PERTURBED TRAVELING WAVE SOLUTIONS OF THE CDGKS EQUATION AND ITS DYNAMICS CHARACTERISTICS

by

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Based on the traveling wave reduction method with a perturbed initial solution and the F-expansion method, a class of explicit exact solutions of the (2+1)-dimensional CDGKS equation are obtained through the symbolic computation. Moreover, both the interaction behavior between parameters and the perturbation degree of periodic wave and Gauss wave to rational pulse wave, and the correlation of parameters to the superposition degree of the interaction energy between solitary wave and rational pulse wave are discussed. Finally, numerical simulations are shown to demonstrate the mechanism of the above solutions.

Key words: *(2+1)-dimensional CDGKS equation, F-expansion method, perturbed traveling wave solution, dynamics characteristics*

Introduction

Non-linear evolution equations, such as the Caudrey-Dodd-Gibbon-Kotera-Sawada (CDGKS) equation, the Korteweg de Vries equation, the Kadomtsev-Petviashvili equation have attracted much attention in non-linear mathematical and physical fields, because solitary wave solutions of the equations play an important role in understanding the mechanism of many nature phenomena. To study soliton solutions of these equations, many classical methods, for example, the inverse scatter method, Darboux transformation method and Hirota bilinear method have been proposed and some new methods are still being developed [1, 2].

In this paper, we focus on the (2+1)-dimensional CDGKS equation:

$$36u_{xt}+u_{x^6}+15(u_xu_{x^3})_x+45u_x^2u_{x^2}-5u_{x^3y}-15(u_xu_y)_x-5u_{y^2}=0 \tag{1}$$

which was firstly proposed by Konopelchenko and Dubrovsky [3]. The equation has widely applied in non-linear sciences such as the conservative flow of Liouville equation, dimensional gauge field theory of quantum gravity and theory of conformal field, and many investigations [4-7] are conducted on it. Recently, through Hirota bilinear form of eq. (1), some lump solutions, a strip soliton, a pair of resonance solitons as well as the rogue wave have been obtained in [8] and Deng *et al*. [9] studied the interaction phenomenon between the lump waves and stripe solitons. They obtained the lump-single stripe soliton interaction solutions, and showed that the one stripe soliton can split into a lump and a stripe soliton. Based on the conjugate transformation and long-wave limit method, diverse soliton and interaction solutions of some solitons were obtained and their dynamic behaviors were analyzed by Zhuang *et al*. [10]. Interestingly, Fang *et al*. [11]

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found the fusion and fission phenomena in the interaction between lump solitons and one-stripe soliton of eq. (1). By means of Bernoulli sub-equation function method, exact traveling-wave and some new oscillating solutions to CDGSK are also observed [12].

In this work, we discuss the perturbed traveling wave solutions of CDGKS eq. (1) by applying the traveling wave reduction method with a perturbed initial solution and the F-expansion method. Meanwhile, numerical simulations have shown the dynamics evolution characteristics of the solutions.

Reduction of the CDGKS eq. (1)

Assume that eq. (1) has the traveling wave solution:

$$u = ax + by + h(t) \tag{2}$$

and consider the traveling wave transformation:

$$u = v(\xi) + ax + by + h(t), \ \ \xi = k_1 x + k_2 y - k_3 t \tag{3}$$

where a, b are different perturbed parameters, respectively, and k_i (i = 1, 2, 3) also are different constants. Substituting eq. (3) into eq. (1), one can get the non-linear ordinary differential equation for function $v(\xi)$:

$$15k_1^4(v_\xi^{\ 3})_\xi + k_1^6 v_{\xi^6} + 15k_1^5(v_\xi v_{\xi^3})_\xi + 5k_1^2(3ak_1 - k_2)\left[k_1 v_{\xi^4} + 3(\mathrm{v}_\xi^{\ 2})_\xi\right] +$$
$$+(45a^2k_1^2 - 15ak_1k_2 - 15bk_1^2 - 36k_1k_3 - 5k_2^2)v_{\xi^2} = 0$$

