BAYESIAN INFERENCE FOR A SIMPLE STEP-STRESS ACCELERATED DEPENDENT COMPETING FAILURE MODEL

by

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The Copula approach can be used to describe the dependence structure between variables. In this paper, by using a Bivariate Clayton copula, we discuss the statistical analysis of a simple step-stress accelerated dependent competing failure model under progressively Type-II censoring sample. With the assumption of cumulative exposure, the Bayesian estimations of the model parameters are derived. Based on Monte-Carlo simulation, the precision of the estimates is assessed. Finally, the statistical analysis of an actual data set of a solar lighting insulation system has been presented for illustrative purposes.

Key words: *step-stress accelerated life test, dependent competing risk model, cumulative exposure model, bivariate clayton copula, Bayesian estimations*

Introduction

High reliability products are often expected to survive for a long time, and it is difficult to assess reliability during the normal working condition. One effective solution to this problem is the accelerated life testing (ALT). Stress can be applied in many ways such as constant-stress, step-stress and progressive-stress [1]. The step-stress accelerated life testing (S-SALT) is a special type of ALT which has advantage in yielding more failure data in a limited testing time and changing the stress level at a prefixed time or a prefixed number of failures during the testing. To analyze the modeling data from S-SALT, one requires a model relating the failure lifetimes under different stress levels, such as the cumulative exposure model (CEM). The CEM is the most studied one in the literature, which was first introduced by Sedyakin [2]. The S-SALT under the assumption of CEM has attracted great attention, see for examples, Balakrishnan and Han [3], Sun and Shi [4], Kohl and Kateri [5], Ramzan *et al.* [6], Zheng [7], and Liu *et al.* [8]. For the complexity of the internal structure and the external working environment, the products failure may be caused by one of failure modes, so the competing failure model is a common model in reliability research, such as Balakrishnan and Han [9], Liu and Shi [10], Zhang *et al.* [11], and Varhgese and Vaidyanathan [12]. In most of the studies of S-SALT with competing failures are mainly based on the assumption that the competing failure modes are independent. However, the failure models are usually dependent in practice. Therefore, the dependent competing failures model in S-SALT has become increasingly popular. Copula [13] is one of the popular models to release the restriction that the joint distribution is constructed from the same family of marginal distribution. However, only

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few literature has applied copula functions to S-SALT, for examples, Bai *et al.* [14], Cai *et al.* [15], and Ghaly *et al.* [16]. Therefore, we focus on the Clayton copula [13] in our model as it is more appropriate to describe the strong dependence in the tail of the lifetime distribution and is simpler than other copulas.

In this paper, we discuss the Bayesian inference for a simple step-stress accelerated dependent competing failure model based on the Copula theory under the progressively Type-II censoring.

Model description and basic assumption

Model description

Under the progressively Type-II censored (PT-IIC) scheme, the simple S-SALT is described. The n test units are placed on the test under the initial stress level S_1. At the first failure $t_{1:n}$, R_1 units are progressively removed from the remaining $n – 1$ units and recording data ($t_{1:n}$, δ_1, R_1). Similarly, the test continues until time $t_{N_1:n}$, R_{N_1} units are progressively removed. Then we increase the stress level to S_2, and the remaining ($n – N_1 – R_1 – … – R_{N_1}$) units continue to be tested. At the time of $(N_1 + 1)^{\text{th}}$ failure, R_{N_1+1} units are progressively removed and we get the sample ($t_{N_1+1:n}$, δ_{N_1+1}, R_{N_1+1}), the test continues until the $(N_1 + N_2)^{\text{th}}$ failure is observed, $R_{N_1+N_2}$ units are removed and the test terminates. Here, N_1, N_2, R_1, … , $R_{N_1+N_2}$, ($N_1 + N_2 + R_1 + … + R_{N_1+N_2} = n$) are prefixed constants, where $t_{1:n}$, …, $t_{N_1+N_2:n}$ are order statistics, $\delta_i \in \{1, 2\}, i = 1, 2, \cdots, N_1 + N_2$.

Basic assumption

To describe the simple S-SALT clearly, some assumptions are made in this paper.

– Just one of the two competing failures leads to the unit failure. The dependence structure among failure modes is described by Bivariate Clayton Copula (BCC):

$$C_\theta(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-\frac{1}{\theta}} \tag{1}$$

and the conditional dependence measure, Kendall's tau [13] of BCC is $\tau = \theta/(\theta+2)$. The failure time is $T = \min\{T_1, T_2\}$.

