# RESEARCH ON GEOMETRIC INVERSE PROBLEM BASED ON THE FIREFLY CONJUGATE GRADIENT METHOD

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In this paper the 2-D steady-state heat transfer geometric inverse problem is solved using the finite element method, conjugate gradient method, and firefly algorithm. Based on the finite element method for solving the forward heat transfer model, and based on continuous iterative optimisation of the conjugate gradient method, the accuracy of the error function of the measured and estimated values is kept within a certain range, so that the geometry of the object under test can be calculated in an inverse way. In the study of the forward problem, the temperature field distribution is solved using circular pipes as the research objects, and the feasibility of applying the finite element method to heat transfer problems is verified. The inverse problem takes the circular tube as the object and considers two different corrosion defects on the inner wall of the circular tube. Simultaneously, the firefly algorithm is introduced based on the conjugate gradient method to stochastically optimize the temperature data and suppress the fluctuation of the inversion result. Numerical experiment results indicate that the method in this paper can perform more accurate geometric shape recognition when there is a certain temperature measurement error or the temperature measurement information is relatively complete.

Key words: inverse heat transfer problem, geometric shape recognition, conjugate gradient method, finite element method, firefly algorithm

## Introduction

The geometric inverse problem of heat transfer has broad application prospects. It has applications in the fields such as metallurgical engineering, biological lesion detection, non-destructive testing, and equipment geometry optimization, and has received extensive attention from scholars worldwide [1-5]. At present, in the research of heat transfer problems [6-8], forward problems are usually solved by finite difference, boundary element method, finite element method, and finite volume method. The inversion methods of geometric shapes are mainly based on the Levenberg-Marquardt algorithm, conjugate gradient method (CGM), steepest descent method, and decentralized fuzzy inference method (DFIM). Jiang *et al.* [9] conducted the shape recognition of the inner wall of a 2-D cylinder based on boundary element and DFIM. Luo *et al.* [10] studied the fuzzy inversion of the shape of the inner wall of an industrial kiln. Wang [11] conducted an inversion study on the geometry of the third type of boundary conditions. Based on the finite element method (FEM), Li [12] proposed

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a new non-iterative inversion algorithm to identify the boundary conditions and geometric shapes in the multi-dimensional steady-state heat conduction problem. Wang et al. [13] used the social particle swarm and conjugate gradient algorithm to identify the 2-D steady-state boundary shape. Mei et al. [14] used the FEM to investigate defect detection. Besides, depending on finite element thermal structure analysis, Sun et al. [15] presented an iterative algorithm to solve the inverse calculation of the cold state of the blade geometry. Fazeli et al. [16, 17] identified the shape and location of the 2-D defect in the heat conduction inverse problem using the FEM and gradient optimization algorithm, and further developed the algorithm and applied it to the 2-D multi-connection geometric shape recognition problem. Li et al. [18] used the boundary element method and fuzzy inference method to explore the inverse problem of the inner geometry of the 2-D heat conduction system. Cheng et al. [19] discussed the application of the heat transfer inversion method in shape recognition, and considered several examples of shape recognition of solid internal voids based on solid surface temperature data. Mosavati et al. [20-22] solved using a simplified back-ward Monte-Carlo method (MCM) for the inverse radiation boundary problem and combined natural-convection-radiation considering specular reflectivity and participating media is solved. Chen et al. [23] employed the CGM to invert the shape of the fouling layer in the 2-D forced convection pipe-line and discussed the influence of the initial value and error on the inversion result. Fan et al. [24] established a transient heat transfer model for a pipe with internal wall defects with internal heat-passing fluid and researched the influence of temperature measurement errors on the identification of the internal wall boundary through simulation experiments. Partridge et al. [25] reverse-coupled the double reciprocity method and genetic algorithm to obtain the size and location of skin tumors from skin surface temperature measurement.

The geometry of the pipe is identified as a typical class of heat transfer inverse problems, the solution of which consists of two main components: the creation and solution of the heat transfer temperature field, and the coupling of the inverse algorithm to the positive problem to solve for the unknown parameters. For forward heat transfer, the FEM used in this paper solves for the method, which is more adaptable to irregular boundary discretization. The solution of inverse problems has long been a focus of scholarly research. In the Introductory section, the methods mentioned in this paper are broadly classified into two categories: statistical class algorithms and deterministic algorithms to solve heat transfer inverse problems. For example, the classical conjugate gradient algorithm is a deterministic algorithm. This type of algorithmic operation mainly uses the gradient information of the objective function, which can solve the inverse problem well, but relies on the influence of the selection and error of the initial value. The biggest advantage of the statistical class of algorithms is that they do not rely on initial value guesses and theoretically do not fall into local extreme value problems in the inverse process, but the algorithms themselves are large in size and take a long time to operate. The hybrid algorithm proposed in this paper, using the firefly algorithm (FA) to optimise the CGM, is a good solution the problems that exist in these two classes of algorithms, and has a much better performance and stability for solving heat transfer inverse problems.

