneous application of the iteration method. By replacing the derivatives in the system of equations with the corresponding relations of finite differences, a system of linear derivative algebraic equations is obtained, which is solved by the three-diagonal method. As a concrete example of the introduced method, the effects of heat transfer in the MHD boundary layers were considered, in the case of mixed convection, over a horizontal circular cylinder. The boundary conditions for temperature are defined by linear functions of longitudinal coordinates and time. Numerical results for different  $Ec$ ,  $Sc$  and expanded  $Pr$  numbers and values: magnetic, dynamic, thermal parameters, temperature and buoyancy parameters were obtained and presented. The obtained results were analyzed through the diagrams of changes in velocity and temperature and the diagrams of integral and differential characteristics of boundary layers and the corresponding conclusions were given.*

**Keywords:** *MHD Dynamic and Thermal boundary layer, Mixed convection, Heat transfer, Suction/injection, Source/sink and Heat radiation, Circular horizontal cylinder.*

## 1. Introduction

The study of MHD flow of incompressible electrically conductive fluid is of significant scientific and research interest in possible technical and technological applications of the obtained results to the development of numerous devices, apparatus and technological processes. Many processes, in which heat transfer, takes place need to study the effects of external magnetic fields on thermal-physical processes in MHD dynamic and temperature boundary layers [1-5]. The flow in the MHD boundary layers has several properties different from the boundary layer properties of the nonconductive fluid. By the action of the magnetic field, it is possible to change the amount of friction, heat transfer and move the separation point of the boundary layer. This possibility, management and control of the dynamic and thermal characteristics of boundary layers, has attracted the attention of a large number of researchers who have tried to study as many special cases as possible, to create a broader basis for using such effects for practical purposes.

The flow of electrically conductive fluid in MHD boundary layers, especially in cases when it is necessary to know the temperature field in addition to the velocity field, and when there are many external influences on their development, is described by a very complex system of nonlinear partial differential equations associated with significant difficulties. Analytical and numerical solutions of the equations of the non-stationary MHD laminar boundary layer of an incompressible conductive fluid exist for a relatively small number of special cases, [6-9], which generally do not cover all those significant problems for engineering and technology that need to solve such a system of MHD equations. Taking into account the all intensive development of numerous, new, modern fields of technique and technology in which the movement of Newtonian and non-Newtonian conductive fluids, very complex physical and chemical characteristics is studied, with the presence of numerous heat transfer effects, chemical reactions, etc. to deal more with the study of non-auto model problems of body obstruction of a more complex shape [10-13].

In solving the MHD problem, analogous to the study of boundary layers of a non-conducting fluid, many accurate and approximate methods developed in the classical boundary layer theory were used. Thus, many known methods have been transferred to the study of more complex tasks of MHD boundary layers [14-19]. The significant presence of non-stationary problems in technical-technological practice, and their insufficient research, especially in the field of MHD boundary layers, indicate the need for their research. In the last twenty years, several papers have been published, that investigate these problems [15,17,19,21]. However, most of these works investigated mostly special cases: horizontal or vertical plate circulation, wedge circulation, and vertical or horizontal circular cylinder, [20-25], with different initial and boundary conditions for body surface temperature and external current [26-28]. The study of phenomena that occur in the non-stationary flow of electrically conductive fluid in MHD boundary layers, especially in cases where it is necessary, in addition to the velocity field, to know the temperature field, and when many external influences on their development are present, give a complex system of nonlinear partial differential equations whose solution is associated with significant difficulties.

This paper discusses the complex physical problem of the magnetic field on the flow of a horizontally circular cylinder, in simultaneous non-stationary MHD dynamic and temperature boundary layers, under the action of buoyancy forces due to temperature differences, suction/injection of fluid, the influence of sink/source and radiation heat and at arbitrary boundary and initial conditions, external velocities and temperatures. The system of equations describing the flow of conductive fluid in the MHD dynamic and temperature boundary layer was solved in a new approach [30-31], which by its nature can be classified into new methods for calculating the equations of MHD boundary layers. After the introduction of new variable similarities, as well as a series of dimensionless similarity parameters: dynamic, magnetic, thermal buoyancy, suction/injection, thermal parameter and temperature parameters, the system of obtained dimensionless equations of MHD boundary layers and momentum equations was solved numerically using the iteration method. At the end of the paper, a graphical presentation is given and the profile of dimensionless velocity and temperature is analyzed, as well as curves of changes in differential and integral characteristics of dynamic and temperature boundary layer, for Eckart and extended Prantl numbers and for different values of introduced similarity parameters