– The lifetime follows an exponential distribution with scale parameter λ_{ij}. The cumulative distribution function (CDF) is:

$$F_{ij}(t; \lambda_{ij}) = 1 - \exp(-\lambda_{ij} t) \tag{2}$$

where $t \ge 0$, $\lambda_{ij} > 0$ and $i, j = 1, 2$, the probability density function (PDF) $f_{ij}(t; \lambda_{ij})$ is easy to get.

– The scale parameter λ_{ij} agrees with a log-linear function of stress:

$$\ln \lambda_{ij} = a_j + b_j \varphi(S_i) \tag{3}$$

where a_j and b_j are unknown parameters, and $\varphi(S_i) = 1/S_i$ that is the Arrhenius model [1].

– Lifetime distribution at different stress levels is related by assuming CEM:

$$F_1(t_1) = F_2(t_2) \tag{4}$$

Under this assumption, the CDF and the corresponding PDF of the lifetime of the test unit failed due to cause $j(j = 1, 2)$ are given by:

$$F_j(t) = F_j(t;\lambda_{1j},\lambda_{2j}) = \begin{cases} F(t,\lambda_{1j}) & (0 \le t \le \tau) \\ F\left(\dfrac{\lambda_{1j}}{\lambda_{2j}}\tau - \tau + t, \lambda_{2j}\right) & (t > \tau) \end{cases}$$

$$F_j(t;\lambda_{1j},\lambda_{2j}) = \begin{cases} F_{1j}(t) = 1 - \exp(-\lambda_{1j}t) & (0 \le t \le \tau) \\ F_{2j}(t) = 1 - \exp[-(\lambda_{1j} - \lambda_{2j})\tau - \lambda_{2j}t] & (t > \tau) \end{cases} \tag{5}$$

$$f_j(t;\lambda_{1j},\lambda_{2j}) = \begin{cases} f_{1j}(t) = \lambda_{1j}\exp(-\lambda_{1j}t) & (0 \le t \le \tau) \\ f_{2j}(t) = \lambda_{2j}\exp[-(\lambda_{1j} - \lambda_{2j})\tau - \lambda_{2j}t] & (t > \tau) \end{cases} \tag{6}$$

where $\tau \in (t_{N_1:n}, t_{N_1+1:n})$.

Based on the first failure mode, the CDF and the corresponding PDF of the lifetime of the product under stress level S_i can be obtained:

$$F^{(i1)}(t) = P(T_{i2} > T_{i1}, T_{i1} \le t) \tag{7}$$

$$f^{(i1)}(t) = \frac{\mathrm{d}F^{(i1)}(t)}{\mathrm{d}t} = f_{i1}(t)\frac{\partial C(u,v)}{\partial u}\bigg|_{\substack{u=S_{i1}(t)\\ v=S_{i2}(t)}} \tag{8}$$

By substituting eqs. (1), (5), and (6) into eq. (8), the equations can be obtained:

$$f^{(11)}(t_{i:n}) = \lambda_{11}\exp(\theta\lambda_{11}t_{i:n})\left[\exp(\theta\lambda_{11}t_{i:n}) + \exp(\theta\lambda_{12}t_{i:n}) - 1\right]^{-\frac{1}{\theta}-1} \tag{9}$$

$$f^{(21)}(t_{i:n}) = \lambda_{21}\exp\left[\left(\lambda_{11} - \lambda_{21}\right)\theta\tau + \lambda_{21}\theta t_{i:n}\right]\cdot$$

$$\cdot\left\{\exp\left[\left(\lambda_{11} - \lambda_{21}\right)\tau\theta + \lambda_{21}\theta t_{i:n}\right] + \exp\left[\left(\lambda_{12} - \lambda_{22}\right)\tau\theta + \lambda_{22}\theta t_{i:n}\right] - 1\right\}^{-\frac{1}{\theta}-1} \tag{10}$$

Based on the second failure mode, the CDF and the corresponding PDF of the lifetime of the product under stress level S_i can be obtained:

$$F^{(i2)}(t) = P(T_{i1} > T_{i2}, T_{i2} \le t) \tag{11}$$

$$f^{(i2)}(t) = \frac{\mathrm{d}F^{(i2)}(t)}{\mathrm{d}t} = f_{i2}(t)\frac{\partial C(u,v)}{\partial v}\bigg|_{\substack{u=S_{i1}(t)\\ v=S_{i2}(t)}} \tag{12}$$