Aiming at the geometric shape recognition problem in the heat conduction system, this paper uses the FEM to solve the 2-D steady-state normal problem, address the inverse problem with the conjugate gradient algorithm, and introduce the FA to optimize the temperature information based on the conjugate gradient algorithm. Consequently, the fluctuations in the inversion process are suppressed to a certain extent, and the stability of the inversion is enhanced.

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# The 2-D heat transfer forward problem

Finite element meshing

This paper uses the FEM to solve the 2-D heat transfer forward problem. The FEM does not directly calculate the global domain but accumulates each element to obtain the global quantity. Therefore, this article analyzes the unit volume. Since triangular elements have a strong ability to fit complex boundaries, triangular meshes are often used in FEM to divide the geometric domain of a 2-D system.

Figure 1 illustrates a three-node triangular element. The three vertices in the triangular element are selected as the interpolation nodes of the temperature field.

The three-node element temperature field interpolation function:



where *T* is the temperature at any point in the triangle mesh and  $a_1$ ,  $a_2$ , and  $a_3$  are undetermined coefficients. To determine the three coefficient values in the equation, the temperature data of the three vertices of the triangle mesh is substituted into eq. (1) to obtain the coefficient equation system:

$$T_{i} = a_{1} + a_{2}x_{i} + a_{3}y_{i}$$

$$T_{j} = a_{1} + a_{2}x_{j} + a_{3}y_{j}$$

$$T_{k} = a_{1} + a_{2}x_{k} + a_{3}y_{k}$$
(2)

The solution is:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} a_i & a_j & a_k \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{bmatrix} \begin{bmatrix} T_i \\ T_j \\ T_k \end{bmatrix}$$
(3)

where A is the area of the triangle mesh. Substituting eq. (3) into eq. (1), the temperature interpolation function of the triangular grid can be obtained:

$$T = \frac{1}{2A} \Big[ (a_i + b_i x + c_i y) T_i + (a_j + b_j x + c_j y) T_j + (a_k + b_k x + c_k y) T_k \Big]$$
(4)

Equation (4) can be simplified:

$$T = T_i N_i + T_j N_j + T_k N_k \tag{5}$$

where,  $N_i$ ,  $N_j$ , and  $N_k$  are the vertex shape functions of the triangle mesh. The temperature field interpolation function of eq. (5) can be used in the entire triangular grid domain, and the temperature field interpolation function on the element side can be simplified as shown in fig. 2.

As presented in fig. 2, suppose jk is a boundary edge. The temperature of the point on the edge has nothing to do with  $T_i$  while it has a linear combination of the two points of j, k:

$$T = (1 - g)T_i + gT_k, \ 0 \le g \le 1$$
(6)





where g = 0 corresponds to point *j*, and g = 1 corresponds to point *k*. When the triangle *jk* is the boundary side, the Galerkin weight function on the boundary can be obtained from:

$$\omega_{i} = \frac{\partial T}{\partial T_{i}} = 0$$

$$\omega_{j} = \frac{\partial T}{\partial T_{j}} = 1 - g \qquad (7)$$

$$\omega_{k} = \frac{\partial T}{\partial T_{k}} = g$$

After the domain is discretized by dividing it into each triangular mesh, the weighted integral margin on each element can be obtained:

$$R_{l}^{e} = \iint_{e} \left[ \lambda \left( \frac{\partial \omega_{l}}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial \omega_{l}}{\partial y} \frac{\partial T}{\partial y} \right) - \omega_{l} q + \omega_{l} \rho c \frac{\partial T}{\partial t} \right] dxdy - \oint_{\partial_{e}} \lambda \omega_{l} \frac{\partial T}{\partial n} ds, \ l = 1, 2...n$$
(8)

where *e* is the cell domain,  $\partial_e$  – the cell boundary and  $\omega_l$  – a known weighting function.

