ESTIMATION OF RELIABILITY IN MULTICOMPONENT STRESS-STRENGTH BASED ON EXPONENTIAL FRECHET DISTRIBUTIONS

by

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When strength and stress variables follow the exponential Frechet distribution with different shape parameters and common scale parameters, the multicomponent stress-strength reliability model of an s-out-of-k system is studied in this paper. Based on samples from stress and strength distributions, the maximum likelihood estimation of the model parameters is obtained. The asymptotic confidence interval for the system reliability is also calculated. The comparison of the reliability estimates based on small sample is given by Monte-Carlo simulation.

Key words: *stress-strength model, exponentiated Frechet distribution, maximum likelihood, reliability, confidence intervals*

Introduction

The exponentiated Frechet (EF) distribution as an exponential distribution was proposed by Nadarajah and Kotz [1]. It is an extension of the standard Frechet distribution. The cumulative distribution function (CDF) and probability density function (PDF) of the EF distribution be given, respectively:

$$F(x;\lambda,\sigma,\theta)=1-\left\{1-\exp\left[-\left(\frac{\sigma}{x}\right)^{\lambda}\right]\right\}^{\theta},\quad x>0 \tag{1}$$

and

$$f(x;\lambda,\sigma,\theta)=\theta\lambda\sigma^{\lambda}x^{-(1+\lambda)}\left\{1-\exp\left[-\left(\frac{\sigma}{x}\right)^{\lambda}\right]\right\}^{\theta-1}\exp\left[-\left(\frac{\sigma}{x}\right)^{\lambda}\right],\quad x>0 \tag{2}$$

where $\lambda>0$ is the shape parameter, $\sigma>0$ and $\theta>0$ are scale parameters. This distribution is denoted as EF (λ,σ,θ).

Many applications of the EF distribution were discussed [2]. For example, temperature in thermal science, queues in supermarkets, sea current, wind speed, network design, synthetic membranes, earthquakes and financial matters. The moment of order statistics was calculated for independent non-identically distributed EF random variables by Jamjoom and Al-

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Saiary [3]. The acceptance sampling plan based on censoring life tests for EF distribution was discussed by Al-Nasser and Al-Omari [4]. When the life time of an item follows EF distribution, a two-stage group acceptance sampling plan was proposed for life tests under censoring data by Gadde *et al.* [5]. The basic theory and method of multicomponent stress-strength model were given by Bhattacharyya *et al.* [6]. In this paper, the reliability estimation of multicomponent stress-intensity model is considered when both strength and stress variables follow EF distributions.

For the stress-strength model, the estimation of the reliability $R = P(Y < X)$ of the components or systems is the basic problem in reliability statistics theory. It is a measure of component reliability when the random strength X is limited by random stress Y. For a multicomponent system, the random strengths X_1, X_2, , X_k of k components are independently and identically distributed random variables. In this paper, it is assumed that all components of the system are subjected to a common random stress Y, and its CDF is $G(y)$. The system is indicated survival only if at least s out of $k(1 < s < k)$ strengths exceed the stress. Let the random variables $Y, X_1, X_2, ..., X_k$ be independent and the common CDF of $X_1, X_2, ... ,X_k$ be $F(x)$. The reliability of a multicomponent stress-strength model can be obtained eq. (3) given in [5]:

$$R_{S,K} = \sum_{i=s}^{k}\binom{k}{l}\int_{-\infty}^{+\infty}[1-F(y)]^i[F(y)]^{k-i}\,\mathrm{d}G(y) \tag{3}$$

The model has been widely applied to reliability problems in many practical applications. Various researches have been done on this model under different probability distributions of stress and strength. Norman *et al.* [7] studied Bayesian estimation of reliability of multicomponent stress-strength model with strength and stress being independence and exponential distribution. Pandey and Borhan [8] studied the estimation of the reliability when the stress-strength variables obey the Burr distribution. Kizilaslan and Nadar [9] discussed classical and Bayesian estimations of model reliability when the stress-strength variables follow the Weibull distribution. Based on progressively censored sample, reliability estimation of this model was discussed under Kumaraswamy distribution and unit Gompertz distribution in [10, 11]. For more alternative stress and strength distributions and incomplete sample cases, the reliability analysis of the model can be found in [12-19].