## **2. Mathematical model unsteady MHD boundary layer**

A mathematical model of unsteady two-dimensional dynamics and thermal MHD boundary layer, of an incompressible fluid, with the free stream velocity  $U(x,t)$  and ambient temperature  $T_\infty$ , is defined with the system of four simultaneous equations: continuity equation and equations of dynamic and thermal boundary layer. The uniform external magnetic field  $B(x)$  is applied

perpendicular to the surface of the body and the magnetic Reynolds number is significantly lower than the one-considered problem in induction-less approximation. As in this paper, the problem we consider the problem of with a given suction/injection velocity  $v_w(x,t)$  is present, it is necessary to first introduce the appropriate transverse flow velocity and the stream function  $\psi(x,y,t)$ , as follows:  $v_1 = v - v_w$ ,  $u = \psi_y$ ,  $v_1 = -\psi_x$ . Introduced replacement, the equation of continuity takes form -  $u_x + v_{1y} = 0$ , and the equations of the dynamic and temperature boundary layer, after replacing the change in partial pressure increment transferred from the external flow, take the following form

$$\begin{aligned}
1. \quad & \psi_{yt} + \psi_y \psi_{xy} + (v_1 - \psi_x) \psi_{yy} = \nu \psi_{yyy} + U^t + UU_x - \frac{\sigma B^2}{\rho} (\psi_y - U) + g\beta_T (T - T_\infty) \sin \alpha \\
& T_t + \psi_y T_x + (v_1 - \psi_x) T_y = \frac{\lambda}{\rho c_p} T_{yy} + \frac{\mu}{\rho c_p} (\psi_{yy})^2 + \frac{\sigma B^2}{\rho c_p} (\psi_y - U) \psi_y + \\
2. \quad & + \frac{1}{c_p} (U^t + UU_x) (U - \psi_y) - \frac{\partial q_r}{\partial y} + Q(T - T_\infty)
\end{aligned} \tag{1}$$

with corresponding initial and boundary conditions:

$$\begin{aligned}
& \psi(x, y, t) = 0, \psi_y = 0, T(x, y, t) = T_w(x, t) \text{ for } y = 0, \\
& \psi_y \rightarrow U(x, t), T(x, y, t) \rightarrow T_\infty \text{ for } y \rightarrow \infty, \\
& \psi_y = u_0(x, y), T = T_0(x, y) \text{ for } t = t_0, \psi_y = u_1(t, y), T = T_1(t, y) \text{ for } x = x_0.
\end{aligned} \tag{2}$$

The last condition indicates that to solve the system of equations, for  $x \gg 0$ , it is necessary to know the size of the stream function and temperature in the whole area of the boundary layer in some characteristic cross-section of the boundary layer for  $x = x_0$

The equation (1), with  $q_r^*$  denotes the heat of radiation  $q_r^* = -\partial q_r / \partial y$ , which, using Roseland's approximation for radiative heat flux, can be written  $q_r = -4\sigma_r / 3\mu \cdot \partial T^4 / \partial T$ , where are  $\sigma_r$  and  $\mu$ , Štefan-Boltzmann constant and the absorption coefficient are denoted. Assuming that the temperature differences are small enough, it can be  $T^4$  represented as a linear function of the temperature, or use the development of this function in the Taylor polynom in the environment  $T_\infty$ , from which, by rejecting members of the second and higher order, it is obtained

$$T^4 \cong T_\infty^4 + 4(T - T_\infty)T_\infty^3 \cong 4T_\infty^3 T - 3T_\infty^4, \text{ respectively } q_r^* = \alpha^* T_{yy} \text{ where } \alpha^* = 16\sigma_r T_\infty^3 / 3\mu$$

Obtained partial nonlinear differential equations (1), with boundary and initial conditions (2) can be solved using one new approach [29,30] which to some extent uses elements of modern integral-differential methods of generalized similarity Loitsianskii and which many authors [5,19,21,22] have used to solve many various MHD problems. To this target is introduced a new variable similarity for transverse co-ordinates of similarity  $\eta$  and the corresponding dimensionless velocity  $\phi(x, \eta, t)$  and temperature  $\theta(x, \eta, t)$

$$\eta(x, y, t) = \frac{D_0}{h(x, t)} y, \quad \phi(x, \eta, t) = \frac{D_0}{U(x, t)h(x, t)} \psi(x, y, t), \quad \theta(x, \eta, t) = \frac{T_w - T}{T_w - T_\infty} \quad (3)$$

In the introduced transformations and the introduction of a new dimensionless function  $\phi(x, \eta, t) = \phi_\eta = u/U$ , which represents the relation velocities, the arbitrary linear characteristic of the transverse coordinate is now marked with  $h(x)$ , and  $D_0$  represents a normative constant, which will be determined in a further process of transformations of the starting equations. According to the introduced variables, the system of equations (1) is transformed into the new form:

$$\begin{aligned} 1. \quad & D_0^2 \phi_{\eta\eta} + \frac{f + 2f_1}{2} \phi \phi_\eta + \frac{\eta \dot{Z}}{2} \phi_\eta + D_0 v_0 \phi_\eta + f_1 (1 - \phi^2) + \alpha_1^T (1 - \theta) + (f_0 + g)(1 - \phi) = \\ & = Z \phi_\eta + f_2 (\phi \phi_x - \phi_x \phi_\eta), \\ 2. \quad & \frac{D_0^2}{P_r^*} \theta_{\eta\eta} + \frac{f + 2f_1}{2} \phi \theta_\eta + \frac{\eta \dot{Z}}{2} \theta_\eta + D_0 v_0 \theta_\eta - E_c [f_1 + f_0] (1 - \phi) - E_c g (1 - \phi) \phi - \\ & - D E_c (\phi_{\eta\eta})^2 = [l_0 + l_1 \phi - q] (1 - \theta) = Z \theta + f_2 (\phi \theta_x - \phi_x \theta_\eta), \end{aligned} \quad (4)$$

with initial and boundary conditions:

$$\begin{aligned} \phi = \phi_\eta = \theta = 0 \quad \text{for} \quad \eta = 0, \quad \phi = \theta \rightarrow 1 \quad \text{for} \quad \eta \rightarrow \infty, \\ \phi = \phi_0(\eta), \quad \theta = \theta_0(\eta) \quad \text{for} \quad x = x_0. \end{aligned} \quad (5)$$

