STUDY ON THE PRESSURE WAVE VELOCITY MODEL OF MULTIPHASE FLUID IN THE ANNULUS OF DUAL-GRADIENT DRILLING

by

Xiucheng GUO[a] and Zhe ZHANG[b]*

[a]GWDC Drilling Engineering and Technology Research Institute, CNPC Greatwall Drilling Company, Panjin City, Liaoning, China
[b]Southwest Petroleum University School of Electrical Information, Chengdu City, Sichuan, China

Dual-gradient drilling technology is difficult to detect and deal with well kick, which may cause safety problems such as well control. Once a well kick occurs, the pressure in the narrow annulus increases instantaneously, so it is particularly important to clarify the transmission law of pressure wave. Taking the pressure fluctuation in the annulus of double gradient drilling as an example, considering the boundary between the inner wall of the casing annulus and the outer wall of the drill string, the fluid resistance, the flow field distribution and the multiphase flow, this paper deduces and establishes the multiphase flow pressure wave velocity model in the annulus, and compares the calculation results with the pressure wave velocity model in the circular pipe. The research shows that when the casing and drill string are in the static condition with small roughness and little difference between them, the shear stress on the inner wall of the annulus and the outer wall of the drill string can be treated as equal, and the derivation of the annulus pressure wave velocity model can be simplified. The pressure wave velocity of multiphase flow in annulus is related to casing size, drill string size and multiphase fluid properties. Among them, the gas content is the main influencing factor. With the increase of the gas content, the pressure wave velocity decreases, but the decreasing range will gradually decrease. When the solid content is small, it has little effect on the pressure wave velocity.

Key words*: dual-gradient drilling, annulus, multiphase flow,
        pressure wave velocity*

Introduction

Double-gradient drilling (DGD) technology is to take certain measures to make the fluid density in the riser close to the density of seawater (all pressures are based on the seabed), so that the margin between formation fracture pressure and pore pressure is relatively increased [1-4]. Figure 1 shows the schematic diagram of conventional drilling technology and dual-gradient drilling technology.

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[*]Corresponding author, e-mail: zhangzhe2000@gmx.com

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Compared with conventional drilling, dual-gradient drilling has many advantages. This drilling method can relatively increase the margin between fracture pressure and pore pressure, and reduce well kick and lost circulation accidents. It can reduce the number of casing layers, shorten the well construction period and save drilling costs and it can reduce the requirements for drilling equipment such as drilling platforms and drilling rigs in deepwater drilling operations [5]. However, the dual-gradient drilling technology will have some difficulties in detecting and dealing with well kick, and there will be safety problems such as well control [6]. Once a kick occurs, the pressure in the narrow annulus increases instantaneously, and it is particularly important to clarify the transmission law of pressure waves.
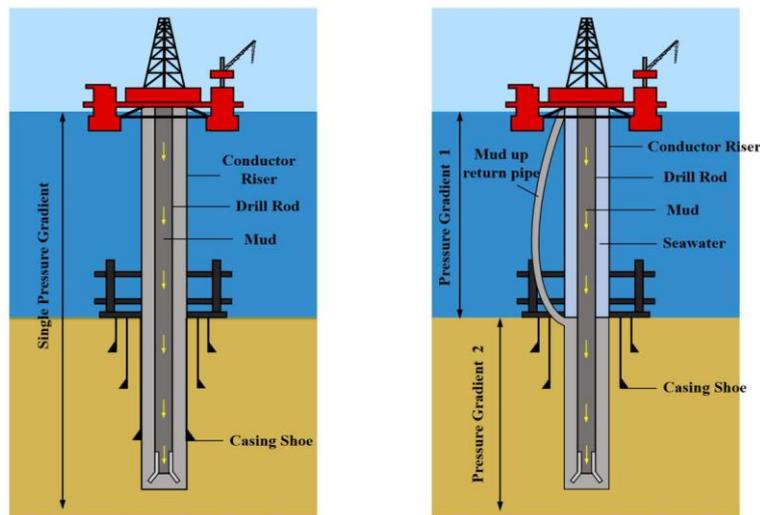
Figure 1. Conventional drilling technology and dual-gradient drilling technology; (a) conventional drilling technology and (b) dual-gradient drilling technology