The main attempt of this paper is to obtain estimates of $R_{s,k}$ when the stress and strength are independent and both obey the EF distribution.

Maximum likelihood estimator of $R_{s,k}$

Let $X \sim \mathrm{EF}(\lambda,\sigma,\alpha)$ and $Y \sim \mathrm{EF}(\lambda,\sigma,\beta)$, the parameters α,β,λ and σ are unknown. Here, X and Y are independent random variables. By using eq. (3), the reliability of multicomponent stress-strength model for EF distribution is expressed:

$$R_{s,k} = \sum_{i=s}^{k}\binom{k}{i}\int_0^{\infty}[1-F(x)]^i[F(x)]^{k-i}\,\mathrm{d}G(x) = \sum_{i=s}^{k}\binom{k}{i}\int_0^{\infty}\left\{1-\exp\left[-\left(\tfrac{\sigma}{x}\right)^{\lambda}\right]\right\}^{\alpha i}\cdot$$

$$\cdot\left(1-\left\{1-\exp\left[-\left(\tfrac{\sigma}{x}\right)^{\lambda}\right]\right\}^{\alpha}\right)^{k-i}\beta\lambda\sigma^{\lambda}x^{-\lambda-1}\left\{1-\exp\left[-\left(\tfrac{\sigma}{x}\right)^{\lambda}\right]\right\}^{\beta-1}\exp\left[-\left(\tfrac{\sigma}{x}\right)^{\lambda}\right]\mathrm{d}x \tag{4}$$

Let $t=\{1-\exp[-(\sigma/x)^{\lambda}]\}^{\beta}$, the eq. (4) is reduced to the following form:

$$R_{s,k}=\sum_{i=s}^{k}\binom{k}{i}\int_0^1 (t^{\alpha/\beta})^1(1-t^{\alpha/\beta})^{k-i}\,\mathrm{d}t$$

Letting $u=t^{\alpha/\beta}$, then:

$$R_{s,k}=\nu\sum_{i=s}^{k}\binom{k}{i}\int_0^1 u^{i+\nu-1}(1-u)^{k-i}\,\mathrm{d}u=\nu\sum_{i=s}^{k}\binom{k}{i}B(\nu+i,k-i+1)$$

where $\nu=\beta/\alpha$ and $B(a,b)$ is the beta function. Since *k* and *i* are integers, we get:

$$R_{s,k}=\nu\sum_{i=s}^{k}\frac{k!}{i!}\left[\prod\nolimits_{j=i}^{k}(\nu+j)\right]^{-1} \tag{5}$$

where $R_{s,k}$ is the reliability of the multicomponent stress-strength model. The α,β,λ, σ and $R_{s,k}$ need to be estimated. In this paper, the model parameters are estimated by ML method, and an estimate of $R_{s,k}$ is obtained using eq. (5). The estimation method is explained.

In order to get MLE of $R_{s,k}$, we first get the MSE of α and β. Suppose $(X_{1j},X_{2j},\cdots,X_{nj})$ is a strength sample from $X_j\sim EF(\lambda,\sigma,\alpha)$, $j=1,2,\cdots,k$, and $(Y_1,Y_2,\cdots,Y_n)$ is a stress sample from $Y\sim EF(\lambda,\sigma,\beta)$. The likelihood function based on the samples is that:

$$L(\alpha,\beta,\lambda,\sigma\,|\,\text{data})=\prod_{i=1}^{n}\left[\prod_{j=1}^{k}f(x_{ij})\right]g(y_i)=\sigma^{(k+1)n\lambda}\lambda^{(k+1)n}\alpha^{kn}\beta^{n}\cdot$$