In the system of the boundary layer equation (4), initial parameters of similarity and characteristic size of the boundary layers were introduced

$$\begin{aligned} f(x, t) = UZ', \quad f_1(x, t) = U'Z, \quad f_0(x, t) = \dot{U}Z/U, \quad f_2(x, t) = UZ, \quad g(x, t) = NZ, \\ v_0(x, t) = -v_w / \sqrt{\nu Z} \cdot Z, \quad \alpha_1^T(x, t) = UZ\alpha^T, \quad q(x, t) = QZ(x, t) \\ l_1(x, t) = \frac{T'_w}{T_w - T_\infty} Z, \quad l_0(x, t) = \frac{\dot{T}_w}{T_w - T_\infty} Z, \quad Z(x, t) = \frac{h^2(x, t)}{\nu}, \quad \alpha^T(x) = \frac{g\beta_T \sin \alpha(x)}{c_p E_{cx}^T} \end{aligned} \quad (6)$$

$$\text{where is: } N(x) = \frac{\sigma(x)B^2(x)}{\rho}, \quad E_c = \frac{U^2}{c_p(T_w - T_\infty)}, \quad P_r^* = \frac{\nu}{\alpha + \alpha^*} \quad (\text{extended Pr number})$$

Normalizing constant  $D_0$ , which is determined from the condition that the first dynamics equation of system of equation (4), is reduced to the case of flow past a flat plate  $\phi_{0\eta\eta} + \phi_0 \phi_{0\eta\eta} = 0$ , results in a constant value -  $D_0 = \sqrt{\xi_0} = 0.469$ .

To the system of equations (4), where the unknown functions are  $\phi(x, t, \eta)$ ,  $\theta(x, t, \eta)$  and variable  $Z(x, t)$  it is necessary to add one of the appropriate integral equations. In this paper as an integral equation, using - the integral impulse equation [32], in which, the momentum thickness is chosen for the characteristic thickness -  $h(x, t) = \delta^{**}$

$$\frac{H^*}{2} \frac{\partial Z}{\partial t} + \frac{U}{2} \frac{\partial Z}{\partial x} = \zeta - \left[ \left( U' + \frac{\dot{U}}{U} + N \right) H^* + 2U' + U\alpha_T H_T \right] Z + v_w \frac{\sqrt{Z}}{\sqrt{v}} \quad (7)$$

with initial and boundary conditions:  $\partial Z/\partial t = 0$ , for  $t = 0$ , and  $\partial Z/\partial x = 0$  for  $x = 0$ .

Where the dimensionless characteristic functions of the boundary layer are introduced in the following form:

$$H^*(x,t) = \frac{\delta^*(x,t)}{\delta^{**}(x,t)}, \quad H_T(x,t) = \frac{\delta_T(x,t)}{\delta^{**}(x,t)}, \quad \zeta = \left[ \frac{\partial(u/U)}{\partial(y/\delta^{**})} \right]_{y=0}$$

$$\delta^*(x,t) = \int_0^\infty \left( 1 - \frac{u}{U} \right) dy, \quad \delta^{**}(x,t) = \int_0^\infty \frac{u}{U} \left( 1 - \frac{u}{U} \right) dy, \quad \delta_T(x,t) = \int_0^\infty (1 - \theta) dy \quad (8)$$

System of equations (4,7), with corresponding boundary conditions, with a group of independent parameters (6), can be now applied to any concrete example, defined profile of the body, given boundary and initial conditions, of temperature on the surface of the body and velocity outer flow.

### 3. Application of the introduced method to flow around a cylinder

As a concrete example of the introduced method, this paper will consider the effects of heat transfer in MHD boundary layers when following a horizontal circular cylinder. Flow analysis is performed with dimensionless quantities, such that the longitudinal coordinate and velocity are given concerning the radius of the cylinder  $r$  and the velocity of the incoming uniform current  $U_\infty$ , and the transverse coordinate and velocity concerning the same quantities divided by the Reynolds number  $\sqrt{R_e}$ . It is also assumed that the intensity of the external magnetic field ( $\hat{N} = rN/U_\infty = const$ ) and the suction/injection velocity are constant values ( $\hat{v}_w = v_w \sqrt{R_e}/U_\infty = const$ ) and that all coefficients are also constant.