Scholars at home and abroad have conducted many studies on pressure wave transmission in annulus. Ma *et al*. [7] designed complex well drilling planning tools with narrow error bounds to account for transient phenomena such as gas inflow during drilling operations. Shirdel *et al*. [8] proposed a three-phase flow model consisting of a liquid-gas two-fluid model and a liquid-phase water-oil drift flow model, which improved the simplified assumptions of the multiphase flow model of oil, water, and gas in the well. Gomes *et al*. [9] proposed a transient model for studying kick behavior by considering the factors of kick and solubility in oil-based mud, and studied the effects of wellbore size, pore pressure and riser geometry on the results. Luo *et al*. [10] established an annular fluid water hammer model caused by shut-in after gas invasion was discovered, and conducted sensitivity analysis on related factors such as the amount of invaded gas, well depth and shut-in time. Galdino *et al*. [11] proposed a transient compressible isothermal mathematical model to predict the pressure propagation problem in the wellbore during gas invasion. Mao *et al*. [12] established a two-phase flow model for sour gas wells, and analyzed the effects of H_2S content, mud displacement, and drilling fluid density on the overflow characteristics of sour gas wells. Fang *et al*. [13] established a theoretical model of the pressure wave velocity and attenuation coefficient of the gas flow, and solved it analytically using the classical fluid mechanics model and the theory of small disturbance.

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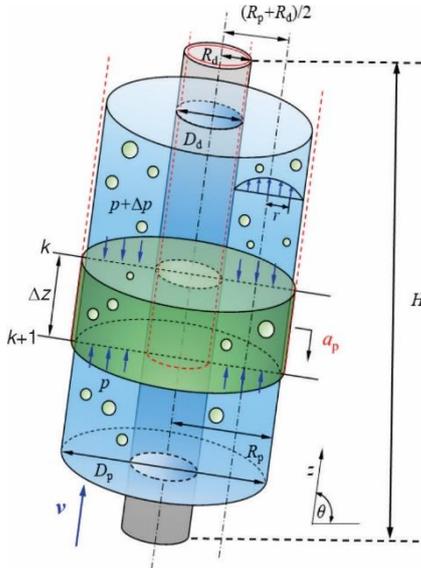
Figure 2. Physical model of annular fluid pressure wave transfer

Krishna *et al.* [14] established a simplified model for predicting tripping excitation pressure and swabbing pressure. Wang *et al.* [15] proposed an auxiliary kick detection method based on the pressure wave attenuation theory and applicable to all well types for the downhole pulsating pressure generated during the rock breaking process.

There is a fundamental difference between annular flow and circular tube flow. There is no obstacle in the center of the circular tube, except that the fluid will form a boundary-layer near the tube wall, other positions are in a fully developed state, and the flow velocity is the largest at the central position of the circular tube. When there is a drill string in the circular tube, the fluid will also form a boundary-layer near the outer wall of the drill string, and the maximum fluid velocity is in the center of the annulus. Due to the essential difference in the distribution of the flow field, it is far from enough to modify the coefficient based on the pressure wave velocity model in the circular tube. It is necessary to establish the pressure wave velocity model of the fluid in the annulus to truly reflect the propagation law of the fluid pressure in the annulus. This has an important guiding role for the study of the transient motion law of fluid in the annulus.

Pressure wave velocity model of annular multiphase flow

Annular pressure wave transmission mechanism

As shown in fig. 2, it is an annular tube, taking the flow section in the annulus as the research object, after Δt, the pressure wave is transmitted from the downstream k-k section to the upstream $k + 1$- $k+ 1$ section, and the distance between the two ends is $\Delta z = a_p\Delta t$, the upstream side of the $k + 1$-$k + 1$ section is not affected by the pressure wave, and within the time Δt, the mass of the fluid flowing from the upstream $k + 1$-$k + 1$ section is $\rho_p A v_p\Delta t$. The mass of the fluid flowing out from the downstream k-k section is $(\rho_p + \Delta\rho_p)(A + \Delta A)(v_p + \Delta v_p)\Delta t$, then the increment of the fluid mass in the flow section caused by the pressure wave in Δt is:

$$\left(\rho_p + \Delta\rho_p\right)\left(A + \Delta A\right)a_p\Delta t - \rho_p A a_p\Delta t = a_p\Delta\left(\rho_p A\right)\Delta t \tag{1}$$

Then the momentum change of the flow segment in the annulus is:

$$\left(\rho_p + \Delta\rho_p\right)\left(A + \Delta A\right)\left(v_p + \Delta v_p\right)a_p\Delta t - \rho_p A v_p a_p\Delta t \approx a_p\Delta\left(\rho_p A v_p\right)\Delta t \tag{2}$$