### Boundary conditions solving

Introduce the first type of boundary conditions:

$$T = f(x, y, t) \tag{9}$$

The second type of boundary conditions:

$$q = -\lambda \frac{\partial T}{\partial n} \tag{10}$$

The third type of boundary conditions:

$$-\lambda \frac{\partial T}{\partial n} = h(T - T_f) \tag{11}$$

where T is the temperature, f(x, y, z, t) – the known mathematical function or constant,  $T_f$  – the surrounding environment, h – the surface heat transfer coefficient,  $\lambda$  – the thermal conductivity of the object, and n – the normal vector outside the boundary.

According to eq. (9), when edge jk is in the first type of boundary condition, the boundary node temperature of the element should be a given value:

$$T_J = \overline{T}_j, \ T_k = \overline{T}_k \tag{12}$$

where  $\overline{T}_j$  and  $\overline{T}_k$  is a known node temperature value, and the two-point values are usually equal. When the edge is in the second type of boundary condition, the node load of the element can be obtained by substituting eqs. (7) and (10) into eq. (8):

$$\Delta p_{i} = \int_{jk} \omega_{i} \lambda \frac{\partial T}{\partial n} ds = -\int_{jk} q \omega_{i} ds = 0$$

$$\Delta p_{j} = \int_{jk} \omega_{j} \lambda \frac{\partial T}{\partial n} ds = -\int_{0}^{1} (1 - g) q s_{i} dg = -\frac{1}{2} q s_{i}$$

$$\longrightarrow$$
(13)

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$$\Delta p_k = \int_{jk} \omega_k \lambda \frac{\partial T}{\partial n} ds = -\int_0^1 g q s_i dg = -\frac{1}{2} q s_i$$
(13)

where q is the boundary heat flux density. When edge jk is in the third type of boundary condition, it can be obtained by substituting eqs. (7) and (11) into eq. (8):

$$\int_{jk} \omega_i \lambda \frac{\partial T}{\partial n} ds = \int_{jk} \omega_i h(T - T_f) ds = 0$$

$$\int_{jk} \omega_j \lambda \frac{\partial T}{\partial n} ds = \int_0^1 (1 - g) h \Big[ (1 - g) T_j + g T_k - T_f \Big] s_i dg = h s_i \Big( \frac{1}{3} T_j + \frac{1}{6} T_k - \frac{1}{2} T_f \Big)$$

$$\int_{jk} \omega_k \lambda \frac{\partial T}{\partial n} ds = \int_0^1 g h \Big[ (1 - g) T_j + g T_k - T_f \Big] s_i dg = h s_i \Big( \frac{1}{3} T_j + \frac{1}{6} T_k - \frac{1}{2} T_f \Big)$$
(14)

## Time domain difference

In this paper, the forward heat transfer simulation of a pipe is a transient heat transfer, and the solution conditions include initial conditions, boundary conditions, and a time difference. If the time is long enough it can be interpreted as steady-state heat transfer, that is the temperature does not change with time, but the process evolves over time, so the time domain difference component is added.

The heat transfer region is discretized into N cells:

$$\Omega \to \sum_{n} \Omega^{e} \tag{15}$$

The temperature field within the cells is interpolated using the temperature at the nodes, that is  $T = NT_e$ , then the variational generalization of the heat transfer problem is:

$$\Pi = \sum_{n} \left( \frac{1}{2} (T)^{T} (\lambda + h) T + [T]^{T} \left( c \frac{\partial T}{\partial t} - P \right) \right)$$
(16)

$$K = \int_{\Omega} \lambda (\nabla N)^{T} (\nabla N) d\Omega, \quad h = \int_{s} h N^{T} N dA$$
(17)

$$C = \int_{\Omega} c\rho N^T N \mathrm{d}\Omega, \ P = \int_{\Omega} q N^T \mathrm{d}\Omega + \int_{s} q N^T \mathrm{d}A + \int_{s} h T_f N^T \mathrm{d}A$$
(18)

Figure 3 exhibits the time domain difference of the FEM, where  $\Delta t$  is the time step. The calculation of the backward difference format of the finite element is written:

$$\frac{\partial T(t)}{\partial t} \approx \frac{T(t) - T(t - \Delta t)}{\Delta t}$$
(19)

Taking the extremes of the generalized variation, that is  $\partial \prod / \partial T = 0$ , obtain the set of finite element equations:  $C(\partial T / \partial t) + KT(t) = P(t)$ . After adopting the backward difference format, the finite element heat transfer model can be written:

$$\left\{T(t)\right\} = \left(K + \frac{C}{\Delta t}\right)^{-1} \left[\left\{P(t)\right\} + \frac{C}{\Delta t}\left\{T(t - \Delta t)\right\}\right]$$
(20)

where K is the temperature stiffness matrix, C – the hot fusion matrix, and P – the node temperature load column vector. After adopting the backward difference format, each step of the calculation process only needs the temperature value of the previous step. In the first step of the calculation, only the initial temperature value can be used to calculate the temperature distribution at each time within the defined domain.