$$\cdot\left(\prod_{i=1}^{n}\prod_{j=1}^{k}x_{ij}^{-(\lambda+1)}\left\{1-\exp\left[-\left(\frac{\sigma}{x_{ij}}\right)^{\lambda}\right]\right\}^{\alpha-1}\right)\exp\left[-\sum_{i=1}^{n}\sum_{j=1}^{k}\left(\frac{\sigma}{x_{ij}}\right)^{\lambda}\right]\left[\prod_{i=1}^{n}y_i^{-(\lambda+1)}\right]\cdot$$

$$\cdot\left(\prod_{i=1}^{n}\left\{1-\exp\left[-\left(\frac{\sigma}{y_i}\right)^{\lambda}\right]\right\}^{\beta-1}\right)\exp\left[-\sum_{i=1}^{n}\left(\frac{\sigma}{y_i}\right)^{\lambda}\right] \tag{6}$$

The log-likelihood function is that:

$$l(\alpha,\beta,\lambda,\sigma\,|\,\text{data})=(k+1)n\lambda\ln\sigma+(k+1)n\ln\lambda+kn\ln\alpha+n\ln\beta-$$

$$-(\lambda+1)\left(\sum_{i=1}^{n}\sum_{j=1}^{k}\ln x_{ij}+\sum_{i=1}^{n}\ln y_i\right)+(\alpha-1)\sum_{i=1}^{n}\sum_{j=1}^{k}\ln\left\{1-\exp\left[-\left(\frac{\sigma}{x_{ij}}\right)^{\lambda}\right]\right\}-$$

$$-\sum_{i=1}^{n}\sum_{j=1}^{k}\left(\frac{\sigma}{y_i}\right)^{\lambda}+(\beta-1)\sum_{i=1}^{n}\ln\left\{1-\exp\left[-\left(\frac{\sigma}{y_i}\right)^{\lambda}\right]\right\}-\sum_{i=1}^{n}\left(\frac{\sigma}{y_i}\right)^{\lambda} \tag{7}$$

The MLE of the model parameters are denoted as $\hat{\alpha},\hat{\beta},\hat{\lambda},$ and $\hat{\sigma}$, respectively. By differentiating (7) with respect to $\alpha,\beta,\lambda,$ and σ, then $\hat{\alpha},\hat{\beta},\hat{\lambda},$ and $\hat{\sigma}$ can be obtained from the following likelihood equations:

$$\frac{\partial l}{\partial \alpha}=\frac{kn}{\alpha}+\sum_{i=1}^{n}\sum_{j=1}^{n}\ln\left\{1-\exp\left[-\left(\frac{\sigma}{x_{ij}}\right)^{\lambda}\right]\right\}=0 \tag{8}$$

$$\frac{\partial l}{\partial \beta}=\frac{n}{\beta}+\sum_{i=1}^{n}\ln\left\{1-\exp\left[-\left(\frac{\sigma}{y_{i}}\right)^{\lambda}\right]\right\}=0 \tag{9}$$

$$\frac{\partial l}{\partial \sigma}=\frac{(k+1)n\lambda}{\sigma}-\lambda\sigma^{\lambda-1}\left(\sum_{i=1}^{n}\sum_{j=1}^{k}\frac{1}{x_{ij}^{\lambda}}+\sum_{i=1}^{n}\frac{1}{y_{i}^{\lambda}}\right)+(\alpha-1)\lambda\sigma^{\lambda-1}\sum_{i=1}^{n}\sum_{j=1}^{k}\frac{\exp\left[-\left(\frac{\sigma}{x_{ij}}\right)^{\lambda}\right]}{x_{ij}^{\lambda}\left\{1-\exp\left[-\left(\frac{\sigma}{x_{ij}}\right)^{\lambda}\right]\right\}}+$$

$$+(\beta-1)\lambda\sigma^{\lambda-1}\sum_{i=1}^{n}\frac{\exp\left[-\left(\frac{\sigma}{y_{i}}\right)^{\lambda}\right]}{y_{i}^{\lambda}\left\{1-\exp\left[-\left(\frac{\sigma}{y_{i}}\right)^{\lambda}\right]\right\}} \tag{10}$$

