Study of a novel ternary second-order viscosity model on Al$_2$O$_3$-H$_2$O nanofluid

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Abstract: In terms of heat dissipation, nanofluids with strong thermal conductivity are becoming more and more common, attracting more and more research and attention to date. In this study, Al$_2$O$_3$-H$_2$O nanofluids were studied using molecular dynamics simulations and model parameterization. The dynamic viscosity distribution patterns were obtained at various temperatures(290K~360K), nanoparticle volume fractions(1.24%~6.2%), and particle sphericity(0.69~1.0), and an efficient ternary second-order polynomial viscosity prediction model was proposed on the basis of these results. The findings demonstrate the model's goodness-of-fit with a coefficient of determination over 0.96 and a root mean square error under 0.05, as well as its high predictive ability with a maximum relative error between simulated and predicted values under 9%. Using this viscosity prediction model, a subsequent parametric sensitivity study shows that the volume fraction had the most significant impact on viscosity, exhibiting not just a second order effect but also an interacting effect with temperature and sphericity. The relative nanofluid viscosity, which is the ratio of nanofluid viscosity to aqueous base fluid viscosity, exhibits a convex parabolic growth at constant temperature and sphericity and increases more quickly at the same volume fraction the higher the temperature. The viscosity of the nanofluid increases by up to 34% when the volume fraction is equal to 6.28% and the particle sphericity is equal to 1. An efficient viscosity prediction model makes it easier to control important variables to reduce energy consumption during flow and increase its capacity to dissipate heat.

Keywords: dynamic viscosity; alumina-water nanofluid; prediction model; goodness-of-fit

1. Introduction

The amount of thermal power used per unit volume of electronic devices likewise rises quickly as a result of their increasing high integration, miniaturization, and sophisticated functionalities. As of right now, the chip's greatest operating temperature can even reach 200°C, and its heat flow is growing by 7% annually[1, 2]. The reliability and performance of electronic components will be impacted by high temperatures. This calls for the development of effective cooling technology. If not, heat failure will occur throughout the entire electrical product[3-5].

Due to their high thermal conductivity and difficulty in blocking, nanofluids—a novel type of heat transfer medium—have gained increasing attention in the field of heat transfer over the past few years[6-9]. They are created by suspending a specific concentration of solid nanoparticles with a size of less than 100 nm in a base fluid. One of the most crucial thermal qualities, viscosity, controls the pumping effort and pressure drop during flow as well as the nanofluid's flow state. Viscosity is an
important thermal property which influences the fluid’s resistance and leads to an increase in pump work as the viscosity increases. Therefore, a precise prediction of viscosity is essential for the use of heat transfer fluid medium in these and other fields[10] to build the thermal system and calculate the necessary pumping power[11].

Currently, numerical simulations and experimental research are the two main methods used to study viscosity. F Garoosi[12] put forward two new empirical formulas to calculate the effective dynamic viscosity of nanofluids and presented the relationship between the effective dynamic viscosity and particle volume fraction (0 ≤ ψ ≤ 12%), particle size (10nm ≤ dp ≤ 5μm), particle type (Ag, Cu, Al2O3, TiO2, CuO, SiO2, etc.), base liquid temperature and thermophysical properties, and the data for establishing the equations depended on a large number of experimental data available in the literature. Furtherly, Which were found that the two models were more accurate than the frequently used classical models (eg. Brinkman and Maxwell-Garnett models) in literature. Georgiana Madalina Moldoveanu et al.[13] got the conclusion that Al2O3 water-based nanofluids possessed a non-Newtonian behavior and the viscosity of which was higher compared with SiO2 nanofluid. And an equation that varied with temperature was proposed, with a relationship that the viscosity decreased with the increase of temperature. Georgiana Madalina Moldoveanu et al.[14] obtained the relationship between relative viscosity and the volume fraction of nanofluids using regression analysis and curve fitting methods in the experimental study on the viscosity of stabilized Al2O3, TiO2 nanofluids and their hybrid. Andrey V. Minakov et al.[15] systematically studied the viscosity of more than 30 kinds of nanofluids such as Al2O3 and TiO2. And among them, the base fluids were distilled water, ethylene glycol and engine oil respectively, the volume concentration of nanoparticles was 0.25 to 8 percent, the size of nanoparticles ranged from 5 to 150nm, and the temperature changed from 25 to 60℃. The results showed that the viscosity of nanofluids increased with the decrease of nanoparticle size, and the higher the viscosity of the base fluid, the higher the increment of the viscosity of nanofluids. OA Alawi[16] studied the thermal conductivity and viscosity of various metal oxides with the 1-5% volume concentration of nanoparticles at the temperature of 300-320K. The results indicated that the viscosity increased significantly with the increase of nanoparticle concentration. Elif Begum Elcioglu et al.[17] carried out an experimental study on Al2O3 nanofluids by Taguchi experimental design plan. Data analysis made clear that the interaction between temperature and nanoparticle volume fraction on viscosity was obvious, and which was reflected in the viscosity equation. Ali Vakilinejad et al.[18] measured the kinematic viscosities of three nanofluids including Al2O3, TiO2 and graphene water-based nanofluids, and calculated the viscosity ratio to deionized water of all liquids. And polynomial models were proposed for each nanofluid, the percentage error can be as low as 0.06% for Al2O3 nanofluid among them. Mohammad Hemmat Esfe[19] used the experimental data of MWCNT (10%)-Al2O3 (90%) hybrid nanofluid to establish an artificial neural network model for predicting viscosity. It was found that temperature had the greatest influence on which and the R-squared values for the ANN was 0.9998.

