MAGIC RAMIE ROPE FOR THE TUG-OF-WAR GAME

by

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The tug of war is a sport known for strength, however a weaker team can also win the game by a suitable team co-operation. A mathematical model is established, showing that the team co-operation or rhythmical frequency plays an important role in victory. A team can win even the rope is pulled to the opposite direction depending upon the rhythmical frequency. A criterion for rhythmical frequency is obtain to guarantee victory when the strength is almost same for both teams. Additionally the rope pulling can be also used for moving a heavy weight object, the principle might be used for building the great pyramids in ancient Egypt. Finally magic ramie ropes with special thermoplastic properties and controllable frequency are discussed.

Key words: *non-linear vibration, forced oscillation, ramie rope, humidity, thermoplastic property*

Introduction

As one of the longest, strongest and oldest natural fibers, ramie is an important material for textile manufacturing [1, 2], and it is widely used for fabrication of ropes for the tug-of-war sport.

The tug of war is a famous sport of a rope [3], each team pulls on an end of the rope, with a goal being to pull the rope to a fixed distance against the opposing team. The Olympic Games once included the sport from 1900 to 1920, but it is still a famous part of sports in various games. It was generally considered as a sport of strength, but the rhythmical co-operation of team members plays an equally important role in victory, if not more than their physical strength. In this paper we will reveal the secret of success in a tug of war using the vibration theory. A vibration phenomenon arising in everywhere from nanofiber fabrication [4], nano/micro-electromechanical system [5-7] to architectural engineering [8], but the vibration of the pulling rope has never been studied before.

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Mathematical model for tug-of-war

The most ropes used in the tug of war are made of ramie, which is one of strongest natural fibers commercialized widely in China and often called as China grass [9]. The strengthening and stiffening of ramie ropes can be adjusted by a cyclic load treatment [10]. The rope vibration property is important in a winding hoist [11] and suspension bridges, but no attention was paid so far on the tug-of-war sport.

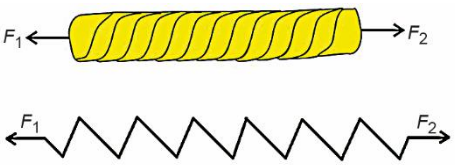

Figure 1. A spring model for the tug of war

We consider the rope as a linear spring as illustrated in fig. 1. Two teams pull oppositely on two ends with forces F_1 and F_2.

We assume F_1 and F_2 can be expressed, respectively:

$$F_1 = a_1 + b_1 \sin(\omega_1 t + \varphi_1) \tag{1}$$

$$F_2 = a_2 + b_2 \sin(\omega_2 t + \varphi_2) \tag{2}$$

where a_1 and a_2 are average forces of two teams pulling on the rope, respectively, ω_1 and ω_2 – the rhythmical frequencies of the two teams, respectively, and b_1 and b_2 are amplitudes. The two teams have a phase difference $\varphi_1 - \varphi_2$. Assuming that the rope has an elastic coefficient of k, we have the following mathematical model for the tug of war:

$$m\ddot{x}(t) + kx = F_1 - F_2 \tag{3}$$

The forced oscillator of the rope can be written:

$$\ddot{x}(t) + \omega_0^2 x = \frac{a_1 - a_2}{m} + \frac{b_1}{m}\sin(\omega_1 t + \varphi_1) - \frac{b_2}{m}\sin(\omega_2 t + \varphi_2) \tag{4}$$

where ω_0 is the frequency of the rope:

$$\omega_0 = \sqrt{\frac{k}{m}} \tag{5}$$

The general solution of eq. (4) is:

$$x = A\sin(\omega_0 t + \varphi_0) + \frac{a_1 - a_2}{\omega_0^2 m} + \frac{b_1}{m(\omega_0^2 - \omega_1^2)}\sin(\omega_1 t + \varphi_1) - \frac{b_2}{m(\omega_0^2 - \omega_2^2)}\sin(\omega_2 t + \varphi_2) \tag{6}$$

where A and φ_0 can be determined by the initial conditions.

