A FRACTAL-FRACTIONAL MODEL FOR COMPLEX FLUID-FLOW WITH NANOPARTICLES

by

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Nanoparticles play an important role in nanofluids to enhance thermal conduction, and can be easily controlled by a magnetic force, so it can be widely used in nano/micro devices. This paper gives two mathematical models to describe the nanofluid flow, one is an approximate continuum model, in which the homotopy matching is used to deal the discontinuity between the fluid and nanoparticles, and the other is to use the conservation laws in a fractal space. The models give new physical insight into the particle fluid-flow.

Key words: *nanoparticles, nanoscale flow, nanofluid, fractal derivative, homotopy matching*

Introduction

The motion of nanoparticles in a thin film or a long and narrow tube plays an important role in mass and energy transfer in a nanofluid, the nanoparticles can greatly enhance heat transfer through the boundary [1-8], as a result the nanofluid has been widely applied in various fields, for examples, lithium batteries [9], energy harvesting devices [10, 11], 3-D integrated circuits [12], micro/nano devices [13, 14], nanofiber or nanoscale membrane fabrication [15, 16], and the melt filling process [17, 18].

The micro/nano devices are now the footstone for nanoindustriation. A magnetic field is widely applied to control the motion of the nanoparticles in a nanofluid [19, 20], so it is extremely useful for blood vessels cleaning. However the local motion law of the nanoparticles in the flow has never been reported, the vibration of the nanoparticles will greatly affect the interaction among the particles [21, 22], and the heat or charge transfer on the boundary will be greatly affected. It was also reported that an addition of nanoparticles in a spun solution or in a printed mortar can greatly enhance the mechanical property, chemical property and electronic property of the nanofibers by either electrospinning or 3-D printing process [23, 24], this is because the nanoparticles in the moving jet will affect their distribution in the jet. The nanoparticles can also produce a high geometric potential, so that the surface property will be greatly affected [25, 26]. In this paper, we will establish two mathematical models for the complex nanofluids considering nanoparticles interaction with the fluid.

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Governing equations on a large scale

We consider a control volume as illustrated in fig. 1, where a nanoparticle is involved. The nanoparticle will interact with the fluid in the control volume.

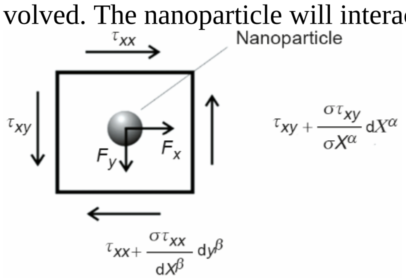

Figure 1. A control volume with a nanoparticle

The governing equations can be obtained according to mass, moment and energy conservation laws.

The mass conservation equation

We assume that the control volume can be considered as an approximate continuum, so the mass equation can be expressed:

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \tag{1}$$

where u, v, and w are, respectively, the velocities in x-, y- and z-directions and ρ – the density, which can be expressed:

$$\rho=\alpha\rho_{\mathrm{n}}+(1-\alpha)\rho_{\mathrm{f}} \tag{2}$$

where the subscripts n and f refer to the nanoparticle and the fluid, respectively and α – the homotopy parameter, it depends upon the mass concentration of the nanoparticles. When α = 0, the control volume contains no nanoparticles, it is a pure fluid mechanics problem and on the other extreme condition when α = 0, the control volume contains no nanoparticles, it is a pure fluid mechanics problem; and on the other extreme condition when α = 0, no fluid is involved in the control volume. This homotopy matching is also used in the homotopy perturbation method [27, 28].

The moment equations

Considering the nanoparticle interaction, the moment equations can be written:

$$\rho\frac{Du}{Dt}=\rho g_x-\frac{\partial p}{\partial x}+\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}-\rho_{\mathrm{n}}a_x-F_x \tag{3}$$

$$\rho\frac{Dv}{Dt}=\rho g_y-\frac{\partial p}{\partial y}+\frac{\partial\tau_{yx}}{\partial x}+\frac{\partial\tau_{yy}}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}-\rho_{\mathrm{n}}a_y-F_y \tag{4}$$