As mentioned above, some research has been done on predictive models for viscosity up until this point. However, the form and even the parameters of these viscosity model expressions vary. Some studies only look at the effect of volume fraction on viscosity and ignore the temperature dependence[20-25]; others look at volume concentration and temperature but ignore the effect of particle size[26-30]; still others don’t look at the effect of particle shape on viscosity[31-34], and so on. Further investigation is required to precisely anticipate the viscosity of Al2O3 nanofluids, which will facilitate their application.
Therefore, the study aimed to present a three-variable second-order polynomial model with good predictive accuracy. Which consists of three parts mainly, including Molecular dynamics simulation on the viscosity of Al2O3-H2O nanofluid, Construction of a viscosity prediction model and results discussion. The following will be developed and studied in detail in the text.

2. Viscosity study on Al2O3-H2O nanofluid by molecular dynamics simulations

2.1 Al2O3-H2O nanofluid Molecular dynamics simulation

With the ongoing advancement of the theory, methodology, and computer technology of molecular simulation, it has become the third way to comprehend the universe from the molecular and atomic levels, treating it as a virtual laboratory[35]. In this study, the numerical analysis of Al2O3-H2O nanofluid viscosity is carried out using the molecular dynamics (MD) approach with the classic molecular dynamics software Lammps. Here a box of size 3×3×6 nm is constructed and the particle size of nanoparticles is 10.86Å. The simulations are performed using periodic boundaries with a truncation radius of 9Å. And the potential function used is the water molecule SPC/E model parameter and the CLAYFF force field is used for the Al2O3 nanoparticles. The model was emulated using the NPT and NVT system synthesis.

Fig. 1 shows the Al2O3 particles microstructure by SEM, and the structural properties and parameters are listed in Tab. 1.

Fig. 1 The microstructure of Al2O3 particles by SEM

<table>
<thead>
<tr>
<th>Unite</th>
<th>Value</th>
<th>Unite</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>O mass</td>
<td>15.999 /g mol-1</td>
<td>LJ sigma of</td>
<td>3.013/Å</td>
</tr>
<tr>
<td>H mass</td>
<td>1.008 /g mol-1</td>
<td>LJ epsilon of</td>
<td>0.096/kcal</td>
</tr>
<tr>
<td>Al mass</td>
<td>26.982 /g mol-1</td>
<td>LJ sigma of</td>
<td>3.447/Å</td>
</tr>
<tr>
<td>O3 Charge</td>
<td>-0.840 (e)</td>
<td>LJ epsilon of</td>
<td>0.040/kcal</td>
</tr>
<tr>
<td>H Charge</td>
<td>0.420 (e)</td>
<td>LJ sigma of</td>
<td>4.053</td>
</tr>
<tr>
<td>LJ epsilon of</td>
<td>0.228/kcal mol-1</td>
<td>LJ epsilon of</td>
<td>0.078/kcal</td>
</tr>
</tbody>
</table>
And the equilibrium molecular dynamics (EMD) method is employed to calculate the viscosity with the Green-Kubo equations as below Eq. (1).

$$
\mu_s = \frac{V}{k_BT} \int_0^\infty \langle P_{\alpha\beta}(t)P_{\alpha\beta}(0) \rangle dt
$$

(1)

Where, \( \mu_s \) is the shear viscosity, \( V \) is the molecular model box volume, \( k_B \) is Boltzmann constant, \( T \) is temperature, and \( P_{\alpha\beta} \) is off-diagonal components of the pressure tensor.

### 2.2 Method verification

The viscosity of water is computed using the same formula and the MD analysis technique to confirm the reliability of the EMD approach. The MD simulation data at temperatures between 300K and 360K at atmospheric pressure were used to determine the viscosity of water. Fig.2 shows a comparison of the water viscosity simulation data with the experimental data and the NIST (National Institute of Standards and Technology of US) data, where the viscosity experimental data are obtained by rotational viscometer DHJ-9S at the same temperature. It is discovered that the water viscosity simulation value closely matches the experimental value and the NIST value, with a maximum relative error between the two values of no more than 5%. It also demonstrates how the viscosity of nanofluids can be determined using the EMD approach and the Green-Kubo equation.