$$\frac{\partial l}{\partial \lambda}=\frac{(k+1)n}{\lambda}+\left(k+1\right)n\ln\sigma-\sum_{i=1}^{n}\sum_{j=1}^{k}\ln x_{ij}-\sum_{i=1}^{n}\ln y_{i}-\sum_{i=1}^{n}\sum_{j=1}^{k}\left(\frac{\sigma}{x_{ij}}\right)^{\lambda}\ln\left(\frac{\sigma}{x_{ij}}\right)-$$

$$-\sum_{i=1}^{n}\left(\frac{\sigma}{y_{i}}\right)^{\lambda}\ln\frac{\sigma}{y_{i}}+\sigma^{\lambda}(\alpha-1)\sum_{i=1}^{n}\sum_{j=1}^{k}\frac{\ln\left(\frac{\sigma}{x_{ij}}\right)}{x_{ij}^{\lambda}\left\{1-\exp\left[-\left(\frac{\sigma}{x_{ij}}\right)^{\lambda}\right]\right\}\exp\left(\frac{\sigma}{x_{ij}}\right)^{\lambda}}+$$

$$+\sigma^{\lambda}(\beta-1)\sum_{i=1}^{n}\frac{\ln\left(\frac{\sigma}{x_{ij}}\right)}{y_{i}^{\lambda}\left\{1-\exp\left[-\left(\frac{\sigma}{y_{i}}\right)^{\lambda}\right]\right\}\exp\left(\frac{\sigma}{y_{i}}\right)^{\lambda}} \tag{11}$$

From $\hat{\alpha}$ and $\hat{\beta}$, we obtain the MLE of $R_{s,k}$:

$$\hat{R}_{s,k}=\hat{\nu}\sum_{i=s}^{k}\frac{k!}{i!}\left[\prod_{j=1}^{k}(\hat{\nu}+j)\right]^{-1} \tag{12}$$

where $\hat{\nu}=\hat{\beta}/\hat{\alpha}$.

Considering the asymptotical confidence interval for $\hat{R}_{s,k}$, we calculate firstly asymptotic variances of $\hat{\alpha}$ and $\hat{\beta}$ which are given, respectively:

$$Var(\hat{\alpha})=\left[E\left(-\frac{\partial^2 l}{\partial\alpha^2}\right)\right]^{-1}=\frac{\alpha^2}{kn} \text{ and } Var(\hat{\beta})=\left[E\left(-\frac{\partial^2 l}{\partial\beta^2}\right)\right]^{-1}=\frac{\beta^2}{n}$$

Because $\hat{R}_{s,k}$ is a function of $\hat{\alpha}$ and $\hat{\beta}$, by using the asymptotic normality theorem and delta method, we can get the asymptotic variance of $\hat{R}_{s,k}$.

$$Var(\hat{R}_{s,k})=Var(\hat{\alpha})\left(\frac{\partial R_{s,k}}{\partial\alpha}\right)^2+Var(\hat{\beta})\left(\frac{\partial R_{s,k}}{\partial\beta}\right)^2 \tag{13}$$

Hence, form eq. (13), the asymptotic variance (AV) can be obtained.

To obtain derivatives of $\hat{R}_{s,k}$ for (s, k) = (1, 4) and (2, 5), we have:

$$\hat{R}_{1,4}=\frac{\hat{v}^4+10\hat{v}^3+35\hat{v}^2+50\hat{v}}{\hat{v}^4+10\hat{v}^3+35\hat{v}^2+50\hat{v}+24} \quad \text{and} \quad \hat{R}_{2,5}=\frac{\hat{v}^4+14\hat{v}^3+71\hat{v}^2+154\hat{v}}{\hat{v}^4+14\hat{v}^3+71\hat{v}^2+154\hat{v}+120}$$