Then integrating it with ξ, we have:

$$15k_1^4 v_\xi^{\ 3} + 15k_1^2(3ak_1 - k_2)v_\xi^{\ 2} + (15k_1^5 v_{\xi^3} + 45a^2k_1^2 - 15ak_1k_2 - 15bk_1^2 - 36k_1k_3 - 5k_2^2)v_\xi +$$
$$+5k_1^3(3ak_1 - k_2)v_{\xi^3} + k_1^6 v_{\xi^5} + C = 0 \tag{4}$$

where C is the constant of integration. Setting $v'(\xi) = w(\xi)$, we get:

$$15k_1^4 w^3 + 15k_1^2(3ak_1 - k_2)w^2 + k_1^6 w^{(4)} + 5k_1^3(3ak_1 - k_2 + 3k_1^2 w)w'' +$$
$$+(45a^2k_1^2 - 15ak_1k_2 - 15bk_1^2 - 36k_1k_3 - 5k_2^2)w + C = 0 \tag{5}$$

and expand $w(\xi)$:

$$w(\xi) = p_0 + p_1 f(\xi) + p_2 f(\xi)^2 \tag{6}$$

where p_i, (i = 0, 1, 2) are the constant to be determined later, $p_2 \neq 0$, and the function $f(\xi)$ satisfies:

$$[f'(\xi)]^2 = s_0 + s_1 f(\xi) + s_2 f(\xi)^2 + s_3 f(\xi)^3 + s_4 f(\xi)^4 \tag{7}$$

where $s_1 \in R$, (i = 0, 1, 2, 3, 4)

Next, substituting eqs. (6) and (7) into eq. (5), we obtain the equation for the undetermined function $f(\xi)$:

$$c_0 + c_1 f(\xi) + c_2 f(\xi)^2 + c_3 f(\xi)^3 + c_4 f(\xi)^4 + c_5 f(\xi)^5 + c_6 f(\xi)^6 = 0 \tag{8}$$

where

$$c_0 = k_1^6 p_1\left(3s_0s_3 + \frac{1}{2}s_1s_2\right) + k_1^6 p_2\left(8s_0s_2 + \frac{3}{2}s_1^2\right) + 15k_1^5 p_0\left(\frac{1}{2}p_1s_1 + 2p_2s_0\right) +$$
$$+15k_1^4\left(\frac{1}{2}ap_1s_1 + 2ap_2s_0 + p_0^3\right) - 5k_1^3\left(-9ap_0^2 + \frac{1}{2}k_2p_1s_1 + 2k_2p_2s_0\right) + 15k_1^2 p_0\left(3a^2 - b - k_2p_0\right) -$$
$$-3k_1p_0\left(5ak_2 + 12k_3\right) - 5k_2^2 p_0 + C$$

$$c_1 = k_1^6 p_1\left(12s_0s_4 + \frac{9}{2}s_1s_3 + s_2^2\right) + 15k_1^6 p_2(2s_0s_3 + s_1s_2) + 15k_1^5 p_0(p_1s_2 + 3p_2s_1) +$$
$$+15k_1^5 p_1\left(\frac{1}{2}p_1s_1 + 2p_2s_0\right) + 15k_1^4(3ap_2s_1 + 3ap_2s_1 + 3p_0^2p_1) -$$
$$-5k_1^3(k_2p_1s_2 - 18ap_0p_1 + 3k_2p_2s_1) - 15k_1^2p_1(b - 3a^2 + 2k_2p_0) - 3k_1p_1(5ak_2 + 12k_3) - 5k_2^2p_1$$