Figure 4. Schematic diagram of circular pipe

## Forward problem verification

The 2-D circular pipe heat transfer model provided in fig. 4 is used to verify the forward problem. The  $\Gamma_1$  is the outer surface of the circular pipe,  $\Gamma_2$  is the inner surface of the pipe. The outer radius of the circular pipe is 100 m, the inner radius is 80 m, and the thermal conductivity of the material is  $\lambda = 25$  W/mK, density is 2600 kg/m<sup>3</sup>, specific heat capacity is 4200 J/kgK. Initial temperature  $T_0 = 293.15$  K (20 °C) for both the inner and outer surfaces of the circular tube for Type I boundary conditions. The inner surface temperature of the circular tube is constant and the outer surface temperature varies with the length of the tube wall, the expression is  $30/(-1 \cdot 10^6) \cdot h + 50$ , h takes values from 0-1000 m.

The mathematical description of the 2-D tube heat transfer model:

$$\begin{cases} \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0\\ T = f_1(x, y, z, t) & \Gamma_1\\ T = f_2(x, y, z, t) & \Gamma_2\\ T = T_0 & \Omega \end{cases}$$
(21)

Figure 5 shows the temperature field information solved for when the temperature information functions in the  $\Gamma_1$  and  $\Gamma_2$  planes are known. Figure 5(a) shows the 3-D model created, with a circular tube 1000 m long, fig. 5(b) shows the solved temperature using the FEM, the meshing of the circular tube model, fig. 5(c) shows the solved temperature field information and fig. 5(d) shows the solved circular tube isotherm information.

From the solution of the circular tube, it is feasible to use the FEM to realize the solution and analysis of the temperature field information distribution.



Figure 5. Solving for the temperature field of a circular tube using finite elements; (a) circular tube 1000 m long, (b) solved temperature using the FEM, meshing of the circular tube model, (c) solved temperature field information, and (d) solved circular tube isotherm information

# The 2-D inverse heat transfer problem Inverse problem model

After verifying the feasibility of the FEM using rectangular flat plates and circular tubes, the inverse problem follows the circular tube model described in the forward problem for inversion identify defects on the inner wall of the pipe-line. The idea of the inverse problem is to solve the geometry of the inner wall of the pipeline when the temperature information parameters of the outer wall of the pipe-line are known. In the calculation of the inverse problem, a third type of boundary condition is added to both the inner and outer walls of the pipe, the surface heat transfer coefficient on the boundary of the circular tube and between the surrounding fluid is specified, as well as the temperature of the surrounding fluid, as shown in fig. 6.



Figure 6. Schematic diagram of the corroded pipe-line

The mathematical model of the 2-D inverse heat transfer problem is described:

$$\frac{\partial^2 T}{\partial (\rho \cos \theta)^2} + \frac{\partial^2 T}{\partial (\rho \sin \theta)^2} = 0$$
  
- $\lambda \left( \frac{\partial T}{\partial n_i} \right)_2 = h_i (T - T_f)$   
- $\lambda \left( \frac{\partial T}{\partial n_o} \right)_1 = h_o (T - T_a)$  (22)

where  $\lambda$  is the thermal conductivity of the tube,  $n_i$ - the unit normal vector of the inner surface of the tube,  $n_o$  - the unit normal vector of the outer surface of the tube,  $h_i$  - the surface heat transfer coefficient of the inner surface of the tube,  $h_o$  - the surface heat transfer coefficient of the outer surface of the tube,  $T_f$ - the temperature of the fluid inside the tube, and  $T_a$  - the temperature of the fluid outside the tube. These thermophysical parameters are all known quantities.