Substituting eqs. (1), (5), and (6) into eq. (12) results in:

$$f^{(12)}(t_{i:n}) = \lambda_{12}\exp(\theta\lambda_{12}t_{i:n})\left[\exp(\theta\lambda_{11}t_{i:n}) + \exp(\theta\lambda_{12}t_{i:n}) - 1\right]^{-\frac{1}{\theta}-1} \tag{13}$$

$$f^{(22)}(t_{i:n})=\lambda_{22}\exp\left[\left(\lambda_{12}-\lambda_{22}\right)\theta\tau+\lambda_{22}\theta t_{i:n}\right]\cdot$$
$$\cdot\left\{\exp\left[\left(\lambda_{11}-\lambda_{21}\right)\tau\theta+\lambda_{21}\theta t_{i:n}\right]+\exp\left[\left(\lambda_{12}-\lambda_{22}\right)\tau\theta+\lambda_{22}\theta t_{i:n}\right]-1\right\}^{-\frac{1}{\theta}-1} \tag{14}$$

Let:

$$I_j(\delta_i)=\begin{cases}1, \delta_i=j\\ 0, \delta_i\neq j\end{cases}$$

be the indicator function. According to assumptions (1) and (2), we can obtain the survival function of the lifetime under stress S_i:

$$S_1(t)=\left[\exp\left(\theta\lambda_{11}t\right)+\exp\left(\theta\lambda_{12}t\right)-1\right]^{-\frac{1}{\theta}} \tag{15}$$

$$S_2(t)=\left\{\exp\left[\left(\lambda_{11}-\lambda_{21}\right)\theta\tau+\theta\lambda_{21}t\right]+\exp\left[\left(\lambda_{12}-\lambda_{22}\right)\theta\tau+\theta\lambda_{22}t\right]-1\right\}^{-\frac{1}{\theta}} \tag{16}$$

Then, the likelihood function of the failure sample is:

$$L\left(t\middle|\lambda_{ij},\theta\right)\propto\prod_{i=1}^{N_1}\left\{\left[f^{(11)}(t_{i:n})\right]^{I_1(\delta_i)}\left[f^{(12)}(t_{i:n})\right]^{I_2(\delta_i)}S_1\left(t_{i:n}\right)^{R_i}\right\}\cdot$$
$$\cdot\prod_{i=N_1+1}^{N_1+N_2}\left\{\left[f^{(21)}(t_{i:n})\right]^{I_1(\delta_i)}\left[f^{(22)}(t_{i:n})\right]^{I_2(\delta_i)}S_2\left(t_{i:n}\right)^{R_i}\right\} \tag{17}$$

Let $\mathbf{\Theta}=\left(\lambda_{11},\lambda_{12},\lambda_{21},\lambda_{22},\theta\right)$, by substituting eqs. (9)-(16) into eq. (17), the likelihood function can be written:

$$L\left(t\middle|\mathbf{\Theta}\right)\propto\lambda_{11}^{n_{11}}\lambda_{12}^{n_{12}}\lambda_{21}^{n_{21}}\lambda_{22}^{n_{22}}\exp\left(\theta\lambda_{11}T_{11}\right)\exp\left(\theta\lambda_{12}T_{12}\right)\cdot$$
$$\cdot\exp\left(\lambda_{21}\theta T_{21}\right)\exp\left(\lambda_{22}\theta T_{22}\right)\prod_{i=1}^{N_1}\left[\exp(\theta\lambda_{11}t_{i:n})+\exp(\theta\lambda_{12}t_{i:n})-1\right]^{-\frac{1+\theta+R_i}{\theta}}\cdot$$
$$\cdot\prod_{i=N_1+1}^{N_1+N_2}\left\{\exp\left[\left(\lambda_{11}-\lambda_{21}\right)\tau\theta+\lambda_{21}\theta t_{i:n}\right]+\exp\left[\left(\lambda_{12}-\lambda_{22}\right)\tau\theta+\lambda_{22}\theta t_{i:n}\right]-1\right\}^{-\frac{1+\theta+R_i}{\theta}} \tag{18}$$

where

$$n_{1j}=\sum_{i=1}^{N_1}I_j(\delta_i),\quad n_{2j}=\sum_{i=N_1+1}^{N_1+N_2}I_j(\delta_i),\quad T_{1j}=\sum_{i=1}^{N_1}t_{i:n}\,I_j(\delta_i)+\tau n_{2j}$$

$$T_{2j}=\sum_{i=N_1+1}^{N_1+N_2}t_{i:n}I_j(\delta_i)-\tau n_{2j},\quad j=1,2$$

According to basic assumption (3), we put $\hat{\lambda}_{i1}$ and $\hat{\lambda}_{i2}$ into eq. (3) and obtain the least squares estimators of a_j and b_j from the Gauss-Markov theorem are:

$$\hat{a}_j=\frac{\ln\hat{\lambda}_{1j}\varphi(S_2)-\ln\hat{\lambda}_{2j}\varphi(S_1)}{\varphi(S_2)-\varphi(S_1)}\quad\text{and}\quad\hat{b}_j=\frac{\ln\hat{\lambda}_{2j}-\ln\hat{\lambda}_{1j}}{\varphi(S_2)-\varphi(S_1)}$$