$$c_2 = 15k_1^6 p_1\left(s_1s_4 + \frac{1}{2}s_2s_3\right) + 2k_1^6 p_2(36s_0s_4 + 21s_1s_3 + 8s_2^2) + 15k_1^5\left(p_1^2s_2 + \frac{7}{2}p_1p_2s_1 + 2p_2^2s_0\right) +$$
$$+15k_1^5 p_0\left(\frac{3}{2}p_1s_3 + 4p_2s_2\right) + 15k_1^4\left[a\left(\frac{3}{2}p_1s_3 + 4p_2s_2\right) + 3p_0(p_0p_2 + p_1^2)\right] +$$
$$+45ak_1^3(2p_0p_2 + p_1^2) - 5k_1^3k_2\left(\frac{3}{2}p_1s_3 + 4p_2s_2\right) + 15k_1^2\left[p_2(3a^2 - b) - k_2(2p_0p_2 + p_1^2)\right] -$$
$$-3k_1p_2(5ak_2 + 12k_3) - 5k_2^2p_2$$

$$c_3 = \frac{5}{2}k_1^2[k_1^4(p_1(8s_2s_4 + 3s_3^2) + 2p_2(18s_1s_4 + 13s_2s_3)] +$$
$$+3k_1^3(4p_0p_1s_4 + 10p_0p_2s_3 + 3p_1^2s_3 + 10p_1p_2s_2 + 6p_2^2s_1) +$$
$$+6k_1^2[a(2p_1s_4 + 5p_2s_3) + p_1(6p_0p_2 + p_1^2)] - 2k_1k_2(2p_1s_4 + 5p_2s_3) + 12p_1p_2(3ak_1 - k_2)$$

$$c_4 = \frac{15}{2}k_1[k_1^4(4p_1s_3s_4 + 16p_2s_2s_4 + 7p_2s_3^2) + k_1^3p_1(4p_1s_4 + 13p_2s_3) + 4k_1^3p_2(3p_0s_4 + 2p_2s_2) +$$
$$+6k_1p_2(2ak_1s_4 + ap_2 + k_1p_0p_2 + k_1p_1^2) - 2k_2p_2(2k_1s_4 + p_2)]$$

$$c_5 = 3k_1^4\left[8k_1^2s_4(p_1s_4 + 7p_2s_3) + 5k_1p_2(8p_1s_4 + 5p_2s_3) + 15p_1p_2^2\right]$$

and

$$c_6 = 15k_1^4p_2(p_2 + 4k_1s_4)(p_2 + 2k_1s_4)$$

Since $f(\xi)^i$ (i = 1, 2, 3, 4, 5, 6) is independent of linearity, p_i (i = 0, 1, 2) and k_i (i = 0, 1, 2) would satisfy:

$$c_0 = c_1 = c_2 = c_3 = c_4 = c_5 = c_6 = 0 \tag{9}$$

When

$$C + k_1^4\{k_1^3[s_0(s_2s_4 - 3s_3^2) + s_1(2s_2s_3 - 3s_1s_4)] +$$
$$+k_1p_0(3k_1[s_1s_3 - 4s_0s_4) + s_2(15p_0 + 4s_2)] + 15p_0^3\} = 0$$

$C \in R$, and $s_3^2 + s_4^2 \neq 0$, solving the eq. (9) leads:

$$p_1 = -k_1s_3,\ \ p_2 = -2k_1s_4,\ \ k_2 = k_1(3a + 3k_1p_0 + k_1^2s_2)$$
$$k_3 = -\frac{k_1}{12}\left\{k_1^4(s_1s_3 - 4s_0s_4 + 3s_2^2) + 5\left[k_1^2(4p_0k_1s_2 + 6p_0^2 + 3as_2) + 3a(3k_1p_0 + a) + b\right]\right\} \tag{10}$$

where a, b, p_0, s_0, s_1, s_2 are different parameters, $k_1 \neq 0$, $s_3^2 + s_4^2 \neq 0$. Therefore, we get:

$$w(\xi) = p_0 - k_1s_3f(\xi) - 2k_1s_4f(\xi)^2 \tag{11}$$

where

$$\xi = k_1 x + k_1(3a + 3k_1 p_0 + k_1^2 s_2) y + \frac{k_1}{12}\left\{k_1^4(s_1 s_3 - 4 s_0 s_4 + 3 s_2^2) + \right.$$
$$\left. +5\left[k_1^2(4 p_0 k_1 s_2 + 6 p_0^2 + 3 a s_2) + 3a(3k_1 p_0 + a) + b\right]\right\} t$$

Perturbed traveling wave solutions of the CDGKS equation

For the simplicity, here we only discuss some traveling wave solutions with an initial perturbation for $s_3 = 0$, $s_4 \neq 0$.