It can be observed from the aforementioned pipe-line model and the schematic diagram that there is only one unknown quantity in eq. (22), that is, the geometric shape of the inner Surface 1 of the pipe-line after corrosion, and the remaining parameters are known. Due to the complex geometry of the inner wall after corrosion, it is difficult to solve in the Cartesian co-ordinate system. To facilitate the following research, the following will adopt the polar co-ordinate system to describe the geometry of the inner wall, denoted by  $P(\theta)$ , and the temperature distribution on the outer surface of the tube is represented by the vector  $Y = (Y_i, i = 1, 2, ..., m)$ . Therefore, the solution of the inverse problem can be understood as a process to make the error function accuracy close enough, that is, there is a sufficiently small search objective function S(p):

$$S_{(p)} = \sum_{i=1}^{m} (Y_i - T_i)^2$$
(23)

where p is the vector composed of the discrete radius r of the inner wall of the tube, that is, the estimated value of the function  $P(\theta)$ , m – the number of sampling points for the round tube,  $T_i$  – the temperature field calculated from p, and  $Y_i$  – the true value of the temperature field.

# Solving 2-D inverse heat transfer problem based on the conjugate gradient method

In the solution of the inverse heat transfer problem, the more classic CGM is used [26, 27]. The CGM is a method between the steepest descent method and the Newton method. It improves the lack of slow convergence of the steepest descent method while avoiding the shortcomings of Newton's method of occupying large storage and complicated calculations.

In the process of iterative algorithm solving, there are several parameter conditions that need to be determined:

 Determine the conjugate search direction. It is a linear combination of the gradient direction of the objective function and the previous search direction:

$$P^{n}(\theta) = \nabla S(p^{n}) + \gamma^{n} P^{n-1}$$
(24)

where  $\nabla S(p^n)$  is the descending gradient of the objective function and  $\gamma^n$  – the conjugate coefficient when iterating to *n* steps:

$$\gamma^{n} = \begin{cases} \frac{\sum_{i=1}^{m} \left[ \nabla S(p^{n}) \right]_{i}^{2}}{\sum_{i=1}^{m} \left[ \nabla S(p^{n-1}) \right]_{i}^{2}} & n > 0 \\ \sum_{i=1}^{m} \left[ \nabla S(p^{n-1}) \right]_{i}^{2} & n > 0 \end{cases}$$

$$\nabla S(p^{n}) = \begin{pmatrix} \frac{\partial J}{\partial p_{1}}, \frac{\partial J}{\partial p_{2}}, ..., \frac{\partial J}{\partial p_{m}} \end{pmatrix}$$
(25)

- Determine the search step factor. Its calculation formula is:

$$\beta^{n} = \frac{\sum_{i=1}^{m} \left[ (T_{i} - Y_{i}) \left( P_{n} \frac{\partial T_{i}}{\partial p^{n}} \right) \right]}{\sum_{i=1}^{i=m} \left( P_{n} \frac{\partial T_{i}}{\partial p^{n}} \right)^{2}}$$
(26)

- Determine the termination condition of the iteration. Its calculation formula is:

$$S(P) = \sum_{i=1}^{m} (Y_i - T_i)^2 < \varepsilon$$
(27)

where  $\varepsilon$  is the allowable error of discriminating convergence, generally a very small forward number. In actual engineering problems, errors are often difficult to avoid. Therefore, the termination condition with error can be written:

$$S(P) = \sum_{i=1}^{m} (Y_i - T_i)^2 < m\sigma^2$$
(28)

where  $\sigma$  is the standard deviation of the temperature measurement error. After determining the aforementioned parameters, the formula for identifying the internal defects of the corroded tube can be written:

$$P^{n+1} = P^n - \beta^n P^n \tag{29}$$

Therefore, the geometric shape of the inner wall of the corroded tube can be solved iteratively through eq. (26).

# The introduction of the firefly algorithm

Considering the large fluctuations in the data of some sampling points, the FA [28] is introduced to randomly optimize the data [29-31].

Mathematical description of FA:

Relative fluorescence brightness of fireflies:

$$I = I_0 e^{-\omega r_{ij}} \tag{30}$$

where  $I_0$  is the brightness of the brightest firefly, that is, the brightness of itself (where r = 0),  $\omega$  – the light absorption coefficient, which can be set as a constant, and  $r_{ij}$  – the distance between firefly *i* and *j*.

Mutual attraction:

$$v(r) = v_0 e^{-\gamma r_{ij}^2}$$
(31)

where  $V_0$  is the maximum attraction, that is, the attraction at the light source (r = 0). - Optimal target iteration:

$$adi(t+1) = adi(t) + v[adj(t) - adi(t)] + \alpha \left(rand - \frac{1}{2}\right)$$
(32)

where *adi* and *adj* are the spatial positions of the two fireflies *i* and *j*, respectively,  $\alpha$  is the spatial step size, and *rand* – the random factor that obeys a uniform distribution on [0, 1].