So

$$\hat{\lambda}_{0\mathrm{j}}=\exp\{\hat{a}_j+\hat{b}_j\varphi(S_0)\}$$

Then, we get the CDF of unit with competing failures under S_0:

$$F_0\left(t\right)=1-\left[\exp\left(\hat{\theta}\hat{\lambda}_{01}t\right)+\exp\left(\hat{\theta}\hat{\lambda}_{02}t\right)-1\right]^{-\frac{1}{\hat{\theta}}} \tag{19}$$

Bayesian analysis

In this section, we will consider the Bayesian estimate of model parameters λ_{ij} based on the SELF. As the conjugate prior, an independent gamma is chosen as prior distribution $G(\alpha_{ij}, \beta_{ij})$ for λ_{ij}, and the prior distribution of θ is no informative prior $\pi(\theta) \propto 1/\theta, \theta > 0$, so:

$$\pi(\lambda_{ij} | \alpha_{ij}, \beta_{ij}) \sim Ga(\alpha_{ij}, \beta_{ij}) = \frac{\beta_{ij}^{\alpha_{ij}}}{\Gamma(\alpha_{ij})} \lambda_{ij}^{\alpha_{ij}-1} e^{-\beta_{ij}\lambda_{ij}} \propto \lambda_{ij}^{\alpha_{ij}-1} e^{-\beta_{ij}\lambda_{ij}} \ (\lambda_{ij} \geq 0) \tag{20}$$

The joint posterior density function of $\mathbf{\Theta}$ is:

$$\pi(\lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}, \theta | t) = \frac{L(t|\mathbf{\Theta}) \cdot \pi(\lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}, \theta)}{\int \cdots \int L(t|\mathbf{\Theta}) \cdot \pi(\lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}, \theta) \mathrm{d}\lambda_{11} \mathrm{d}\lambda_{12} \mathrm{d}\lambda_{21} \mathrm{d}\lambda_{22} \mathrm{d}\theta} \tag{21}$$

The marginal posterior distribution of $\mathbf{\Theta}$ is:

$$\pi(\lambda_{11} | \lambda_{12}, \lambda_{21}, \lambda_{22}, \theta) \propto \lambda_{11}^{\alpha_{11}+n_{11}-1} \exp(\theta\lambda_{11}T_{11} - \beta_{11}\lambda_{11}) \prod_{i=1}^{N_1} [\exp(\theta\lambda_{11}t_{i:n}) + \exp(\theta\lambda_{12}t_{i:n}) - 1]^{-\frac{1+\theta+R_i}{\theta}} \cdot$$

$$\cdot \prod_{i=N_1+1}^{N_1+N_2} \{\exp[(\lambda_{11} - \lambda_{21})\tau\theta + \lambda_{21}\theta t_{i:n}] + \exp[(\lambda_{12} - \lambda_{22})\tau\theta + \lambda_{22}\theta t_{i:n}] - 1\}^{-\frac{1+\theta+R_i}{\theta}} \tag{22}$$

$$\pi(\lambda_{12} | \lambda_{11}, \lambda_{21}, \lambda_{22}, \theta) \propto \lambda_{12}^{\alpha_{12}+n_{12}-1} \exp(\theta\lambda_{12}T_{12} - \beta_{12}\lambda_{12}) \prod_{i=1}^{N_1} [\exp(\theta\lambda_{11}t_{i:n}) + \exp(\theta\lambda_{12}t_{i:n}) - 1]^{-\frac{1+\theta+R_i}{\theta}} \cdot$$

$$\cdot \prod_{i=N_1+1}^{N_1+N_2} \{\exp[(\lambda_{11} - \lambda_{21})\tau\theta + \lambda_{21}\theta t_{i:n}] + \exp[(\lambda_{12} - \lambda_{22})\tau\theta + \lambda_{22}\theta t_{i:n}] - 1\}^{-\frac{1+\theta+R_i}{\theta}} \tag{23}$$