Therefore:

$$\frac{\partial\hat{R}_{1,4}}{\partial\alpha}=\frac{-48\hat{v}(2\hat{v}^3+15\hat{v}^2+35\hat{v}+25)}{\alpha\left[(1+\hat{v})(2+\hat{v})(3+\hat{v})(4+\hat{v})\right]^2} \text{ and } \frac{\partial\hat{R}_{1,4}}{\partial\beta}=\frac{48(2\hat{v}^3+15\hat{v}^2+35\hat{v}+25)}{\alpha\left[(1+\hat{v})(2+\hat{v})(3+\hat{v})(4+\hat{v})\right]^2}.$$

$$\frac{\partial\hat{R}_{2,5}}{\partial\alpha}=\frac{-240\hat{v}(2\hat{v}^3+21\hat{v}^2+71\hat{v}+77)}{\alpha\left[(2+\hat{v})(3+\hat{v})(4+\hat{v})(5+\hat{v})\right]^2} \text{ and } \frac{\partial\hat{R}_{2,5}}{\partial\beta}=\frac{240(2\hat{v}^3+21\hat{v}^2+71\hat{v}+77)}{\alpha\left[(2+\hat{v})(3+\hat{v})(4+\hat{v})(5+\hat{v})\right]^2}.$$

Therefore, according to eq. (13), we obtain:

$$AV(\hat{R}_{1,4})=\frac{48^2\hat{v}^2(2\hat{v}^3+15\hat{v}^2+35\hat{v}+25)^2}{\left[(1+\hat{v})(2+\hat{v})(3+\hat{v})(4+\hat{v})\right]^4}\left(\frac{1}{kn}+\frac{1}{n}\right)$$

$$AV(\hat{R}_{2,5})=\frac{240^2\hat{v}^2(2\hat{v}^3+21\hat{v}^2+71\hat{v}+77)^2}{\left[(2+\hat{v})(3+\hat{v})(4+\hat{v})(5+\hat{v})\right]^4}\left(\frac{1}{kn}+\frac{1}{n}\right).$$

As

$$n\to\infty, \frac{\hat{R}_{s,k}-R_{s,k}}{\sqrt{AV(\hat{R}_{s,k})}}\to N(0,1),$$

a $100(1-r)$ confidence interval for $R_{s,k}$ is:

$$R_{s,k}\mp z_{r/2}\sqrt{AV(\hat{R}_{s,k})},$$

where $z_{r/2}$ is the upper $r/2^{\text{th}}$ percentile of standard normal distribution.

The 95% asymptotic confidence interval of $R_{s,k}$ can be obtained:

$$\left[\hat{R}_{s,k}-1.96\sqrt{AV(\hat{R}_{s,k})},\ \hat{R}_{s,k}+1.96\sqrt{AV(\hat{R}_{s,k})}\right]$$

The asymptotic 95% confidence interval for $R_{1,4}$ is:

$$\hat{R}_{1,4}\mp 1.96\frac{48\hat{v}(2\hat{v}^3+15\hat{v}^2+35\hat{v}+25)}{\left[(1+\hat{v})(2+\hat{v})(3+\hat{v})(4+\hat{v})\right]^2}\sqrt{\frac{1}{kn}+\frac{1}{n}}$$

The 95% asymptotic confidence interval for $R_{2,5}$ is:

$$\hat{R}_{2,5} \mp 1.96 \frac{240\hat{v}(2\hat{v}^3 + 21\hat{v}^2 + 71\hat{v} + 77)}{\left[(2+\hat{v})(3+\hat{v})(4+\hat{v})(5+\hat{v})\right]^2} \sqrt{\frac{1}{kn} + \frac{1}{n}}$$