When performing stochastic optimisation of temperature information, some basic parameters of the firefly optimisation algorithm were set the population size was set to 100, the initial fluorescein concentration was set to 5, the volatility factor of fluorescein was set to 0.4, the enhancement factor of fluorescein was set to 0.5, the domain change rate was set to 0.06, and the adaptation extraction ratio was set to 0.06.

Flow chart of the hybrid algorithm after the introduction of FA in presented in fig. 7.



Figure 7. Algorithm flow chart

### Numerical experiment and analysis

In this paper, the aforementioned circular tube model is used for numerical experiments, two kinds of corrosion of the inner wall surface are considered, and the influence of the different number of measurement points and measurement errors on the experiment is analyzed [32]. The radius of the outer wall of the pipe is 0.18 m, the radius of the inner wall is 0.13 m, the thermal conductivity of the material is  $\lambda = 25$  W/mK, and the convective heat transfer coefficient of the inner wall is  $h_i = 1000$  W/m<sup>2</sup>K. The outer wall is natural-convection. Assuming that the inner wall parameters after two kinds of corrosion are.

The first type of inner wall corrosion defect:

$$r_{1} = \begin{cases} 0.130 \quad \left(0 \le \theta < \frac{2\pi}{3} \bigcup \frac{4\pi}{3} < \theta \le 2\pi\right) \\ 0.165 \qquad \left(\frac{2\pi}{3} \le \theta \le \frac{4\pi}{3}\right) \end{cases}$$
(33)

The second type of inner wall corrosion defect:

$$r_2 = 0.03\sin(\theta) + 0.130, \ (0 \le \theta \le 2\pi)$$
 (34)

Measurement error,  $\sigma$ 

fig. 9.

Take the first type of corrosion of the inner wall boundary as an example, at the number of measurement points N = 20, consider the effect of different measurement errors on the experimental results, respectively take  $\sigma = 0.01, 0.2, 0.4$  for the simulation experiments.

The identification results using the CGM are shown in fig. 8.

It can be observed from tab. 1 and fig. 8 that the inversion accuracy is highest when the measurement error is  $\sigma = 0.01$ . As the measurement error increases, the fluctuation of the inversion value increases. When  $\sigma = 0.4$ , the geometric shape of the inner wall can only be roughly recognized.

0.01



Table 1. The error value of the inversion result of a single algorithm under different measurement errors

The recognition result after introducing FA for random optimization is illustrated in

It can be revealed from the comparison of tabs. 1 and 2 and figs. 8 and 9 that after the introduction of FA, the fluctuation of some data values is suppressed to a certain extent. Particularly, the image changes when  $\sigma = 0.4$  is the most significant, and the inversion accuracy is greatly improved. However, the inversion value still fluctuates greatly when the shape of the inner wall defect occurs abruptly, thus, it needs to be further improved.

0.4

0.2

In the following content, the second type of corrosion of the inner wall boundary is used as an example (introducing FA), and the influence of the number of measured nodes N on the experimental results is considered. Perform simulation experiments when N = 14, 28, 56, respectively.



Table 2. Error values of inversion results after introducingFA under different measurement errors

As indicated from tab. 3 and fig. 10, the measurement information is deficient when the number of measurement points N is 14. The inversion result has certain fluctuations when  $\sigma = 0.05$ . The inversion result fluctuates more and the deterioration magnifies significantly when  $\sigma = 0.2$ . Moreover, a more accurate inversion result is obtained when the number of measuring points N is 28. The inversion accuracy is further improved when the number of measuring points N changes from 28-56.

Table 3. Error values of inversion results under different number of measuring points

Number of measuring point, N	14		28		56	
Measurement error, $\sigma$	0.05	0.2	0.2	0.05	0.2	0.05
Relative mean error, $\eta\%$	2.84	6.13	0.04	2.57	0.03	1.95



# Conclusions

In this paper, the geometric shape of the inner wall of the 2-D steady-state circular pipe is inverted by the FEM and the CGM optimized, are as follows.