$$\pi(\lambda_{21} | \lambda_{11}, \lambda_{12}, \lambda_{22}, \theta) \propto \lambda_{21}^{\alpha_{21}+n_{21}-1} \exp(\theta\lambda_{21}T_{21} - \beta_{21}\lambda_{21}) \prod_{i=N_1+1}^{N_1+N_2} \{\exp[(\lambda_{11} - \lambda_{21})\tau\theta + \lambda_{21}\theta t_{i:n}] +$$

$$+ \exp[(\lambda_{12} - \lambda_{22})\tau\theta + \lambda_{22}\theta t_{i:n}] - 1\}^{-\frac{1+\theta+R_i}{\theta}} \tag{24}$$

$$\pi(\lambda_{22} | \lambda_{11}, \lambda_{12}, \lambda_{21}, \theta) \propto \lambda_{22}^{\alpha_{22}+n_{22}-1} \exp(\theta\lambda_{22}T_{22} - \beta_{22}\lambda_{22}) \prod_{i=N_1+1}^{N_1+N_2} \{\exp[(\lambda_{11} - \lambda_{21})\tau\theta + \lambda_{21}\theta t_{i:n}] +$$

$$+ \exp[(\lambda_{12} - \lambda_{22})\tau\theta + \lambda_{22}\theta t_{i:n}] - 1\}^{-\frac{1+\theta+R_i}{\theta}} \tag{25}$$

Simulation study and data analysis

In this section, we use Monte-Carlo simulations to compare different methods for different sample size and progressive censoring schemes which are shown in tab. 1. Suppose the normal stress level and the accelerated stress levels are S_0 = 293 K, S_1 = 323 K, and S_2 = 353 K. The initial values of the scale parameters are $\lambda_{11} = 1.0$, $\lambda_{12} = 0.5$, $\lambda_{21} = 2.0$, and $\lambda_{22} = 1.0$. Let the parameter of BCC $\theta = 1$, $\theta = 2$, and $\theta = 3$, and equivalently $\tau = 1/3$, $\tau = 1/2$, and $\tau = 3/5$. Base on the experimental schemes in tab. 1 and 1000 simulation, we obtain the average estimates (AE) and the mean square errors (MSE) of the BE. Furthermore, the simu-

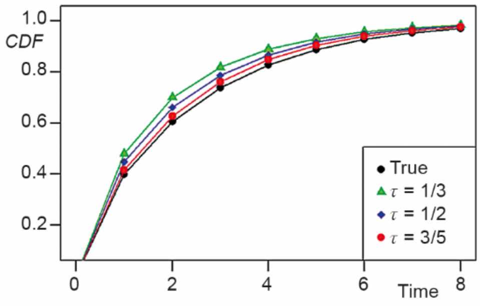

Figure 1. The cumulative distribution functions under different Kendall's Tau coefficients and the true cumulative distribution function

lation results are shown in tab. 2. Based on the eq. (19), the estimates of the parameters at the normal stress level are shown in tab. 3. Figure 1 show the CDF of the BE result under different θ and true value.

From the results of the simulations, we can obtain the conclusions.

– Form tab. 2, we observe that the estimation of $\hat{\lambda}_{ijB}$ are closed to the true values and the MSE of parameters are smaller as N increasing under $\theta = 1$, $\theta = 2$, and $\theta = 3$, respectively.
– Form tab. 2, we observe that the estimation of $\hat{\lambda}_{ijB}$ are closed to the true values and the MSE of parameters are smaller as θ increasing, which indicates that the correlation coefficient between failure mechanisms will affect the estimation accuracy of parameters.
– From fig. 1, the CDF of the BE are closed to the true CDF as θ increasing.

Real data analysis

In this section, we will apply the previous method to the data set of solar lighting devices from Han and Kundu [17]. There are two failure modes: Capacitor failure and Controller failure, and denote as Model 1 and Model 2, respectively. There are 35 solar lighting devices are put into S-SALT, $S_1 = 293$ K and $S_2 = 353$ K, the stress changed time point $\tau = 5$ (in hundred hours). The normal temperature is $S_0 = 273$ K and the original data set list in tab. 4. Using the mentioned approach, the BE of unknown parameters is obtained, and the results are shown in tab. 5.