In the k-$k + 1$ section, the following forces exist in the flow direction: the pressure on the *j* section is pA, the pressure on the k + 1 section is $(p + \Delta p)(A + \Delta A)$. The component of the fluid gravity in the micro-body along the axis of the pipe is $G_z = \rho_p g A\Delta z\sin\theta$. The frictional resistance of fluid flowing in the annulus is $F_t = \tau_p\chi_p\Delta z + \tau_d\chi_d\Delta z$. Then there are:

$$-\Delta\left(pA\right) + \rho_p g A\Delta z\sin\theta - \left(\tau_p\chi_p\Delta z + \tau_d\chi_d\Delta z\right) = a_p\Delta\left(\rho_p A v_p\right) \tag{3}$$

Simplify eqs. (4) and (5) and separately propose annular fluid pressure wave velocity, a_p:

$$a_p = \frac{\sqrt{\dfrac{p}{A}\dfrac{dA}{dp} + 1 - \dfrac{\left(\rho_p g \sin\theta - \dfrac{\tau_p \chi_p + \tau_d \chi_d}{A}\right)}{\dfrac{\partial p}{\partial z}}}}{\sqrt{\rho_p \left(\dfrac{1}{A}\dfrac{dA}{p} + \dfrac{1}{\rho_p}\dfrac{d\rho_p}{dp}\right)}} \tag{4}$$

If the pressure wave velocity, a, in the circular tube is used to define the denominator part of the eq. (4), that is:

$$a = \frac{1}{\sqrt{\rho_p \left(\dfrac{1}{A}\dfrac{dA}{dp} + \dfrac{1}{\rho_p}\dfrac{d\rho_p}{dp}\right)}} \tag{5}$$

Then the calculation formula of the fluid pressure wave velocity in the annulus can be expressed as:

$$_p = a\sqrt{\dfrac{p}{A}\dfrac{dA}{dp} + 1 - \dfrac{\rho_p g \sin\theta - \dfrac{\tau_p \chi_p + \tau_d \chi_d}{A}}{\dfrac{\partial p}{\partial z}}} \tag{6}$$

If it is assumed that the roughness of the inner wall of the annulus and the outer wall of the drill string are approximately equal, the shear stress of the inner wall of the annulus and the outer wall of the drill string are also approximately equal, that is:

$$\tau_p = \rho_p g \frac{h_f}{\Delta z}\frac{\left(D_p - D_d\right)}{4} \quad \rho_p g R J \tag{7}$$

Substitute eqs. (4)-(14) into eqs. (4)-(9) to get:

$$a_p = a\sqrt{\dfrac{p}{A}\dfrac{dA}{dp} + 1 - \dfrac{\rho_p g \sin\theta - \rho_p g J}{\dfrac{\partial p}{\partial z}}} \tag{8}$$

Because

$$\frac{\partial p}{z} \approx \rho_p g \frac{\partial H}{\partial z} + \rho_p g \sin\theta \tag{9}$$

$$J = -\frac{\partial\left[H + \left(\dfrac{v_p^2}{2g}\right)\right]}{\partial z} \tag{10}$$

Then the following is derived:

$$a_p = a \sqrt{\frac{\dfrac{dA}{A}}{\dfrac{dp}{p}} - \frac{\dfrac{\partial}{\partial z}\left(\dfrac{v_p^2}{2g}\right)}{\dfrac{\partial}{\partial z}\left(\dfrac{p}{\rho_p g}\right)}} \tag{11}$$

Equation (11) is the relational expression of the pressure wave velocity of the annular pipeline. The partial derivative terms represent the rate of change of fluid kinetic energy and pressure energy in the flow direction, respectively. Due to the complex flow pattern structure of the fluid in the annulus, the pressure wave velocity and any variable are not purely linear.

It should be noted that the shear stress on the inner wall of the annulus and the outer wall of the drill string is treated as equal, which is only suitable for static conditions where the roughness of the outer casing and the drill string are both small and the difference between the two is not large. If the roughness of the outer pipe and the inner drill string is quite different, or if the relative motion conditions such as drilling and drilling are encountered, then these two shear stresses need to be calculated separately at this time.