- For the forward heat conduction problem in this paper, after the feasibility of the algorithm is proved by taking circular pipe as objects, the FEM is used to solve the analysis. Simulation results show that the algorithm has good stability and improves the efficiency of numerical experiments.
- The analysis and solution of numerical experiments verify that the method in this paper has high accuracy in the inversion process. Meanwhile, the influence of the different number of measuring points and measuring error on the experimental results is discussed. It is demonstrated that two different types of corrosion on the inner wall of the tube can be effectively identified when there is a certain measurement error or basic temperature information of the measuring point.
- The conjugate gradient algorithm is used to identify the geometric shapes of the corroded inner walls of two pipe-lines. After finding that the recognition effect is not particularly ideal at the mutation of the objective function, the firefly algorithm is introduced to optimize the processing of temperature information based on the conjugate gradient algorithm. This further improves the stability of the inversion results compared to the CGM.

#### Nomenclature

- $A \text{area of mesh}, [m^2]$
- a undetermined coefficient, [–]
- C hot fusion matrix, [–]
- $\partial_e$  cell boundary, [–]
- h heat transfer coefficient, [Wm<sup>-2</sup>K<sup>-1</sup>] K – temperature stiffness matrix, [–]
- jk boundary edge, [m]
- N functions of the mesh, [–]
- n normal vector, [–]
- P node temperature load column vector, [–]
- q heat flux density, [Jm<sup>-</sup>2s<sup>-1</sup>]  $\nabla S(p^n)$  – descending gradient, [–]
- T temperature, [K]  $T_a$  – temperature of the fluid, [K]
- $T_f$  ambient temperature, [K]
- $\Delta t$  time step, [–]

## Greek symbols

- $\Gamma$  surface of the pipe, [–]
- conjugate coefficient, [-] γ
- relative mean error, [-] n
- thermal conductivity, [Wm<sup>-1</sup>K<sup>-1</sup>] λ
- $\rho$  density, [kgm<sup>-3</sup>]
- $\sigma$  measurement error, [–]
- $\omega_l$  weighting function, [–]

### **Subscripts**

- e cell domain
- i, j, k mesh vertex position

# Acronyms

- FA - firefly algorithm
- finite element method FEM
- CGM - conjugate gradient method
- DFIM decentralized fuzzy inference method

## **References**

- Wang, S. B., et al., Solving of 2-D Unsteady-State Heat transfer Inverse Problem Using Finite Difference [1] Method and Model Prediction Control Method, Complexity, 2019 (2019), 7, pp. 1-12
- Gonzalez, M., et al., Inverse Geometry Heat Transfer Problem Based on a Radial Basis Functions Ge-[2] ometry Representation, International Journal for Numerical Methods in Engineering, 65 (2006), 8, pp. 1243-1268
- [3] Lu, T., et al., A 2-D Inverse Heat Conduction Problem in Estimating the Fluid Temperature in a Pipe-Line, Applied Thermal Engineering, 30 (2010), 13, pp. 1574-1579
- [4] Szenasi, S., et al., Configuring Genetic Algorithm to Solve the Inverse Heat Conduction Problem, Acta Polytechnica Hungarica, 14 (2017), 6, pp. 133-152
- Zhang, T., et al., Seeking Heat Source in Inverse Heat Conduction Problem by Using Particle Swarm [5] Optimization, University of Shanghai for Science and Technology, 35 (2013), 4, pp. 377-381
- [6] Peker, H. A., et al., Application of Kashuri Fundo Transform and Homotopy Perturbation Methods to Fractional Heat Transfer and Porous Media Equations, Thermal Science, 26 (2022), 4A, pp. 2877-2884
- [7] Cuha, F. A., et al., Solution of Abel's Integral Equation by Kashuri Fundo Transform, Thermal Science, 26 (2022), 4A, pp. 3003-3010
- [8] Peker, H. A., et al., Solving Steady Heat Transfer Problems Via Kashuri Fundo Transform, Thermal Science, 26 (2022), 4A, pp. 3011-3017
- Jiang, S., et al., Geometry Estimation of Furnace Inner Wall Based on BEM and Decentralized Fuzzy [9] Inference Method, World Sci-Tech R and D, 65 (2006), 8, pp. 122-126
- [10] Luo, Z., et al., Fuzzy Estimation for Irregular Configuration of Furnace, Journal of Engineering Thermophysics, 10 (2013), 8, pp. 1906-1909
- [11] Wang, D., Thermal Conducting Geometric Inversion Method Based on the Third Boundary Condition (in Chinese), Ph. D. thesis, Harbin Institute of Technology, Harbin, China, 2018
- [12] Li, Y., Non-Iterative Algorithm for Identifying Boundary Conditions and Geometry Shapes in Heat Conduction Problems (in Chinese), M. Sc. thesis, Hefei University of Technology, Hefei, China, 2018
- [13] Wang, S., et al., The 2-D Steady-State Boundary Shape Inversion of CGM-SPSO Algorithm on Temperature Information, Advances in Materials Science and Engineering, 2017 (2017), ID2461498
- [14] Mei, L., et al., A New Method to Evaluate the Subsurface Defect by Thermal Non-destructive Testing, Infrared Millim Waves, 19 (2000), 6, pp. 457-459
- [15] Sun, S., et al., Back Calculation for Cold-state Geometric Shape of Turbine Blade Based on Modification of Nodal Co-Ordinates and Mesh Smoothing, Journal Of Mechanical Engineering, 50 (2014), 10, pp. 143-148
- [16] Fazeli, H., et al., Shape Identification Problems on Detecting of Defects in a Solid Body Using Inverse Heat Conduction Approach, Journal of Mechanical Science and Technology, 26 (2012), pp. 3681-3690