Table 1. The prefixed sample sizes

Scheme	N	N_1	N_2	$\Sigma_{i=1}^{N_1} R_i$	$\Sigma_{i=N_1+1}^{N_2} R_i$	$(R_1, \ldots, R_{N_1})(R_{N_1+1}, \ldots, R_{N_1+N_2})$
1	40	20	10	5	5	(0,…,0,1,2,2) (0,…,0,1,2,2)
2	40	10	20	5	5	(0,…,0,1,2,2) (0,…,0,1,2,2)
3	60	30	16	8	6	(0,…,0,2,2,2,2) (0,…,0,2,2,2,2)
4	60	16	30	6	8	(0,…,0,2,2,2) (0,…,0,2,2,2,2)

Discussion and conclusion

The present method is applied to the solar lighting device with great success, and it can be extended to other devices, *e.g.* gas turbines [18], and to fuzzy environment [19].

Under progressively Type-II censoring scheme, the statistical analysis and reliability estimation of simple step-stress accelerated dependent competitive failure model are studied, and the dependence of failure mechanism is described by Clayton Copula function. In this paper, the Bayesian method is used to estimate the model parameters. It calculates the model parameters under the normal stress and predicts the remaining life of products. The simulation results show that: first, Copula theory plays an important role in studying the correlation of

Table 2. The AE and MSE (in parentheses) of the λ_{ij}

Parameter of BBC	Scheme	$\hat{\lambda}_{11B}$	$\hat{\lambda}_{12B}$	$\hat{\lambda}_{21B}$	$\hat{\lambda}_{22B}$
		AE(MSE)	AE(MSE)	AE(MSE)	AE(MSE)
$\theta = 1$	1	1.092(0.116)	0.518(0.099)	2.106(0.221)	0.940(0.239)
	2	1.087(0.118)	0.512(0.100)	2.101(0.197)	0.939(0.229)
	3	1.089(0.120)	0.516(0.089)	2.103(0.204)	0.941(0.237)
	4	1.091(0.124)	0.483(0.090)	2.109(0.207)	0.950(0.180)
$\theta = 2$	1	1.082(0.117)	0.516(0.090)	2.088(0.209)	0.941(0.232)
	2	1.078(0.124)	0.470(0.091)	2.070(0.212)	0.940(0.224)
	3	1.078(0.110)	0.511(0.083)	2.082(0.200)	0.943(0.219)
	4	1.075(0.119)	0.487(0.089)	2.107(0.203)	0.952(0.198)
$\theta = 3$	1	1.063(0.115)	0.490(0.089)	2.069(0.197)	0.948(0.219)
	2	1.087(0.119)	0.482(0.093)	2.077(0.204)	0.940(0.214)
	3	1.051(0.109)	0.495(0.081)	2.063(0.189)	0.951(0.209)
	4	1.065(0.117)	0.496(0.088)	2.097(0.199)	0.960(0.187)

Table 3. The estimates of the scales parameters at the normal stress level under Bayes

Scheme	$\hat{\lambda}_{01}$				$\hat{\lambda}_{02}$		
	$\theta = 1$	$\theta = 2$	$\theta = 3$		$\theta = 1$	$\theta = 2$	$\theta = 3$
1	0.4949	0.4900	0.4765		0.2526	0.2501	0.2213
2	0.4913	0.4911	0.4982		0.2465	0.2039	0.2155
3	0.4928	0.4878	0.4663		0.2502	0.2443	0.2254
4	0.4931	0.4778	0.4708		0.2138	0.2171	0.2238

Table 4. The data of solar lighting devices on a step-stress life test

Temperature	Failure times and failure causes (1 = capacitor; 2 = controller)
293 K	0.140(1), 0.738(2), 1.324(2), 1.582(1), 1.716(2), 1.794(2), 1.883(2), 2.293(2) 2.660(2), 2.674(2), 2.725(2), 3.085(2), 3.924(2), 4.396(2), 4.612(1),4.892(2)
353 K	5.002(1), 5.022(2), 5.082(2), 5.112(1), 5.147(1), 5.238(1), 5.244(1), 5.247(1) 5.305(1), 5.337(2), 5.407(1), 5.408(2), 5.445(1), 5.483(1), 5.717(2)

Table 5. The Bayes estimation of parameter

Parameter	λ_{11}	λ_{12}	λ_{21}	λ_{22}	a_1	b_1	a_2	b_2
BE	0.025	0.082	1.846	0.732	21.6	–7415	10.3	–3773

competitive failure mechanism, second, Bayesian method improves the estimation accuracy of model parameters due to combine the prior information, and finally, the data of solar lighting equipment is given as an example. For future research, we will discuss the optimal design of the simple S-SALT under different censoring schemes.

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