Derivation of annular pressure wave velocity model

For the cross-section of the annular pipe, there is:

$$dA = \frac{\pi}{2}\left(D_p dD_p - D_d dD_d\right) \tag{12}$$

Therefore:

$$\frac{1}{A}\frac{dA}{dp} = \frac{2D_p^2}{D_p^2 - D_d^2}\frac{dD_p}{D_p}\frac{1}{dp} - \frac{2D_d^2}{D_p^2 - D_d^2}\frac{dD_d}{D}\frac{1}{dp} \tag{13}$$

The elastic modulus of the inner and outer ring pipe wall materials of the annular pipe is the ratio of stress to strain. Let the stress change of the outer ring pipe wall material be $d\sigma_a$, the stress change of the inner ring pipe wall material be $d\sigma_t$, and the corresponding strains are dD_a/D_a and dD_t/D_t, then:

$$\frac{1}{A}\frac{dA}{dp} = \frac{D_p^3}{D_p^2 - D_d^2}\frac{1}{E_p \delta_p} - \frac{D_d^3}{D_p^2 - D_d^2}\frac{1}{E_d \delta_d} \tag{14}$$

Taking the single-phase flow in the annulus as an example, considering the compressibility of the medium, the pressure wave velocity, a_p, of the fluid in the annulus can be obtained as:

$$a_p = \sqrt{\frac{E_l}{\rho_p\left[1+\left(\dfrac{D_p^3}{D_p^2 - D_d^2}\dfrac{E_l}{\,_p\delta_p} - \dfrac{D_d^3}{D_p^2 - D_d^2}\dfrac{E}{E_d \delta_d}\right)\right]}} \tag{15}$$

where E_l [MPa] is the bulk modulus of the fluid in the annulus.

The pressure wave velocity derived from the mechanism of fluid flowing in the annulus is not only related to the properties of the fluid itself, pipe diameter and wall thickness, but also to the diameter and wall thickness of the inner cylinder that constitutes the annulus.

When considering the multiphase flow factor, the density, ρ_{pm}, of the multiphase mixture in the annulus is defined as:

$$\rho_{pm} = \rho_g X_g + \rho_s X_s + \rho_l \left(1 - X_g - X_s\right) \tag{16}$$

Then the pressure wave velocity, a_{pm}, under multiphase conditions should be defined as:

$$a_{pm} = \sqrt{\frac{1}{\rho_{pm}\left(\varepsilon_1 + \varepsilon_2\right)}}, \quad \varepsilon_1 = \frac{1}{E_g}X_g + \frac{1}{E_s}X_s + \frac{1}{E_l}\left(1 - X_g - X_s\right)$$

$$\varepsilon_2 = \frac{D_p^3}{D_p^2 \, D_d^2}\frac{1}{E_p \delta_p} - \frac{D_d^3}{D_p^2 - D_d^2}\frac{1}{_{d} \quad _{d}} \tag{17}$$

Influence analysis of pressure wave velocity

When the drilling fluid-flows in the annulus, in addition to contact with the inner wall of the casing, there will also be friction with the outer wall of the drill string that constitutes the annulus to form a viscous force. It is inferred from this that the pressure wave velocity in the annulus should be smaller than that in the hollow casing. In this section, the sensitivity analysis and calculation of the pressure wave velocity of the annulus fluid are carried out based on the actual drilling data of an example well, and the comparison with the wave velocity model in the hollow casing is made. The influence of casing, drill string size and annulus medium on the pressure wave velocity is discussed, tab. 1.