$$\rho\frac{Dw}{Dt}=\rho g_z-\frac{\partial p}{\partial z}+\frac{\partial\tau_{zx}}{\partial x}+\frac{\partial\tau_{zy}}{\partial y}+\frac{\partial\tau_{zz}}{\partial z}-\rho_{\mathrm{n}}a_z-F_z \tag{5}$$

where D/Dt is the material derivative, g_x, g_y, and g_z are volume forces due to gravity or electromagnetic force in x-, y- and z-directions, respectively, τ_{ij} (i = x, y, z; j = x, y, z) are shear stresses as illustrated in fig. 1, $\rho_{\mathrm{n}}a_x$, $\rho_{\mathrm{n}}a_y$, and $\rho_{\mathrm{n}}a_z$ are inertia forces which the nanoparticle acts on the control volume; F_x, F_y, and F_z are resistance between the nanoparticle and the fluid.

If the fluid is incompossible, eqs. (3)-(5) become:

$$\rho\frac{Du}{Dt}=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}\right)-\rho_{\mathrm{n}}a_x-F_x \tag{6}$$

$$\rho \frac{Dv}{Dt} = \rho g_y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \rho_\mathrm{n} a_y - F_y \tag{7}$$

$$\rho \frac{Dw}{Dt} = \rho g_z - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \rho_\mathrm{n} a_z - F_z \tag{8}$$

where μ is the viscosity coefficient can be written:

$$\mu = \beta \mu_\mathrm{n} + (1-\beta)\mu_\mathrm{f} \tag{9}$$

where β is the homotopy parameter and subscripts n and f refer to the nanoparticle and the fluid in the control volume, respectively.

The resistance between the nanoparticle and the fluid can be expressed:

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k} \tag{10}$$

$$F = \zeta (u^2 + v^2 + w^2) \tag{11}$$

where ζ is the friction coefficient.

The energy equation

The energy equation can be written:

$$\rho \frac{DT}{Dt} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial w^2} \right) + S \tag{12}$$

where T is the temperature, S – the source term, and k – the thermal conduction coefficient, which can be expressed:

$$k = \gamma k_\mathrm{n} + (1-\gamma) k_\mathrm{f} \tag{13}$$

where γ is a homotopy parameter and subscripts n and f refer to the nanoparticle and the fluid in the control volume, respectively.

Governing equations in a fractal space

In the previous derivation, the continuum assumption is still used, in this section, we use the two-scale fractal calculus [29] to establish a mathematical model for nanofluids. The fractal theory becomes a useful tool to many complex problems, for examples, the fluidity of a cement mortar [30], thermal conduction in a porous concrete [31], the stress-slip model for 3-D printed concretes [32], and fractal vibration systems [33, 34].

Figure 2 is the control volume in a fractal space, and the fluid follows the conservation laws in a fractal space.

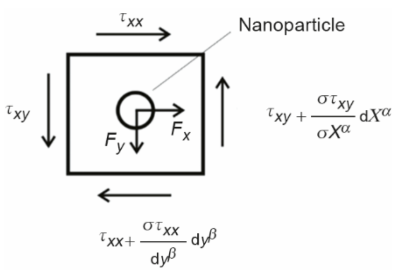

Figure 2. Control volume in a fractal space

The mass conservation equation

Wang *et al.* [35] suggested a mass conservation law in a fractal space, the mass equation in a fractal space can be written:

$$\frac{\partial \rho_{\mathrm{f}}}{\partial t^{\sigma}}+\frac{\partial}{\partial x^{\alpha}}(\rho_{\mathrm{f}} u)+\frac{\partial}{\partial y^{\beta}}(\rho_{\mathrm{f}} v)+\frac{\partial}{\partial z^{\gamma}}(\rho_{\mathrm{f}} w)=0 \tag{14}$$

where σ, α, β, and γ are the fractal dimensions for time, x-, y-, and z-directions, respectively, the fractal derivative is given [36-40]:

$$\frac{\partial \rho}{\partial t^{\sigma}}=\Gamma(1+\sigma) \lim_{\substack{t-t_0 \to \Delta t \\ \Delta t \neq 0}} \frac{u(t^{\sigma})-u(t_0^{\sigma})}{(\Delta t)^{\sigma}} \tag{15}$$

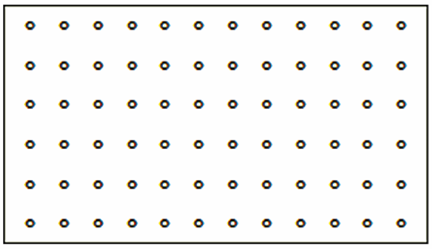

Figure 3. The nanoparticles distribution in the fluid in a fractal pattern

Using the fractal derivative model, Li *et al.* [41] established a fractal two-phase flow model for the fiber motion in a polymer filling process, and some amazing properties were first revealed.