We assume that $\varphi_1 = 0$ for simplicity, eq. (6) becomes:

$$x = A\sin(\omega_0 t + \varphi_0) + \frac{a_1 - a_2}{\omega_0^2 m} + \frac{b_1}{m(\omega_0^2 - \omega_1^2)}\sin(\omega_1 t) - \frac{b_2}{m(\omega_0^2 - \omega_2^2)}\sin(\omega_2 t + \Delta\varphi) \tag{7}$$

For the team 1, it would win over the team 2 at time $t = T_1/2$, where $T_1 = 2\pi/\omega_1$, when:

$$x\frac{T_1}{2} = A\sin\left(\omega_0 \frac{T_1}{2} + \varphi_0\right) + \frac{a_1 - a_2}{\omega_0^2 m} +$$
$$+ \frac{b_1}{m(\omega_0^2 - \omega_1^2)}\sin\left(\omega_1 \frac{T_1}{2}\right) - \frac{b_2}{m(\omega_0^2 - \omega_2^2)}\sin\left(\omega_2 \frac{T_1}{2} + \Delta\varphi\right) > L \tag{8}$$

where L is the distance for the victory pulling one direction against the opposite team.

This requires:

$$\frac{b_1}{m(\omega_0^2-\omega_1^2)}\sin\left(\omega_1\frac{T_1}{2}\right)>0 \text{ for } 0\le t\le\frac{T_1}{2} \tag{9}$$

and

$$\frac{b_2}{m(\omega_0^2-\omega_2^2)}\sin\left(\omega_2\frac{T_1}{2}+\Delta\varphi\right)<0 \quad 0\le t\le\frac{T_1}{2} \tag{10}$$

Equation (10) can not be always satisfied during $0\le t\le T_1/2,$ we have the following criterion:

$$\omega_2\frac{T_1}{2}+\Delta\varphi>\frac{T_2}{2} \tag{11}$$

Now we consider a special case that:

$$\Delta\varphi=\pi \tag{12}$$

Equation (8) becomes:

$$x\frac{T_1}{2}=A\sin\left(\omega_0\frac{T_1}{2}+\phi_0\right)+\frac{a_1-a_2}{\omega_0^2 m}+$$

$$+\frac{b_1}{m(\omega_0^2-\omega_1^2)}\sin\left(\omega_1\frac{T_1}{2}\right)+\frac{b_2}{m(\omega_0^2-\omega_2^2)}\sin\left(\omega_1\frac{T_1}{2}\right)>L \tag{13}$$

Another criterion for victory of the team 1 is the denominator of the coefficient of $\sin(\omega_1 t)$ should be as small as possible, and:

$$\omega_0-\omega_1<\omega_0-\omega_2 \tag{14}$$

That means:

$$\frac{\omega_1}{\omega_2}>1 \tag{15}$$

Equation (15) reveals that a higher rhythmical frequency plays an extremely important role in victory.

Figure 2 shows the effect of frequencies on the tug of war, the red lines are the fixed distances for success. Two terms have the same strength, and the team with a rhythmical frequency closer to the rope frequency will win the game. Figures 3 and 4 show that a weaker team has the possibility to win over the stronger one if its frequency is closer to the rope frequency.

Rope pulling for moving a heavy object

We consider a special case when b_2 = 0 and $a_2 = \mu W$, eq. (7) becomes:

$$x=A\sin(\omega_0 t+\varphi_0)+\frac{a_1-\mu W}{\omega_0^2 m}+\frac{b_1}{m(\omega_0^2-\omega_1^2)}\sin(\omega_1 t) \tag{16}$$

