

## TEMPERATURE-DEPENDENT CAPILLARY RISE AND ITS EFFECTS ON FABRIC CLEANING AND PERMEABILITY

by

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*A fabric can be considered as a porous medium, its porosity size and temperature will greatly affect air/moisture permeability and thermal comfort. This paper studies the effect of temperature on the capillary rise experimentally, the experimental data reveal that a higher temperature results in a higher capillary rise, as a result, a better air/moisture permeability is predicted. This paper elucidates also a higher temperature is favorable for effective washing.*

Key words: *capillary pressure, capillary rise, geometric potential theory, temperature, sportswear*

### Introduction

Capillary phenomenon is common in everyday life and is observed in the smallest blood vessels in the living body. The properties of a porous medium are greatly affected by the capillary effect, for examples, thermodynamic property of a porous concrete [1], mechanical property of a 3-D printed concrete [2], electronic property of a composite [3], and sliding capillary contact [4]. The capillary force will also affect greatly the nano/micro devices, for examples, micro-electromechanical systems [5-8], micro-mirrors [9], and micro-fluidics [10]. The capillary oscillation plays an important role in ions release in a small tube [11, 12] and capillary action is the mechanism for moisture transmission in a porous medium [13-16], furthermore, it can be used for sensors [17].

A fabric can be considered as a porous medium, and the capillary effect will greatly affect the fabric's surface clearness, air permeability and perspiration, in this paper we will study the effect of temperature on capillary rise.

### Modified Young-Laplace equation

The height  $h$  of the capillary rise can be derived by Young-Laplace equation [18]:

$$h = \frac{2\sigma\cos\theta}{\rho gr} \quad (1)$$

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where  $\sigma$  is the liquid-air surface tension,  $g$  – the gravity acceleration,  $r$  – the radius of the capillary tube,  $\theta$  – the contact angle, and  $\rho$  – the density of the solution.

The capillary phenomenon can be explained through the geometric potential theory [19], which assumes that any a boundary can produce a force perpendicular to the surface.

For a sphere surface, the geometric potential,  $E$ , is inversely proportion to the surface radius:

$$E \propto \frac{1}{r} \quad (2)$$

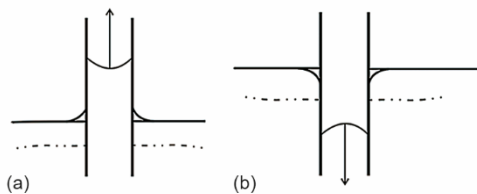
and the boundary-induced force,  $F$ , is:

$$F = -\frac{dE}{dr} \propto \frac{1}{r^2} \quad (3)$$

For a non-sphere surface, eq. (3) can be modified as:

$$F \propto \frac{1}{r^n} \quad (4)$$

where  $n$  is shape factor. For the capillary phenomenon,  $n = 1$ . Equation (4) shows that nanoscale surface can produce a high force as that for the gecko adhesion [20], virulence of virus [21], fractal diffusion-reaction process [22], smart adhesion [23], and adsorption [24]. Using the geometric potential theory [25], Fan *et al.* explained the cell orientation in a nanofiber membrane [22]. Song *et al.* explained the permeability and wetting properties of nanofiber membranes [26]. Mei *et al.* revealed adsorption property of bioretention systems [27]. He and Qian elucidated the fractal diffusion [28].



**Figure 1. Capillary effect in a small tube;**  
**(a) water and (b) mercury**

Figure 1 shows the capillary effect in a small tube. When a small tube is put into either water surface or mercury surface, the geometrical potential produces a force perpendicular to the surface, so water is pulled upward, fig. 1(a). Mercury is pushed downward, fig. 1(b).

Temperature will greatly affect the surface tension, the density of the solution, as a result, the capillary rise will be affected [29-32]. The capillary pressure can be written in the form:

$$P = \frac{2\sigma\cos\theta}{r} \quad (5)$$

where  $P$  is capillary pressure.

Using the relationship:

$$P = \rho gh \quad (6)$$

We can obtain eq. (1) from eq. (5).

Gardner *et al.* [33] and Faybishenko *et al.* [34] suggested that the effect of temperature on capillary pressure could be described by:

$$P = a_p + b_p T \quad (7)$$

where  $a_p$  and  $b_p$  are empirical constants.

Similarly surface tension and contact angle can be written in forms:

$$\sigma = a_\sigma + b_\sigma T \quad (8)$$

$$\theta = a_\theta + b_\theta T \quad (9)$$

where  $a_\sigma$ ,  $b_\sigma$ ,  $a_\theta$ , and  $b_\theta$  are empirical constants. The modified Young-Laplace equation can be written:

$$P = a_p + b_p T = \frac{2(a_\sigma + b_\sigma T) \cos(a_\theta + b_\theta T)}{r} = \rho g h \quad (10)$$

The effect of temperature on capillary rise could be described by:

$$h = \frac{a_p + b_p T}{\rho g} \quad (11)$$

Equation (11) is an empirical one. Alternatively, we can use Boltzmann factor to modify the Young-Laplace equation. As temperature increases, the capillary energy increases, which can be described by the Boltzmann factor [35] in the form:

$$h = \frac{2\sigma \cos \theta}{\rho g r} \exp\left(-\frac{E}{kT}\right) \quad (12)$$

where  $E$  is the capillary energy,  $T$  [K] – the absolute temperature, and  $k$  – the Boltzmann constant, which is a physical constant relating the average kinetic energy of the solution in the capillary tube.