The solution is searched under the following boundary conditions

$$\hat{U}(\hat{x}, \hat{t}) = (1 + a\hat{t}^n) \sin \hat{x}, \quad [T_w(\hat{x}, \hat{t}) - T_\infty] / \Delta T_{w\infty} = (1 + a_{2x} \hat{x}^m)(1 + a_{2t} \hat{t}^m) \quad (9)$$

and for the following values of quantities, which define the influence of buoyancy force

$$\alpha_{10}^T = \hat{U} \hat{Z} \hat{\alpha}^T, \quad \hat{\alpha}^T(\hat{x}, \hat{t}) = \frac{ag\beta_T \sin \hat{x}}{c_p E_{cx}^T} = \hat{\alpha}^{cT} \sin \hat{x}, \quad -(\hat{\alpha}^T = a\alpha^T)$$

where -  $\hat{x} = x/r$  is the longitudinal angular coordinate, measured from the front stop point in radians,  $\hat{t} = U_\infty t/a$  - the dimensionless values of time measured from zero, and,  $\hat{Z} = U_\infty Z/r$ ,  $\hat{Q} = U_\infty Q/r$ .

Quantities  $a, a_{2x}, a_{2t}$ , can be negative or positive dimensionless constants. The positive value of the constant  $a$  corresponds to the accelerated external flow, and the negative, to the slowed flow of the fluid. Positive or negative values of constants  $a_{2x}$ , indicate an increase or decrease in temperature along the body, and positive or negative values of constants  $a_{2t}$ , indicate an increase or decrease in temperature over time. Degrees  $n, m$  can be positive or negative integer constants. In the case of such initial and boundary conditions, for the flow of a horizontal circular cylinder, the introduced dimensionless similarity parameters (6) take the following form:

$$\begin{aligned}
f(x, t) &= (1 + at^n) \sin x Z'(x, t), \quad f_1(x, t) = (1 + at^n) \cos x Z(x, t) \\
f_0(x, t) &= \frac{nat^{n-1}}{1 + at^n} Z(x, t), \quad f_2(x, t) = (1 + at^n) \sin x \cdot Z(x, t), \quad g(x, t) = N(x, t) Z(x, t) \\
\alpha_1^T &= \widehat{U} Z \alpha^T, \quad \alpha^T(\widehat{x}, \widehat{t}) = \frac{g \beta_T \sin \widehat{x}}{c_p E_{cx}} = \alpha^{cT} \sin \widehat{x}, \quad q(x, t) = Q Z(x, t) \\
l_1(x, t) &= \frac{ma_{2x} x^{m-1}}{1 + a_{2x} x^m} (1 + at^n) \sin x Z(x, t), \quad l_0(x, t) = \frac{m_1 a_{2t} t^{m_1-1}}{1 + a_{2t} t^{m_1}} Z(x, t) \quad (10)
\end{aligned}$$

in which, for the simple writing, for all introduced dimensionless quantities  $\widehat{x}, \widehat{t}, \widehat{U}, \widehat{N}, \widehat{v}_w, \widehat{\alpha}^{cT}, \widehat{Q}$ , use previously introduced notations.

#### 4. Application of the finite difference method and analysis of the results

As the application of numerical integration results in high accuracy, the system of partial nonlinear differential equations (4) and the momentum equation (7), is numerically solved by applying the finite difference method, combined with the iteration method, in the space of independent coordinates  $(x, \eta, t)$ . On the introduced integration networks  $(x_n, \eta_m, t_k)$ , discrete quantities of dependent and independent variables were defined, as well as introduced similarity parameters. Then, a five-point spatial integration network was set up and an indirect-implicit scheme of finite differences was introduced. By replacing the derivatives in equations (4), and the impulse equation (7), the corresponding ratios of finite differences, where, due to the expected rapid changes in velocity and temperature, perpendicular to the boundary layer, i.e. in the direction  $\eta$ -axis, central schemes of differences are used second order of accuracy, and in the direction along the boundary layer  $x$  and time  $t$ , where minor changes are expected, the schemes of the first order of accuracy are used. In this way, the initial system is approximated by a system of three sets of differential-algebraic linear equations of dynamic and temperature boundary layer and impulse equation [29]. The special form of matrices of the system of equations thus obtained, matrices are three-diagonal, enabling simple direct methods to be used for their solution, which do not require the formation of

inverse matrices for their application. In that sense, the obtained recurrent systems of algebraic equations were solved by applying the three-diagonal method.

The size of the integration space varied depending on the parameter values  $a, v_w, a_{cT}, N$ . The step in the  $t$ -axis direction was constant  $\Delta t = 0.01$ , while the step along the  $x$ -axis varied from the value  $\Delta x = 0.01$ , front stop point, to the value  $\Delta x = 0.005$ , around the boundary layer separation points. The process of solving equations, in the whole integration space, started from the coordinate starting  $x = t = 0$ , the front stop point of the stationary MHD flow, to the point of separation of the boundary layer. The solution of the system of algebraic equations of boundary MHD layers and impulse equation [29], was performed on planes  $t = t_k = k\Delta t$ , parallel to the plane of stationary flow, based on the formed algorithm, and the software program. The iteration process took place until the difference in size  $\varphi, \theta, Z$ , on two successive iterations, received values smaller than an advance-determined value  $-\varepsilon = 10^{-6}$ .