Case I. If $s_0 = s_1 = 0$, $s_2 > 0$, $s_3 < 0$, eq. (7) has the bell-shaped soliton solution:

$$f(\xi) = \sqrt{\frac{-s_2}{s_4}}\operatorname{sech}\left(\sqrt{s_2}\,\xi\right)$$

According to the transformation eq. (6), the solution of eq. (5):

$$w(\xi) = p_0 + 2k_1 s_2 \operatorname{sech}^2\left(\sqrt{s_2}\,\xi\right)$$

and the corresponding solution of eq. (4):

$$v(\xi) = \int\left[p_0 + 2k_1 s_2 \operatorname{sech}^2\left(\sqrt{s_2}\,\xi\right)\right]\mathrm{d}\xi = p_0\xi + \frac{2k_1\sqrt{s_2}\sinh\left(\sqrt{s_2}\,\xi\right)}{\cosh\left(\sqrt{s_2}\,\xi\right)} + C$$

Thus, one can get the solution of the CDKGS eq. (1):

$$u_1(x,y,t) = p_0\xi + \frac{2k_1\sqrt{s_2}\sinh\left(\sqrt{s_2}\,\xi\right)}{\cosh\left(\sqrt{s_2}\,\xi\right)} + C + ax + by + h(t) \tag{12}$$

where

$$\xi = k_1\left(x + y(3a + 3k_1 p_0 + k_1^2 s_2) + \right.$$
$$\left. + \frac{t}{12}\left\{3k_1^4 s_2^2 + 5\left[k_1^2(4 p_0 k_1 s_2 + 6 p_0^2 + 3 a s_2) + 9 a k_1 p_0 + 3a^2 + b\right]\right\}\right)$$

Case II. If $s_0 = s_1 = s_2 = 0$, $s_4 > 0$, eq. (7) has the rational solution:

$$f(\xi) = -\frac{1}{\sqrt{s_4}\,\xi}$$

It follows from eq. (6) that eq. (5) has the solution $w(\xi) = p_0 - 2k_1/\xi^2$, and the solution of eq. (4) is:

$$v(\xi) = \int\left(p_0 - \frac{2k_1}{\xi^2}\right)\mathrm{d}\xi = p_0\xi + \frac{2k_1}{\xi} + C$$

Certainly, we get the solution of CDKGS eq. (1):

$$u_2(x,y,t) = p_0\xi + \frac{2k_1}{\xi} + C + ax + by + h(t) \tag{13}$$

where

$$\xi = k_1\left[x + y(3a + 3k_1 p_0) + \frac{5t}{12}(6k_1^2 p_0^2 + 9 a k_1 p_0 + 3a^2 + b)\right]$$

Case III. If

$$s_0 = \frac{s_2^2}{(4s_4)}, \; s_1 = 0, \; s_2 < 0, \; s_4 > 0$$

eq. (7) has the kink soliton solution:

$$f(\xi) = \sqrt{\frac{-s_2}{(2s_4)}} \tanh\left(\sqrt{-s_2}\,\frac{\xi}{2}\right)$$

Similarly, from the transformation eq. (6), the solution of eq. (5) is obtained:

$$w(\xi) = p_0 + k_1 s_2 \tanh^2\left(\sqrt{-s_2}\,\frac{\xi}{2}\right)$$

and the solution of eq. (4)

$$v(\xi) = \int\left[p_0 + k_1 s_2 \tanh^2\left(\sqrt{-s_2}\,\frac{\xi}{2}\right)\right]\mathrm{d}\xi = p_0\xi - k_1 s_2 \left\{\frac{\tanh\left(\sqrt{-s_2}\,\frac{\xi}{2}\right)}{\sqrt{\frac{-s_2}{2}}} + \right.$$

$$\left. + \frac{\ln\left[\tanh\left(\sqrt{-s_2}\,\frac{\xi}{2}\right) - 1\right]}{2\sqrt{\frac{-s_2}{2}}} - \frac{\ln\left[\tanh\left(\sqrt{-s_2}\,\frac{\xi}{2}\right) + 1\right]}{2\sqrt{\frac{-s_2}{2}}}\right\} + C$$