Influence of casing size

When the outer diameter of the casing is a certain value, the wall thickness increases and the inner diameter decreases. Regardless of whether there is a drill string, the space inside the casing will decrease accordingly, resulting in an increase in the wave velocity. That is, with the increase of the casing wall thickness, the pressure wave velocity of the fluid in the casing increases. Figure 3 shows the comparison of the results under different casing wall thicknesses when the two methods are used to calculate the pressure wave velocity of the single-phase fluid in the wellbore. The outer diameter of the casing used is 177.8 mm. There are six wall thicknesses of 10.36 mm, 11.51 mm, and 12.65 mm, and the drilling fluid with a density of 1250 kg/m³ is used as the fluid in the pipe.
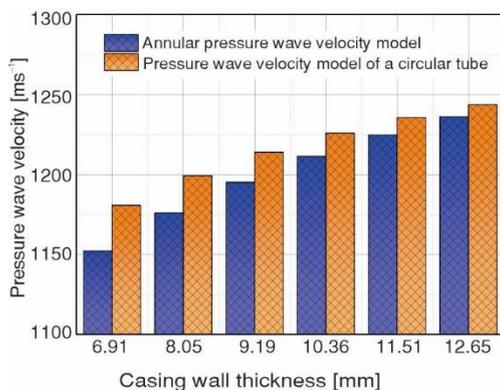
Figure 3. Comparison of two methods for calculating single-phase pressure wave velocity (influence of casing wall thickness)
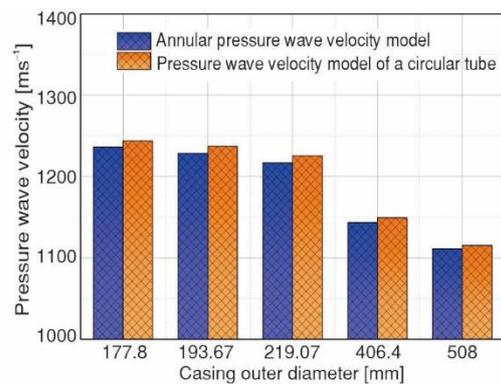
Figure 4. Comparison of two methods for calculating single-phase pressure wave velocity (influence of casing outer diameter)

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Table 1. Specifications and parameters of drill string

Outer diameter [mm]	Steel grade	Wall thickness [mm]	Tensile strength [MPa]	Extrusion strength [MPa]	Internal compressive strength [MPa]	Torsional strength [kNm]
88.9	X-95	9.35	1530.13	123.26	120.52	31.86

It can be seen from the figure that with the increase of the casing wall thickness, the pressure wave velocity calculated by the two methods increases, and the pressure wave velocity in the annulus is smaller than that in the hollow casing, which also confirms the inference. However, it can be found that as the wall thickness increases, the corresponding pressure wave velocity difference becomes smaller and smaller. The reason for this phenomenon is that, according to the calculation of the annular wave velocity model, there is a viscous force between the fluid and the inner wall of the drill string, but the annular area is very small at this time, the pressure wave velocity increases compared with the small wall thickness, and the proportion of this effect is also gradually increased. As the wall thickness continues to increase, when the effect is greater than that of the viscous force, the annular pressure wave velocity will be greater than the pressure wave velocity in the hollow casing.

Figure 4 shows the comparison of the results under different casing outer diameters when the two methods are used to calculate the pressure wave velocity of the single-phase fluid in the pipe. The casing outer diameters used are 177.8 mm, 193.67 mm, 219.07 mm, 406.4 mm, and 508 mm, the corresponding steel grades are C-95, P110, N-80, J-55, and K-55, the casing wall thickness is 12.7 mm ($\varnothing$177.8 is 12.65 mm, $\varnothing$406.4 is 12.57 mm). Drill string specifications and drilling fluid properties are the same as previous.

When the casing wall thickness is not much different, no matter whether there is a drill string or not, the larger the casing outer diameter and the larger the space, the smaller the pressure wave velocity should be. It can be seen from the figure that as the outer diameter of the casing increases, the pressure wave velocity calculated by the two methods decreases. However, due to the viscous force between the fluid and the outer wall of the drill string, the calculated value of the circular tube wave velocity model is still larger than the calculated value of the annular wave velocity model. The smaller the annulus area, the faster the pressure wave velocity. Therefore, with the increase of the outer diameter of the casing, the calculated value of the circular tube wave velocity model decreases, and the calculated value of the annular wave velocity model increases. It is inferred that in a larger casing with the same wall thickness, the calculated value of the circular tube wave velocity model will be smaller than that of the annular wave velocity model.

Influence of drill string size

It can be seen from the previous results that the calculated value of the pressure wave velocity model considering the existence of the drill string is smaller than the calculated value of the circular pipe pressure wave velocity model. So how does the size of the drill string affect the calculation of the pressure wave velocity?

A casing with an outer diameter of 177.8 mm was used, the steel grade was C-90, the wall thickness was 12.7 mm, and the properties of the drilling fluid were the same as above. The drill string specifications are shown in tab. 2. In order to make the trend more reasonable, five sets of drill string specifications with the wall thickness increasing with the outer diameter and all the same materials were selected. From the data in the table, it can be seen that with the increase of drill string size, the value of the calculated pressure wave veloc-

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ity does not have a linear trend. When the outer diameter of the casing remains unchanged, the outer diameter of the drill string increases, which means that the annulus area decreases. However, it also means that the viscous force generated by the existence of the drill string increases, so the phenomenon that the pressure wave velocity first increases and then decreases, and then increases and decreases again.