After determining the dimensionless functions of the ratio of velocity  $\varphi(x, t, \eta)$ , temperature  $\theta(x, t, \eta)$  and variable  $Z(x, t) = \widehat{Z}(\widehat{x}, \widehat{t})$ , they were calculated the tangential stress on the body  $\widehat{\tau}_w$ , the heat transfer coefficient  $\zeta_T$  and the corresponding thickness of the boundary layers  $\widehat{\delta}^*, \widehat{\delta}^{**}, \widehat{\delta}_T$ , for accelerated and decelerated flows ( $a_1 = \pm 1$ ), degree values  $m = 1, n = 2$ , and different values of introduced coefficients  $a_{2x}, a_{2t}$ , quantities-  $Q, \widetilde{N}, \widetilde{v}_w$  and numbers  $P_r^*, E_c$ ,

$$\begin{aligned} \widehat{\delta}^{**} = \widehat{Z}^{1/2} = \frac{\sqrt{R_e}}{a} \delta^{**}, \widehat{\delta}^* = H^* \widehat{\delta}^{**} = \frac{\sqrt{R_e}}{a} \delta^*, \widehat{\tau}_w = \frac{\sqrt{R_e}}{\rho U_\infty^2} \tau_w = D_0 \frac{\widehat{U}(\widehat{x}, \widehat{t})}{\widehat{\delta}^{**}(\widehat{x}, \widehat{t})} \zeta, \\ \widehat{\delta}_T = \frac{\sqrt{R_e}}{a} \delta_T = \widehat{\delta}^{**} H_T = \frac{\widehat{\delta}^{**} \eta_M}{D_0} \int_0^{\eta_M} (1 - \theta) d\eta, N_{u\delta} = -\frac{\delta^{**}}{T_w - T_\infty} \left( \frac{\partial T}{\partial y} \right)_{y=0} = D_0 \zeta_T. \quad (11) \end{aligned}$$

Based on the obtained results for the wall shear stress on the cylinder, the heat transfer coefficient on the cylinder surface (Nuselt number), and for the thicknesses of the dynamic and temperature boundary layer, as well as for the profiles of velocity and dimensionless temperature, in certain cross sections of boundary layers it can be concluded on the possibilities of the introduced influences on the development of MHD boundary layers. Thus, the development of both MHD boundary layers can be successfully controlled by the action of the magnetic field  $-\widetilde{N}$ , the characteristics of buoyancy forces  $-\widetilde{\alpha}^{cT}$ , the magnitude of the non-stationary coefficient  $-a$ , the magnitude of the degree  $-n$  and the magnitude of the dimensionless suction/injection velocity  $\widetilde{v}_w$ . Temperature parameters, values of coefficients  $a_{2x}, a_{2t}$ , and degree  $m$ , values that determine heat sources/sink  $\widehat{Q}$ , extended Prandtl  $P_r^*$  number, which includes the influence of radiation heat and Eckart's  $E_c$  number, can control the development of the temperature boundary layer.

Figures (1-3), show a change in the tangential stream function on the body and position of the boundary layer separation point, concerning the change in the size of the non-stationary coefficient- $a$  and concerning the change of degree  $n$  and time.

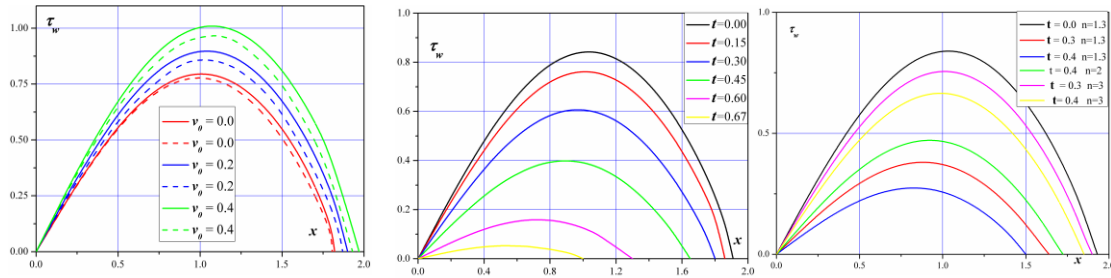
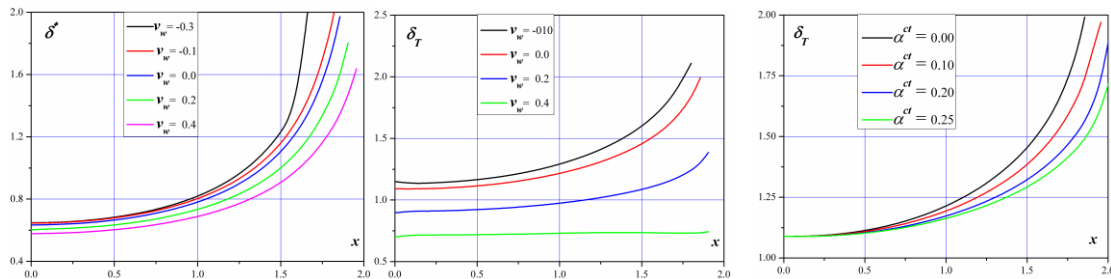