Then, the solution of the CDKGS eq. (1) can be derived:

$$u_3(x,y,t) = p_0\xi - k_1 s_2 \left\{\frac{\tanh\left(\sqrt{-s_2}\,\frac{\xi}{2}\right)}{\sqrt{\frac{-s_2}{2}}} + \frac{\ln\left[\tanh\left(\sqrt{-s_2}\,\frac{\xi}{2}\right) - 1\right]}{2\sqrt{\frac{-s_2}{2}}} - \right.$$

$$\left. - \frac{\ln\left[\tanh\left(\sqrt{-s_2}\,\frac{\xi}{2}\right) + 1\right]}{2\sqrt{\frac{-s_2}{2}}}\right\} + C + ax + by + h(t) \tag{14}$$

where

$$\xi = k_1\Big(x + y(3a + 3k_1 p_0 + k_1{}^2 s_2) +$$

$$+ \frac{t}{12}\left\{2k_1{}^4 s_2{}^2 + 5\left[k_1{}^2(4p_0 k_1 s_2 + 6p_0{}^2 + 3as_2) + 9ak_1 p_0 + 3a^2 + b\right]\right\}\Big)$$

Case IV. If

$$s_0=\frac{s_2^2}{(4s_4)},\ \ s_1=0,\ \ s_2>0,\ \ s_4>0$$

eq. (7) has the triangle solution:

$$f(\xi)=\sqrt{\frac{s_2}{(2s_4)}}\tan\left(\sqrt{s_2}\frac{\xi}{2}\right)$$

In the same way, we have the solution of eq. (5):

$$w(\xi)=p_0-k_1s_2\tan^2\left(\sqrt{s_2}\frac{\xi}{2}\right)$$

and the solution of eq. (4)

$$v(\xi)=\int\left[p_0-k_1s_2\tan^2\left(\sqrt{s_2}\frac{\xi}{2}\right)\right]\mathrm{d}\xi=p_0\xi-k_1\sqrt{2s_2}\left\{\tan\left(\sqrt{s_2}\frac{\xi}{2}\right)-\arctan\left[\tan\left(\sqrt{s_2}\frac{\xi}{2}\right)\right]\right\}+C$$

Further, we also have the solution of CDKGS eq. (1):

$$u_4(x,y,t)=p_0\xi-k_1\sqrt{2s_2}\left\{\tan\left(\sqrt{s_2}\frac{\xi}{2}\right)-\arctan\left[\tan\left(\sqrt{s_2}\frac{\xi}{2}\right)\right]\right\}+C+ax+by+h(t) \tag{15}$$

where

$$\xi=k_1\Big(x+y(3a+3k_1p_0+k_1{}^2s_2)+$$
$$+\frac{t}{12}\left\{2k_1{}^4s_2{}^2+5\left[k_1{}^2(4p_0k_1s_2+6p_0{}^2+3as_2)+9ak_1p_0+3a^2+b\right]\right\}\Big)$$

Case V. If

$$s_0=\frac{s_2^2m^2}{s_4(2m^2+1)^2},\ \ s_1=0,\ \ s_2<0$$

eq. (7) has the periodic solution:

$$f(\xi)=\sqrt{-\frac{s_2m^2}{s_4(m^2+1)}}\mathrm{sn}\left(\sqrt{-\frac{s_2}{m^2+1}}\xi\right)$$

Similarly the solution of eq. (5):

$$w(\xi)=p_0+\frac{2m^2k_1s_2}{m^2+1}\mathrm{sn}^2\left(\sqrt{-\frac{s_2}{m^2+1}}\xi\right)$$

and the solution of eq. (4)

$$v(\xi)=\int\left[p_0+\frac{2m^2k_1s_2}{m^2+1}\mathrm{sn}^2\left(\sqrt{-\frac{s_2}{m^2+1}}\xi\right)\right]\mathrm{d}\xi=p_0\xi+\frac{2m^2k_1s_2}{m^2+1}\int\mathrm{sn}^2\left(\sqrt{-\frac{s_2}{m^2+1}}\xi\right)\mathrm{d}\xi+C$$