Table 2. Influence of drill string of different specifications on annular pressure wave velocity

Outer diameter [mm]	Steel grade	Wall thickness [mm]	Tensile strength [MPa]	Extrusion strength [MPa]	Tensile and compressive strength [MPa]	Torsional strength [kNm]	Pressure wave velocity [ms⁻¹]
60.33	X-95	7.11	778.76	136.23	135.14	10.73	1239.81
73.03	X-95	9.19	1207.7	144.18	144.33	19.84	1245.74
88.9	X-95	9.35	1530.13	123.26	120.52	31.86	1236.07
101.6	X-95	9.65	1826.21	112.63	108.9	44.32	1236.60
114.3	X-95	10.92	2323.38	113.22	109.53	63.37	1231.94

Table 3. Influence of drill string wall thickness on annular pressure wave velocity

Outer diameter [mm]	Steel grade	Wall thickness [mm]	Tensile strength [MPa]	Extrusion strength [MPa]	Tensile and compressive strength [MPa]	Torsional strength [kNm]	Pressure wave velocity [ms⁻¹]
88.9	X-95	6.45	1094.56	83.27	83.19	24.29	1248.88
88.9	X-95	9.35	1530.13	123.26	120.52	31.86	1236.07
88.9	X-95	11.40	1818.64	146.49	147.05	36.21	1231.06

Table 3 shows the effect of wall thickness on the annular pressure wave velocity under the condition of the same outer diameter of the drill string. It can be seen from the data in the table that with the increase of wall thickness, the pressure wave velocity decreases.

Influence of annular medium

During dual gradient drilling, once a kick occurs, a lot of free gas and solid particles will appear in the wellbore annulus. Figure 5 shows the corresponding annular fluid pressure wave velocity at the concentration of different types of solid particles without considering the gas intrusion into the wellbore. The elastic modulus of solid particles is difficult to measure, and the elastic modulus of the same solid type is different in different blocks or layers, so sandstone and limestone are selected as the research objects here, and only one of the corresponding elastic modulus is selected. (The elastic modulus of sand is in the range of 4.8G Pa ~ 84.1 GPa, in order to compare with limestone, the smaller value should be taken), that is, the elastic modulus of sandstone is 16.2 GPa, and the elastic modulus of limestone is 88.5 GPa.

It can be seen from fig. 5 that as the concentration of solid particles increases, the pressure wave velocity of the annular fluid also increases gradually. The reason is that when there are pressure fluctuations in the drilling fluid, the solid particles are also forced to compress. The greater the elastic modulus of the particle, the smaller the elastic deformation caused by the external force, and the less work the particle does to the surrounding drilling fluid when the particle recovers its deformation. Therefore, the lower the energy dissipation of the pressure wave, the lower the wave speed. and this trend becomes more pronounced with the increase of solid particle concentration.

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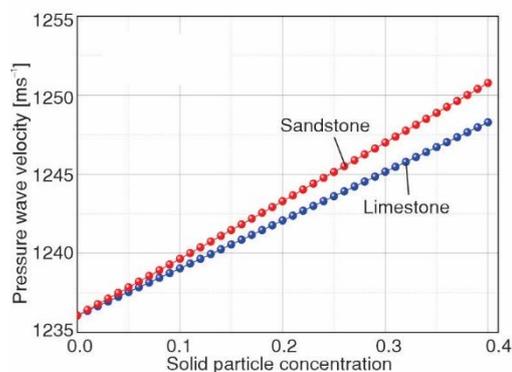
Figure 5. Variation of pressure wave velocity with different solid particle concentrations
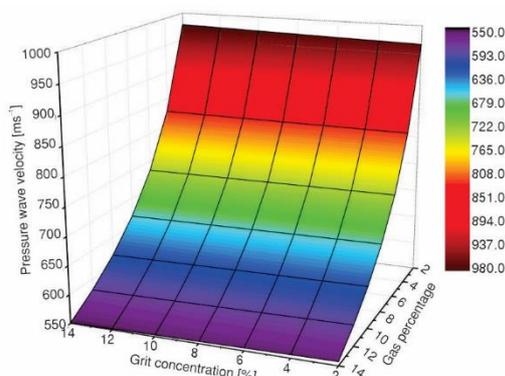
Figure 6. Effects of different gas content and sand concentration on pressure wave velocity of annular fluid

Table 4 shows the pressure wave velocity of annular fluid under the combined influence of different gas content and gravel concentration when the solid particles are gravel.