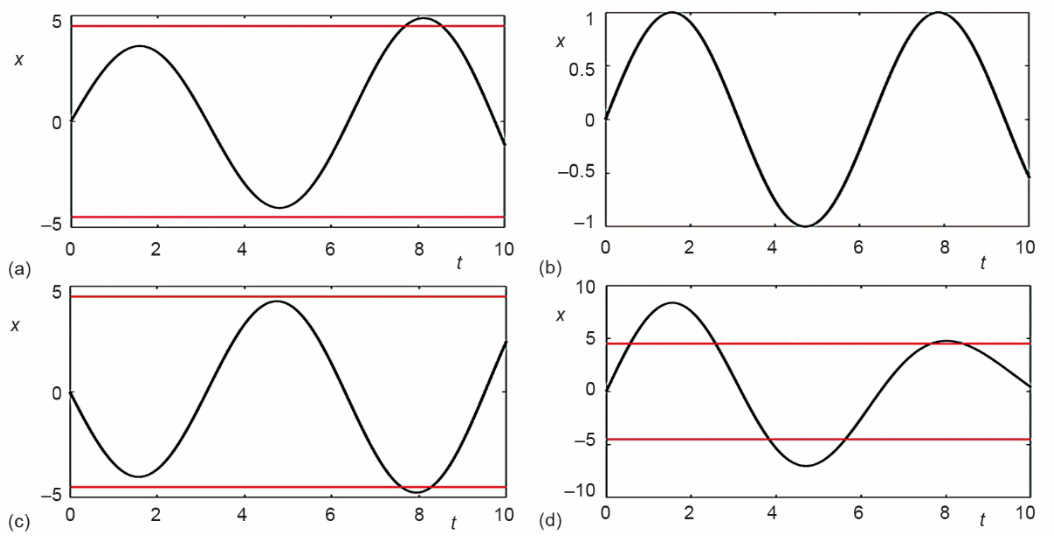

Figure 2. Effect of the rhythmical frequency on the tug of war for the case $A = 1$, $\varphi_0 = \varphi_1 = \varphi_2 = 0$, $a_1 - a_2 = 0$, $b_1 = b_2 = 1$, $\omega_0 = 1$, $\omega_1 = 0.9$; (a) $\omega_2 = 0.8 < \omega_1$, team 1 wins, (b) $\omega_2 = 0.9 = \omega_1$, a no-win game, *(c)* $\omega_2 = 0.95 > \omega_1$, team 2 wins, and (d) $\omega_2 = 1.2 > \omega_1$, team 1 wins

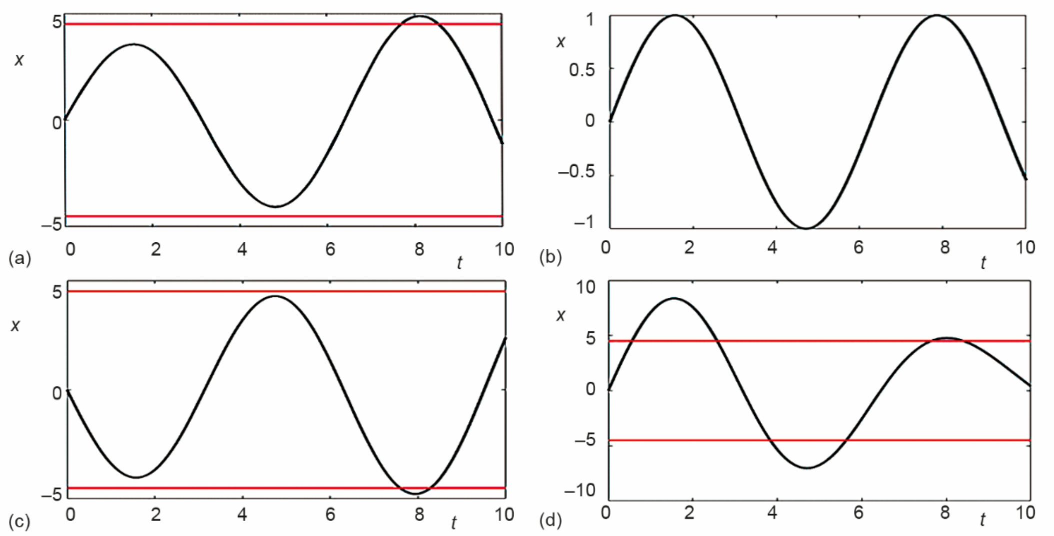

Figure 3. A weaker team vs a stronger team for $A = 1$, $\varphi_0 = \varphi_1 = \varphi_2 = 0$, $a_1 = 1$, $a_2 = 1.2$, $b_1 = b_2 = 1$, $\omega_0 = 1$, $\omega_1 = 0.9$; (a) $\omega_2 = 0.8 < \omega_1$, team 1 wins, (b) $\omega_2 = 0.9 = \omega_1$, a no-win game, (c) $\omega_2 = 0.95 > \omega_1$, team 2 wins, and (d) $\omega_2 = 1.2 > \omega_1$, team 1 wins

This special case can depict the rope pulling in ancient time for moving a heavy object as that for building the great pyramids in ancient Egypt. By suitable fabrication of a rope with given frequency, a stone can be easily moved according to eq. (16), where μW is the friction between the heavy object with the ground. When:

$$\frac{b_1}{\omega_0^2 - \omega_1^2} > \frac{\mu W}{\omega_0^2}$$

the heavy object can be moved forward.