Figure 1-3. Diagrams stream function  $\tau_w$

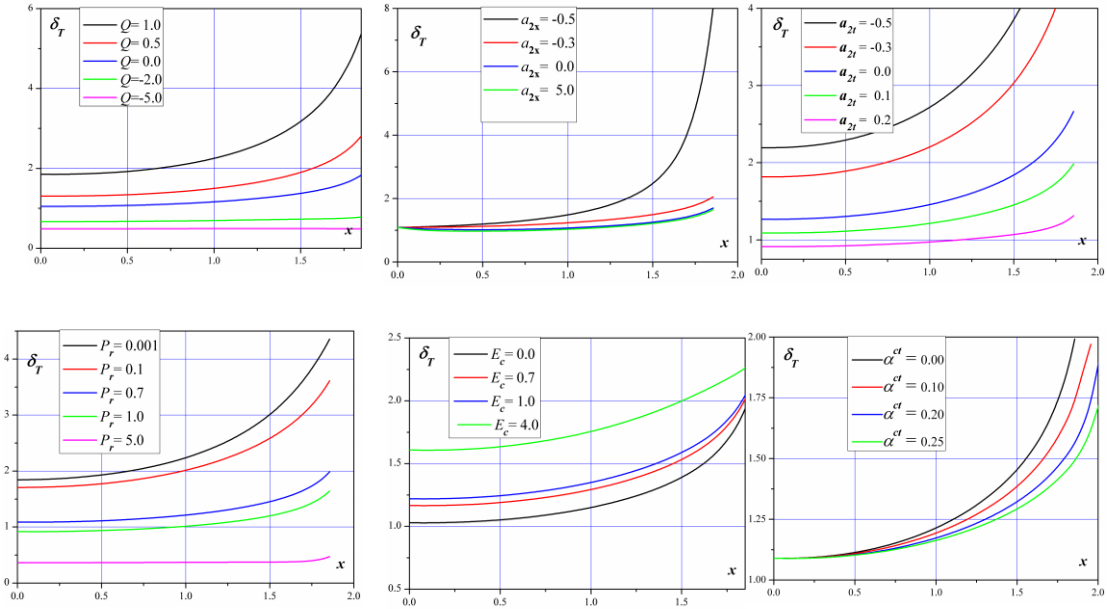
The results compared to the correct results [12], Fig.1 for non-porous ( $v_w = 0$ ), and porous contours ( $v_w \neq 0$ ), for stationary and non-stationary flow non-conductive fluids without action buoyancy forces



Figures 4-5 Diagrams of boundary layers thickness  $\hat{\delta}^*$ ,  $\hat{\delta}_T$

Analysis of the influence of introduced values  $\tilde{v}_w$ , on the development of the displacement thickness  $\hat{\delta}^*(\hat{x}, t)$  and the temperature thickness  $\hat{\delta}_T(\hat{x}, t)$ , is given in Fig. 4-5. The diagrams show that the thicknesses of boundary layers are reduced with the increase in parameters  $\tilde{\alpha}^{cT}$  and suction of fluid ( $\tilde{v}_{00} > 0$ ). The opposite effect, increasing the boundary layer thickness is obtained when injection of fluid ( $\tilde{v}_{00} < 0$ ).





Figures 6-10. Diagrams of temperature boundary layer thickness  $\hat{\delta}_T$

Diagrams in Fig.6-11 show, that the thickness of the temperature boundary layer, reduces the increase  $a_{2x}$  and  $a_{2t}$ , from negative values, via zero and positive, increases the extended coefficient of Prandtl number  $P_r^*$ , (by increasing the heat of radiation), and values  $\tilde{\alpha}^{cT}$ , as well as in cases sink heat ( $\hat{Q} < 0$ ), and it grows in case of increasing the intensity of heat sources ( $\hat{Q} > 0$ ) and increasing the Ekart's number because in these cases increases, i.e. reduces, the temperature in the boundary layer. Figure 6 shows a change in the boundary layer thickness  $\delta_T$ , with time for accelerated and slow flow. It is noticed, to accelerate the flow and flow of time, the thickness of the boundary layer reduces.

Calculation of the thickness of the temperature boundary layer  $\hat{\delta}_T(\hat{x}, t)$  was determined by the product of the momentum thickness  $\hat{\delta}^{**}$  and the appropriate integration (11). In this sense, the increase in the integral (11), in cases of the increase in temperature, resulting from the magnetic field, does not always have to monitor the thickness of the boundary layer  $\hat{\delta}_T(\hat{x}, t)$ . This is because as the value of the magnetic parameter  $N$  increases, at the same time, it reduces the momentum thickness. This means that in these cases, the decrease in thickness  $\hat{\delta}^{**}$  is greater than increasing in the value of integrals. This tendency can be explained through equations of dynamic and temperature boundary layer, i.e., over the size of Lorenzce's force  $-NZ(x, t)(1-\varphi)$  and Joule's heat  $E_c NZ(x, t)(1-\varphi)\varphi$ . Namely, for the values of Ekart's number, the diagrams are smaller