### Experimental verification

In this experiment,  $\text{Na}_2\text{CO}_3$  (Sodium carbonate anhydrous, Aladdin Industrial Corporation, Shanghai, China) was used. A  $\text{Na}_2\text{CO}_3$  solution was prepared using pure water as a solvent with weight ratio  $\text{Na}_2\text{CO}_3 / \text{H}_2\text{O}$  of 3.5 g/45.5 g. The solution was then magnetically stirred at a constant temperature of, respectively, 20 °C, 30 °C, 40 °C, 50 °C, 60 °C, 70 °C, and 80 °C.

Glass capillary with diameter of 1 mm was used to measure the capillary rise for  $\text{Na}_2\text{CO}_3$  solution at different temperatures. The results were listed in tab. 1.

**Table 1. Capillary rise of  $\text{Na}_2\text{CO}_3$  solution and  $\text{H}_2\text{O}$  at different temperature**

$\text{Na}_2\text{CO}_3$		$\text{H}_2\text{O}$	
Temperature, $T$ [K]	Rise height, $h$ [mm]	Temperature, $T$ [K]	Rise height, $h$ [mm]
293.16	15	293.16	20.3
303.16	15.5	303.16	20.6
313.16	15.5	313.16	20.7
323.16	15.4	323.16	21.8
333.16	16.2	333.16	22.0
343.16	16.6	343.16	23.3
353.16	16.8	353.16	24.1

Equation (12) can be written in the form:

$$h = a \exp\left(-\frac{b}{T}\right) \quad (13)$$

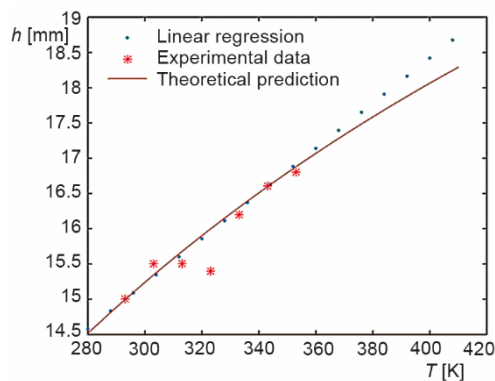
Using the previous data, we obtained  $a = 29.23$ ,  $b = 195.55$  for the  $\text{Na}_2\text{CO}_3$  solution, and  $a = 59.25$ ,  $b = 320.32$  for  $\text{H}_2\text{O}$ .

The linear relevant fitting could be described by:

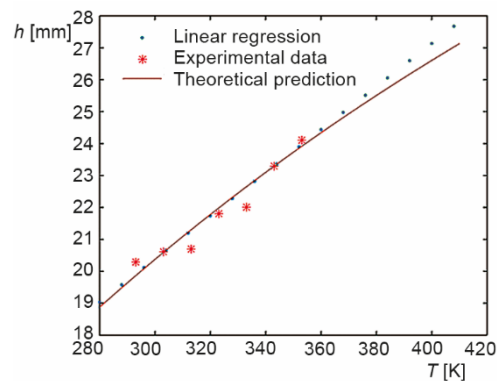
$$h = m + nT \quad (14)$$

where  $m$  and  $n$  could be determined in a similar way as mentioned.

As shown in figs. 2 and 3, with the increase of temperature, the height of the capillary rise gradually increases, and the theoretical calculation value was consistent with the experimental test value.



**Figure 2. Effect of temperature on the capillary rise in the  $\text{Na}_2\text{CO}_3$  solution**



**Figure 3. Effect of temperature on the capillary rise in  $\text{H}_2\text{O}$**

### Effect of temperature on a fabric surface cleanliness, moisture/air permeability

The previous experiment showed that a higher temperature leads to a higher capillary rise, this property can be used for cleaning fabrics at a higher temperature. During a washing process at a high temperature, a high capillary pressure is produced which can pull out the dirt inside the fabric. A higher temperature is also favorable for moisture/air permeability through the fabric, this property can be used to design various kinds of sports dresses. The capillary rise should be larger than the thickness of the fabric, so that sweat can be pulled out through the capillary effect, this makes thermal comfort for sportswear [36, 37].

### Discussion and conclusions

This paper gives an experimental study of the effect of temperature on capillary rise. Temperature affects the surface tension and density of the solution, the geometric potential energy of the liquid surface depends upon temperature, so the temperature will change the boundary-induced force, which will affect the height of the liquid rise in the capillary. Our experiment is important for designing sports wears. Additionally our results are also useful to

design porous bearings [38, 39], and the temperature-dependent capillary effect can be also used for enhanced heat transfer [40, 41]. For unsmooth surface, a fractal modification is needed using the two-scale fractal theory [42-44].

### Authors contributions

Xiao-Xia LI and Ling ZHAO contributed equally.

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