Non-linear vibration

In this paper we consider the rope as a linear spring, actually it is non-linear, eq. (3) can be modified:

$$m\ddot{x}(t) + kx + \varepsilon x^3 = F_1 - F_2 \tag{17}$$

or

$$\ddot{x}(t) + \omega_0^2 x + \frac{\varepsilon}{m} x^3 = \frac{a_1 - a_2}{m} + \frac{b_1}{m}\sin(\omega_1 t + \varphi_1) - \frac{b_2}{m}\sin(\omega_2 t + \varphi_2) \tag{18}$$

Equation (18) is the well-known Duffing oscillator [12, 13], and it can be linearized:

$$\ddot{x}(t) + \Omega_0^2 x = \frac{a_1 - a_2}{m} + \frac{b_1}{m}\sin(\omega_1 t + \varphi_1) - \frac{b_2}{m}\sin(\omega_2 t + \varphi_2) \tag{19}$$

where the linearized frequency can be expressed by:

$$\Omega_0 = \sqrt{\omega_0^2 + \frac{3}{4}\frac{\varepsilon}{m} A^2} \tag{20}$$

Equation (20) can be obtained by the variational iteration method [14, 15], the variational approach [16-18], the homotopy perturbation method [19, 20], or He's frequency formulation [21-26].

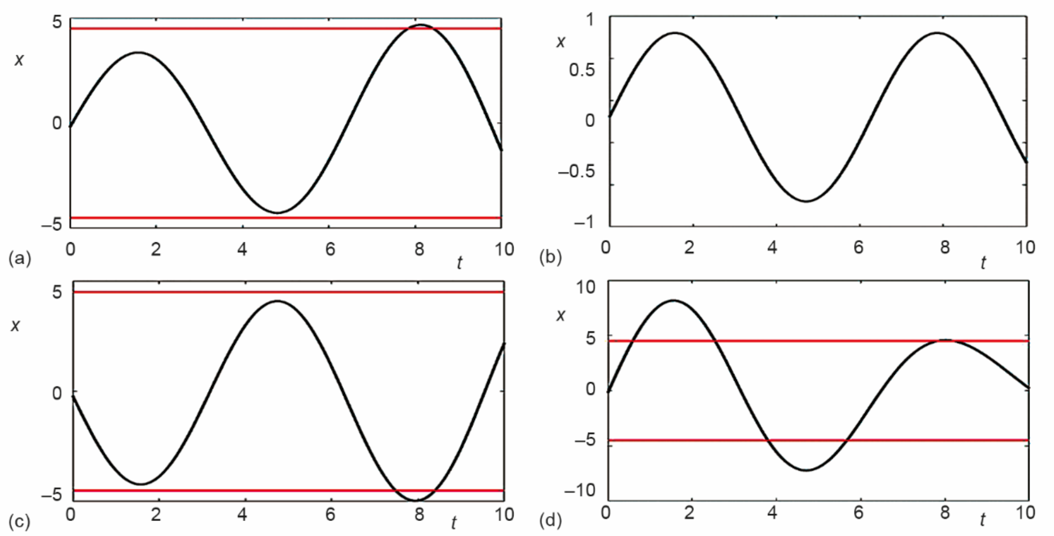

Figure 4. Two teams with different strength and different frequencies $\varphi_0 = \varphi_1 = \varphi_2 = 0$, $a_1 = 1.2$, $a_2 = 1$, $b_1 = b_2 = 1$, $\omega_0 = 1$, $\omega_1 = 0.9$; (a) $\omega_2 = 0.8 < \omega_1$, team 1 wins, (b) $\omega_2 = 0.9 = \omega_1$, a no-win game, (c) $\omega_2 = 0.95 > \omega_1$, team 2 wins, and (d) $\omega_2 = 1.2 > \omega_1$, team 1 wins

Discussion and conclusions

For the term 1 to win over the term 2, it is important the rhythmical frequency should be as close as possible to the rope frequency ω_0.

The rope frequency is an important factor in the tug of war, Chen, *et al.* [27] found that the natural frequency and the vibration amplitude are greatly affected by the ramie fiber content [27]. The thermoplastic properties of the ramie ropes can be adjusted by humidity, temperature and nanofibers content involved in the ropes. Nanotechnology enables the ramie rope to change its natural frequency easily. Carbon nanotube bundle [28] and nanofibers [29, 30] involved in the rope can greatly adjust its vibration property, and now nanofibers can be fabricated by the electrospinning [31-36]. Nanotechnology makes it possible to fabricate a magic rope for rope pulling games. The humidity of the rope will greatly affect its elastic property [37] and its natural frequency as well.

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