In eq. (14), ρ_{f} the fluid's density, which does consider the particles concentration. The nanoparticles distribution in the fluid is considered as a fractal pattern, fig. 3. When no particle is involved in the fluid, $\sigma=\alpha=\beta=\gamma=1$.

The fractal derivative has the following properties [37]:

$$\lim_{\sigma \to 1} \frac{\partial \rho}{\partial t^{\sigma}}=\frac{\partial \rho}{\partial t} \tag{16}$$

The moment equations

The moment equations in a fractal space can be written [29]:

$$\rho_{\mathrm{f}} \frac{Du}{Dt^{\sigma}}=\rho_{\mathrm{f}} g_x-\frac{\partial p}{\partial x^{\alpha}}+\frac{\partial \tau_{xx}}{\partial x^{\alpha}}+\frac{\partial \tau_{xy}}{\partial y^{\beta}}+\frac{\partial \tau_{xz}}{\partial z^{\gamma}}-\rho_{\mathrm{n}} a_x-F_x \tag{17}$$

$$\rho_{\mathrm{f}} \frac{Dv}{Dt^{\sigma}}=\rho_{\mathrm{f}} g_y-\frac{\partial p}{\partial y^{\beta}}+\frac{\partial \tau_{yx}}{\partial x^{\alpha}}+\frac{\partial \tau_{yy}}{\partial y^{\beta}}+\frac{\partial \tau_{yz}}{\partial z^{\gamma}}-\rho_{\mathrm{n}} a_y-F_y \tag{18}$$

$$\rho_{\mathrm{f}} \frac{Dw}{Dt^{\sigma}}=\rho_{\mathrm{f}} g_z-\frac{\partial p}{\partial z^{\gamma}}+\frac{\partial \tau_{zx}}{\partial x^{\alpha}}+\frac{\partial \tau_{zy}}{\partial y^{\beta}}+\frac{\partial \tau_{zz}}{\partial z^{\gamma}}-\rho_{\mathrm{n}} a_z-F_z \tag{19}$$

where D/Dt^{σ} is the material derivative in a fractal space [29].

The energy equation

The energy equation in a fractal space can be written [29]:

$$\rho \frac{DT}{Dt^{\sigma}}=\nabla^{(\alpha,\beta,\gamma)}[k^{(\alpha,\beta,\gamma)}\nabla^{(\alpha,\beta,\gamma)}T]+S \tag{20}$$

where the gradient operator is defined:

$$\nabla^{(\alpha,\beta,\gamma)}T = \frac{\partial T}{\partial x^{\alpha}}\vec{\mathrm{i}}^{\alpha} + \frac{\partial T}{\partial y^{\beta}}\vec{\mathrm{j}}^{\beta} + \frac{\partial T}{\partial w^{\gamma}}\vec{\mathrm{k}}^{\gamma} \tag{21}$$

where $\vec{\mathrm{i}}^{\alpha}$, $\vec{\mathrm{j}}^{\beta}$, and $\vec{\mathrm{k}}^{\gamma}$ are unit vectors. The $k^{(\alpha,\beta,\gamma)}$ is thermal conduction coefficient in the fractal space.

The nanoparticle motion can be modelled:

$$\rho_{\mathrm{n}}V\vec{\mathrm{a}} = \rho_{\mathrm{n}}V\vec{\mathrm{g}} + A\vec{\mathrm{F}} + LA\nabla\vec{\tau} \tag{22}$$

where V is the nanoparticle volume, A – the section area in its motion direction, and L – the width.

Conclusion

This paper establishes two mathematical models for nanofluids [42-44], though they are mathematically rigorous, numerical study and experimental verification of the models are strongly needed, which will be done in a forthcoming article.

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