Table 4. Pressure wave velocity of annular fluid under different gas content and solid particle concentration (solid particles are sandstone)

Solid particle concentration [%] / Gas content [%]	2%	4%	6%	8%	10%	12%	14%
2%	967.29	969.00	970.77	972.60	974.49	976.44	978.46
4%	824.29	824.60	824.94	825.31	825.72	826.15	826.62
6%	733.79	733.48	733.20	732.93	732.69	732.47	732.28
8%	670.40	669.76	669.15	668.55	667.97	667.41	666.86
10%	623.14	622.31	621.50	620.70	619.92	619.15	618.40
12%	586.41	585.46	584.51	583.58	582.67	581.77	580.89
14%	557.01	555.96	554.93	553.90	552.90	551.91	550.93

As shown in tab. 2, taking the gravel concentration as 2%, when the gas content increases from 2% to 4%, the pressure wave velocity decreases from 967.29 m/s to 824.29 m/s, a decrease of 143 m/s. However, when the gas content was increased from 12% to 14%, the pressure wave velocity decreased from 586.41 m/s to 557.01 m/s, which only decreased by 29.4 m/s. According to the data, in the actual dual gradient drilling process, the gas content of the annular multiphase fluid is the main influencing factor. With the increase of gas content, the pressure wave velocity decreases, but the decreasing range will gradually decrease.

Plot the data in tab. 2 into a 3-D graph, as shown in fig. 6. It can be seen from the figure that under the coupled influence of gas-solid content, the increase or decrease of gas content plays a leading role, and when the solid content is small, the pressure wave velocity has little effect.

Conclusions

In this paper, taking the pressure fluctuation condition in the DGD annulus as an example, considering the boundary, fluid resistance, flow field distribution, and multiphase flow of the casing annulus and the outer wall of the drill string, the annulus multiphase flow is deduced and established. The flow pressure wave velocity model was compared with the cal-

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culation results of the pressure wave velocity model in the circular tube, and the following conclusions were drawn.

- When the roughness of casing and drill string are both small and the difference between the two is small, the shear stress on the inner wall of the annulus and the outer wall of the drill string can be treated as equal to simplify the derivation of the annular pressure wave velocity model.
- The pressure wave velocity of annular multiphase flow is related to casing size, drill string size and properties of multiphase fluid.
- The gas content is the main influencing factor. With the increase of the gas content, the pressure wave velocity decreases, but the decreasing range will gradually decrease. When the solid content is small, the influence on the pressure wave velocity is not significant.

Acknowledgment

Nomenclature

A – cross-sectional area of the annulus, [m^2]
a – pressure wave velocity in the circular tube, [ms^{-1}]
a_p – pressure wave velocity in the annulus, [ms^{-1}]
a_{pm} – pressure wave velocity of the annular multiphase flow, [ms^{-1}]
D_d – drill string outer diameter, [mm]
D_p – casing outer diameter, [mm]
E_d – drill string elastic modulus, [MPa]
E_g – gas elastic modulus, [MPa]
E_l – liquid elastic modulus, [MPa]
E_p – casing elastic modulus, [MPa]
E_s – solid elastic modulus, [MPa]
F_t – frictional resistance of the fluid flowing in the annulus, [N]
G_z – component of the fluid gravity in the micro-element along the axis of the pipeline, [N]
H – pressure head, [m]
J – hydraulic slope, [m]
p – annulus pressure, [MPa]

R – inner diameter of the annulus, [mm]
Δt – time step, [s]
v_p – annular flow velocity, [ms^{-1}]
Δz – space step, [m]
X_g – gas percent content, [–]
X_s – solid percent content, [–]

Greek simbol

δ_d – drill string wall thickness, [mm]
δ_p – casing wall thickness, [mm]
ρ_g – gas density, [kgm^{-3}]
ρ_l – liquid density, [kgm^{-3}]
ρ_{pm} – annular mixed fluid density, [kgm^{-3}]
ρ_p – annulus fluid density, [kgm^{-3}], [MPa]
ρ_s – solid density, [kgm^{-3}]
τ_d – shear stress of the outer wall of the drill string, [MPa]
χ_d – wetted circumference of the outer wall of the drill string, [mm]
χ_p – wetted circumference of the inner wall of the annulus, [mm]

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