Consequently, we get the solution of CDKGS eq. (1):

$$u_5(x,y,t)=p_0\xi+\frac{2m^2k_1s_2}{m^2+1}\int\mathrm{sn}^2\left(\sqrt{-\frac{s_2}{m^2+1}}\xi\right)\mathrm{d}\xi+C+ax+by+h(t) \tag{16}$$

where

$$\xi = k_1\Big(x + y(3a + 3k_1p_0 + k_1^2 s_2) + $$
$$+\frac{t}{12}\left\{k_1^4 s_2^2\left[3-\frac{4m^2}{(2m^2+1)^2}\right]+5\left[k_1^2(4p_0k_1s_2+6p_0^2+3as_2)+9ak_1p_0+3a^2+b\right]\right\}\Big)$$

Dynamic evolution characteristics

We mainly describe the dynamic behavior of solution eq. (13) and solution eq. (15) of CDKGS eq. (1) on the plane $y = x$.

Case I. Perturbed structure of the periodic wave to the rational impulse wave.

In eq. (13), if $p_0 = 1$, $a = 0.65$, $b = 2$, $C = 2$, $h(t) = 28\cos(4t)$, and k_1 takes 0.8, 1.6, and 2.4, respectively, the dynamic behavior of the periodic wave to the rational impulse wave is illustrated in fig. 1. It is clear that the perturbation degree of periodic wave to rational pulse wave has significant positive correlation with the parameter k_1:

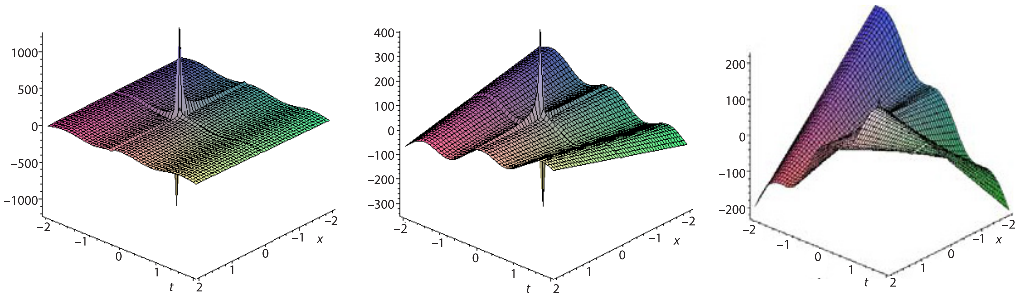

Figure 1. Perturbed structure of the periodic wave to the rational impulse wave with different k_1

Case II. Perturbed structure of the Gauss wave to the rational impulse wave.

In eq. (13), if $k_1 = 0.5$, $a = 2$, $b = 1$, $C = 0$, $h(t) = 24\exp(-t^2/2)$, and p_0 takes 0.5, 1.0, and 1.5, respectively, the dynamic behavior of the Gauss wave to the rational impulse wave is also shown in fig. 2. Obviously, it can be seen that the perturbation degree of Gauss wave to rational pulse wave is negatively correlated with the parameter p_0.

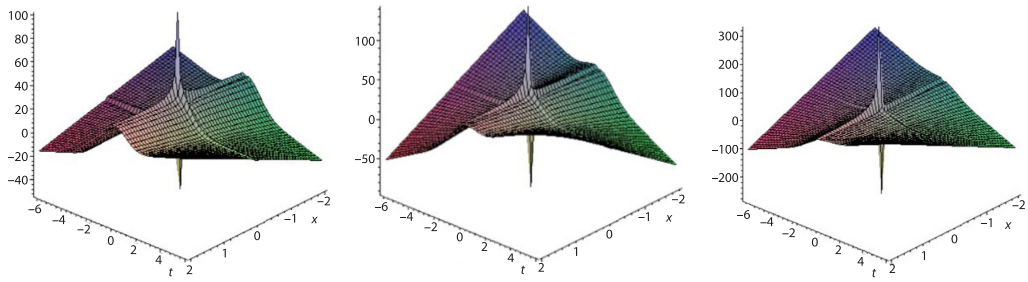

Figure 2. Perturbed structure of the Gauss wave to the rational impulse wave with different p_0

Case III. Perturbed structure of the periodic wave to the periodic wave.