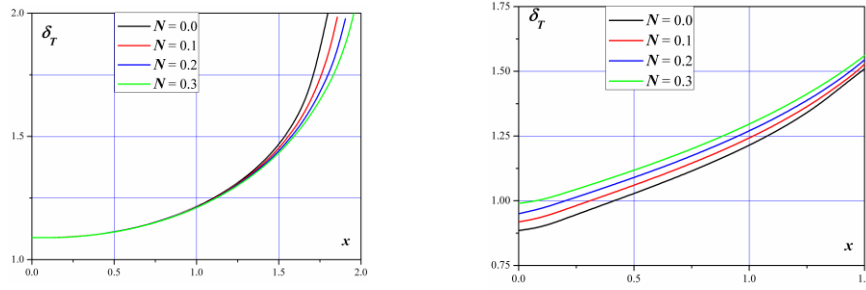
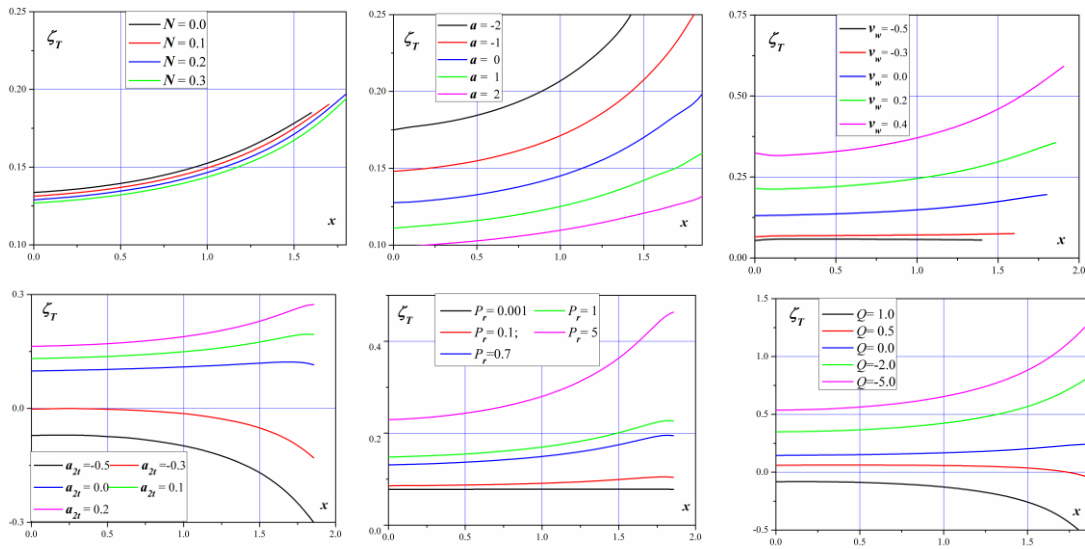


Figure 11-12. Diagrams of the temperature thickness  $\hat{\delta}_T$ , for variable  $N$  and  $E_c$

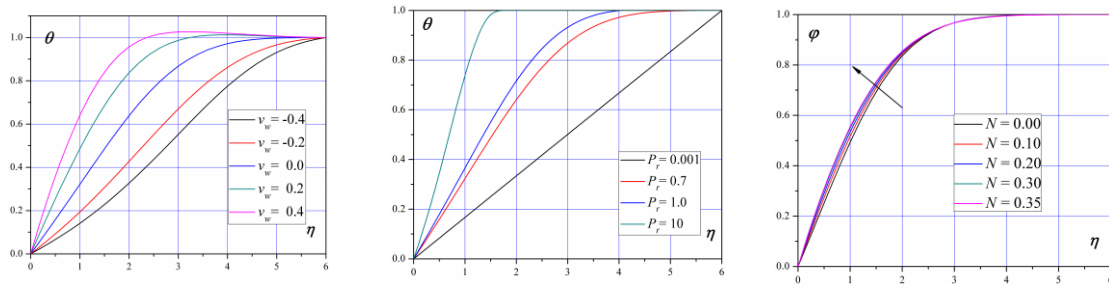
then one (the diagrams are calculated for  $E_c = 0.3$ , faster decreases in the momentum thickness  $\hat{\delta}^{**}$ , than the value of the appropriate integral, so the conclusions of reducing this thickness are correct (Fig 11). However, for the values of Ekart's number, greater than one  $E_c = 4.0$ , Joule's heat grows faster than Lorenz's force, thus, the temperature grows faster, i.e., by increasing the magnetic field parameter, the thickness of the temperature boundary layer is growing (Fig.12).



Figures 13-18. Diagrams of the heat transmission coefficients on the body  $\zeta_T$ ,

By increasing the magnetic parameter  $\tilde{N}$  and the value of the coefficient  $a$  (from negative values, for slow currents, through positive values for accelerated currents), heat transfer on the body decreases, as a consequence of the increase in temperature in the boundary layer (Fig. 13,14). The same effect is achieved by intensifying the suction process, or the opposite, by intensifying the injection of the fluid (Fig. 15). Also Figures (16,17,18), with increasing coefficient values  $a_{2r}$  (from negative to zero and positive), increasing extended Prandtl number  $P_r^*$  and heat sink ( $\hat{Q} < 0$ ), dimensionless heat transfer  $\zeta_T(\hat{x})$  increases, and in the case of heat sources ( $\hat{Q} > 0$ ), decreases. The reason for such changes lies in the fact that with such changes in the above values, the

temperature increases in the boundary layer, i.e. decreases, which results in a decrease in heat transfer from the body to the fluid.