Simulation study

In this section, using the R software, Monte-Carlo simulation method is used to compare the biases and mean square error under different sample sizes. Suppose $\hat{R}_{(s,k)i}$ is the i^{th} estimation of $R_{s,k}$ based on simulated dataset, the bias and MSE for

$$\hat{R}_{s,k} = \frac{1}{N}\sum_{i=1}^{N} \hat{R}_{(s,k)i}$$

are that:

$$\text{Bias} = \frac{1}{N}\sum_{i=1}^{N} |\hat{R}_{(s,k)i} - \hat{R}_{s,k}| \ \text{ and } \ \text{MSE} = \frac{1}{N}\sum_{i=1}^{N} (\hat{R}_{(s,k)i} - \hat{R}_{s,k})^2$$

All results are obtained through N = 800 Monte-Carlo simulations. In simulation settings, different sample sizes n are 50, 60, 80, and 110, respectively, and different parameter $(\sigma, \lambda, \alpha, \beta)$ are (0.8, 1.5, 2, 3), (0.8, 1.5, 2, 2.5) and (0.8, 1.5, 3, 2), respectively, Under the three cases of $(\sigma, \lambda, \alpha, \beta)$ when (s,k) = (1, 4), $\hat{R}_{s,k}$ is 0.88918, 0.85291, and 0.68442, respectively. When (s,k) = (2, 5), $\hat{R}_{s,k}$ is 0.78688, 0.73523, and 0.53591, respectively. The results are given in tab. 1.

Table 1. Bias, MSE and asymptotic confidence of $\hat{R}_{s,k}$ under different cases

$(\sigma, \lambda, \alpha, \beta)$	(s, k)	$R_{s,k}$		Bias	MSE	Asymptotic confidence
(0.8, 1.5, 2, 3)	(1, 4)	0.88918	50	0.03988	0.00219	(0.87395, 0.96431)
			60	0.03679	0.00186	(0.88860, 0.96526)
			80	0.03867	0.00180	(0.88778, 0.95710)
			110	0.03888	0.00178	(0.88116, 0.94522)
	(2, 5)	0.78688	50	0.06290	0.00557	(0.75422, 0.90108)
			60	0.06183	0.00535	(0.72071, 0.86772)
			80	0.06359	0.00496	(0.69236, 0.82877)
			110	0.06202	0.00461	(0.75180, 0.85759)
(0.8, 1.5, 2, 2.5)	(1, 4)	0.85291	50	0.02864	0.00198	(0.79292, 0.92406)
			60	0.02667	0.00164	(0.79075, 0.91364)
			80	0.02850	0.00150	(0.83422, 0.92733)
			110	0.02931	0.00135	(0.85209, 0.92751)
	(2, 5)	0.73523	50	0.04203	0.00414	(0.68797, 0.85697)
			60	0.04031	0.00348	(0.64112, 0.80745)
			80	0.04115	0.00297	(0.70898, 0.84166)

⟶

Table 1. Continuation

$(\sigma,\lambda,\alpha,\beta)$	(s, k)	$R_{s,k}$		Bias	MSE	Asymptotic confidence
			110	0.04311	0.00279	(0.66291, 0.78573)
(0.8, 1.5, 3, 2)	(1, 4)	0.68442	50	–0.07007	0.00734	(0.54757, 0.74314)
			60	–0.07002	0.00717	(0.54841, 0.72753)
			80	–0.06920	0.00643	(0.54233, 0.68876)
			110	–0.06801	0.00547	(0.57930, 0.71116)
	(2, 5)	0.53591	50	–0.06434	0.00625	(0.40091, 0.57443)
			60	–0.06389	0.00615	(0.42739, 0.60391)
			80	–0.06720	0.00595	(0.42852, 0.58045)
			110	–0.06860	0.00576	(0.42087, 0.57875)

Conclusion

In this paper, the reliability of multicomponent stress-strength model is studied when the stress and strength variables obey the exponentiated Frechet distribution. We have obtained MSE of model parameters and estimated asymptotic confidence interval for reliability under the multicomponent stress-strength model. It is easy to see from the tab. 1 that the variance of reliability estimator decreases as expected as the sample size increases. The results show that the proposed method can be applied to practical reliability problems in the fields of thermal science [20], micro-electromechanical systems [21, 22], rotors [23, 24], vibration systems [25, 26], and the milk process [27].

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