In eq. (13), if $k_1 = 0.5$, $s_2 = 0.5$, $a = 0.5$, $b = 0.5$, $C = 0$, $h(t) = 24\cos(t/2)^2$, and p_0 takes 0.06, 0.07, 0.08, 0.09, and 0.10, respectively, the dynamic behavior of the periodic wave to the periodic wave is also given in fig. 3. We can see that in a certain range, the parameters p_0 has certain correlation with the superposition energy evolution of periodic waves in two directions, that is, positive correlation when $p_0 < 0.08$ and negative correlation when $p_0 > 0.08$.

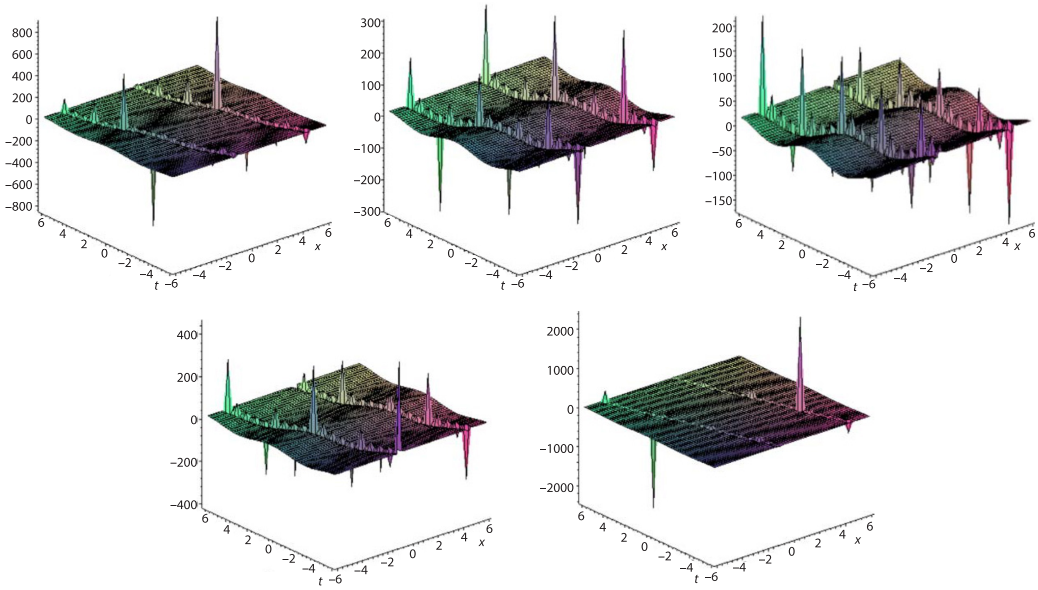

Figure 3. Perturbed structure of the periodic wave to the periodic wave with different p_0

Conclusion

By the traveling wave reduction with the initial perturbation of the (2+1)-dimensional CDGKS equation, the F-expansion of the reduction equation is obtained, and a series of explicit exact solutions of the equation with the initial perturbation are derived. The correlation between the perturbation degree and parameters of the periodic wave, the Gauss wave to the rational pulse wave, and the correlation between parameters and the superposition degree of energy of interaction between the solitary wave and the rational pulse wave are also analyzed, respectively. All solutions obtained in this paper contain the perturbed initial solutions of any time function, which indicates the rich dynamic characteristic of the CDGKS equation.

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Nomenclature

t – time, [second]

x, y, z– co-ordinates, [m]

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