Figures 19-21. Diagrams of the dimensionless temperature  $\theta$  and velocities  $\phi$

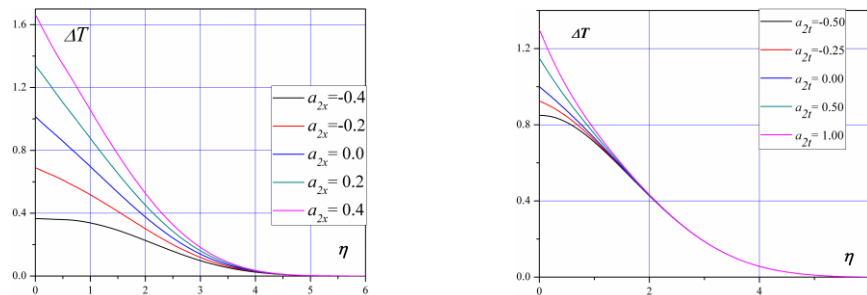
The profiles of dimensionless temperature changes  $\theta$ , Fig.(19-20) show that dimensionless temperatures increase, i.e. temperatures  $T$  decrease, with increasing in case of increased extended Prandtl number  $P_r^*$ , as well as injection ( $v_w < 0$ ). The dimensionless temperature decreases, i.e. the temperature  $T$  increases, with increasing well for the suction of fluid ( $v_w > 0$ ). The profiles of dimensionless velocity changes  $\phi = u/U$ , Fig 21, show that dimensionless velocities increase with increasing magnetic parameters, respectively the intensity of the magnetic field  $\tilde{N}$ .

The conclusions obtained in this paper refer to the heat transfer when circulating a horizontal cylinder with a conducting fluid, in cases when the forces are equal to zero, and which refer to the dynamic boundary layer, correspond to the conclusions obtained in the papers [14,15] for ( $\tilde{v}_{00} = q_{10} = 0, P_r^* = P_r$ ).

As there is a relation between dimensionless and physical-dimensional quantities of temperature

$$\Delta \tilde{T} = [T - T_\infty] / \Delta T_{w\infty} = (1 + a_{2x} \hat{x}^m)(1 + a_{2t} \hat{t}^{m_t})(1 - \theta)$$

It can be stated that the direction of change of temperature values  $T$  is opposite to the direction of change of dimensionless temperature  $\theta$ . This conclusion refers to cases when the temperature is variable, i.e. when the coefficients change  $a_{2x}, a_{2t}$ . In this regard, Figures 22-23 show changes in temperature values, and for higher values of temperature parameters



Figures 22-23. Diagrams of the temperature difference  $\Delta \tilde{T}$ , variable  $a_{2x}, a_{2t}$

In this paper, the derived equations (4), (7) with the corresponding initial and boundary conditions and the set of introduced similarity parameters (6), the procedure shown in the example of the flow of a horizontal circular cylinder, have a universal character and can be applied, to any specific example, a stationary or non-stationary problem, conductive or non-conductive fluid flow, a given body profile - given external flow velocity, with a given initial and boundary temperature conditions, a set value of suction/blowing speed, with or without source/heat sink, that is, with or without the presence of heat radiation.

## 5. Conclusion

This study presents one new method for the solution of heat transfer in unsteady mixed dynamic and thermal MHD boundary layer flow past a horizontal circular cylinder with variable initial and end conditions for velocity and temperature. Fluid is incompressible and electrically conductive. The effects of the magnetic field, thermal buoyancy force, in the presence of radiation and heat source/sink and fluid injection/ suction were analyzed. The system of nonlinear partial differential governing boundary layer equations, integral impulse equations and associated boundary conditions are converted into a dimensionless form using a suitable similarity transformation and similarity parameters. Numerical solutions of dynamic, temperature boundary layer equations, with integral impulse equation, are obtained by using the finite difference and three-diagonal method, combined with the method of iteration. Obtained results of the solutions for the velocity and temperature also differential and integral characteristics of the dynamic and thermal boundary layer, are obtained and presented for different values Eckart and extended Prandtl number and introduced similarity parameters magnetic field, thermal buoyancy force, radiation and source/sink heat, fluid injection/suction and accelerating/deceleration external flow, on the flow development around the horizontal circular cylinder surface.

## Nomenclature

$B$  – induction uniform magnetic field, [T]

$T$  – fluid temperature, [K]

$H$  – characteristic function, [–]

$N$  – characteristic function, [ $s^{-1}$ ]

$t$  – time, [s]

$U$  – free stream velocity, [ $ms^{-1}$ ]

$u, v$  – velocities in the boundary layer, [ $ms^{-1}$ ]

$x, y$  – longitudinal and transversal coordinate, [m]

$Q$  – heat generation/absorption constant, [ $s^{-1}$ ]

$a, m, n$  – constants, [–]

### Subscripts

$x$  – longitudinal coordinate

$t$  – time, [s]

$w$  – surface conductions

$\infty$  – conductions far away from the surface

$0$  – initial time moment  $t = t_0$

$1$  – boundary layer cross-section  $x = x_0$

### Greek symbols

$\zeta$  – characteristic function, [–]

$\phi$  – dimensionless stream function, [–]

$\eta$  – dimensionless transversal coordinate, [–]

$\tau$  – shear stress, [Pa]

$\psi$  – stream function, [ $m^2 s^{-1}$ ]

$\delta$  – boundary layer thickness [m]

$\theta$  – dimensionless temperature [–]

$\sigma$  – the fluid electrical conductivity

$\varphi$  – dimensionless velocities

$\eta$ ,  $x$ ,  $t$  – differentiation with respect to  $\eta$ ,  $x$ ,  $t$

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