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- [17] Fazeli, H., et al., Estimation of Location and Size of Defects in a Solid Body via Inverse Heat Conduction Problem, *Proceedings*, 14<sup>th</sup> International Heat Transfer Conference, Washington DC, USA, 2010, pp. 387-396
- [18] Li, Y., et al., A Decentralized Fuzzy Inference Method for the Inverse Geometry Heat Conduction Problem, Applied Thermal Engineering, 106 (2016), 8, pp. 109-116
- [19] Cheng, C. H., Cheng, M. H., Shape Identification by Inverse Heat Transfer Method, Journal of Heat Transfer, 125 (2013), 2, pp. 224-231
- [20] Mosavati, M., et al., A Novel, Non-iterative Inverse Boundary Design Regularized Solution Technique Using the Backward Monte Carlo Method, *Journal of Heat Transfer*, 135 (2013), 4, pp. 130-136
- [21] Mosavati, B., et al., Solution of Radiative Inverse Boundary Design Problem in a Combined Radiating-Free Convecting Furnace, International Communications in Heat and Mass Transfer, 45 (2013), July, pp. 130-136
- [22] Mosavati, B., et al., Inverse Boundary Design Solution in a Combined Radiating-Free Convecting Furnace Filled with Participating Medium Containing Specularly Reflecting Walls, International Communications in Heat and Mass Transfer, 76 (2016), Aug., pp. 69-76
- [23] Chen, W., Yang, Y., Inverse Estimation for Unknown Fouling-Layer Profiles with Arbitrary Geometries on the Inner Wall of a Forced-Convection Duct, *International Journal of Thermal Sciences*, 49 (2010), 1, pp. 86-98
- [24] Fan, C., et al., Research on the Recognition Algorithm of Irregular Boundary of Inner Wall of Circular Tube Based on Infrared Temperature Measurement, *Journal of Thermal Science and Technology*, 02 (2006), pp. 112-117
- [25] Partridge, P. W., Wrobel, L. C., An Inverse Geometry Problem for the Localisation of Skin Tumours by Thermal Analysis, *Engineering Analysis with Boundary Elements*, 31 (2007), 10, pp. 803-811
- [26] Zhu, L., et al., Estimating Steady Multi-variables Inverse Heat Conduction Problem by Using Conjugate Gradient Method, Proceedings of the CSEE, 31 (2011), 8, pp. 58-61
- [27] Zhang, Y. W., et al., Inverse Heat Conduction Problem of Deducing Inner Wall Temperature by Using Conjugate Gradient Method, Journal Of Engineering Thermophysics, 30 (2009), 7, pp. 1188-1190
- [28] Yang, X. S., Firefly Algorithm, Stochastic Test Functions and Design Optimization, *International Journal of BioInspired Computation*, 2 (2010), 2, pp. 78-84
- [29] Ma, X., et al., Energy-Saving Operation Optimization of High-Speed Trains Using Firefly Algorithm, Journal of Huaqiao University (Natural Science), 4 (2019), pp. 452-456
- [30] Zuo, Z., et al., An Improved Swarm Optimization Alogorithm, Microelectronics and Computer, 35 (2018), 2, pp. 61-66
- [31] Gao, W., Study on the Firefly Algorithm and Application (in Chinese), Lanzhou University, Lanzhou, China, 2013
- [32] Lu, S., et al., Identification of Irregular Pipe-Line Geometry Boundary Using Infrared Transient Inspection Based on Finite Element Discretization, CIESC Journal, 63 (2012), 12, pp. 3805-3811