![Viscosity comparison between the experimental result, the NIST value, and the simulation value](image)

**Fig. 2 Viscosity comparison between the experimental result, the NIST value, and the simulation value**
2.3 Viscosity analysis of Al₂O₃-H₂O nanofluid

The macroscopic characteristics of base fluid and nanoparticles will have an impact on the thermophysical characteristics of nanofluids. And the interactions between the particles, the base liquid, as well as agglomeration, sedimentation, and varied distributions all have an impact on the properties of the material. Here, the effects of temperature (T) from 290 K to 360 K, volume fraction (\(\varphi\)) from 1.24% to 6.28%, and various forms (given in Tab. 2) were explored to determine how viscosity of nanofluids changed. L stands for length, D for diameter, H for height, and W for width.

Fig. 3 depicts the viscosity data distribution according to temperature, and Fig. 4 and Fig. 5 illustrate the distribution state with sphericity (\(\alpha\)) and volume fraction, respectively. The diverse particle forms are classified by sphericity, and the closer a particle is to a ball in shape, the closer its sphericity is to 1. The computation of sphericity is illustrated in Eq. (2).

\[
\alpha = \frac{4\pi \left(\frac{3V_F}{4\pi}\right)^{\frac{2}{3}}}{S_p}
\]  

Where, \(V_F\) is the particle volume, and \(S_p\) is the particle surface area, \(\alpha\) is the particle sphericity.

<table>
<thead>
<tr>
<th>Particle shape</th>
<th>Graphical representation</th>
<th>D/L(Å)</th>
<th>H(Å)</th>
<th>W(Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td><img src="image" alt="Sphere" /></td>
<td>10.858</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.330</td>
<td>8</td>
<td>---</td>
</tr>
<tr>
<td>Columnar</td>
<td><img src="image" alt="Columnar" /></td>
<td>11.930</td>
<td>6</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14.610</td>
<td>4</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.743</td>
<td>8.743</td>
<td>8.743</td>
</tr>
<tr>
<td>Cubic</td>
<td><img src="image" alt="Cubic" /></td>
<td>13.880</td>
<td>6.938</td>
<td>6.938</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20.000</td>
<td>5.780</td>
<td>5.780</td>
</tr>
</tbody>
</table>

According to Fig. 3, viscosity decreases as temperature rises. The decrease is more pronounced at low temperatures between 290 and 320 K and tends to be mild at high temperatures between 320 and 360 K. Even at temperatures as high as 350 K, the trend of viscosity declines slowly, and the rate of change can even approach zero.
Fig. 4 shows that the viscosity increases as the volume fraction increases, with the effect being more pronounced when the volume fraction exceeded 3.72 percent. As the volume fraction and particle count increase, the molecular forces between the particles and the base liquid rise, which causes the friction force of the overall fluid flow to rise. In Fig. 5, the viscosity likewise rises as the particular surface area changes, but the pattern of change is not very predictable. The contact area and hence viscosity increase as the surface area of the nanoparticles increases.

In the process of analyzing viscosity data, it is discovered that numerous parameters interact with one another. Fig. 6 and Fig. 7 illustrate the relationship on $\eta_{nf}$ between temperature, volume fraction, and as well as the relationship between particle sphericity and temperature.

Fig. 3 The distribution of viscosity data under the temperature

Fig. 4 The distribution of viscosity data under $\phi$

Fig. 5 The distribution of viscosity data under $\alpha$

Fig. 6 Effect of volume fraction and temperature on $\eta_{nf}$ at $\alpha = 1$

Fig. 7 Effect of particle sphericity and temperature on $\eta_{nf}$ at $\varphi = 1$
According to Fig. 6, at the same temperature, the viscosity increases with a higher volume fraction, and at a higher temperature, the effect of volume fraction on viscosity decreases. Fig. 7 shows that the effect of particle sphericity on viscosity is nearly constant across a range of temperatures, however there is a bigger fluctuation when the sphericity is less than 0.85.

It is important to process the viscosity data and develop the relational expression between viscosity and factors like temperature, volume fraction, and particle sphericity in order to represent the relationship between them.

3. Viscosity model study on Al$_2$O$_3$-H$_2$O nanofluid

The viscosity of nanofluids can be precisely predicted using a competent parametric model. A parameterized polynomial model of viscosity is preferred as Eq. (3) in light of the aforementioned data distribution. To find the relevant expressions and calculated coefficients, this equation needs to be further examined. And $a_0, a_1, ..., a_{10}$ is the coefficient terms. The specific process is to determine an optimal model expression and the coefficients in the expression using a stepwise forward regression method for the constructed ternary second-order full-effects model, examining the changes in the four indicator terms $R^2$, RMSE, AICc and BIC, as well as the contribution of the introduction of each variable to the least-squared sum of effects, weighing the complexity of the model against the accuracy of the prediction.

$$ \frac{\eta_{nf}}{\eta_f} = a_0 \varphi^2 + a_1 T^2 + a_2 \alpha^2 + a_3 \varphi T + a_4 \varphi \alpha + a_5 \alpha T + a_6 \varphi + a_7 T + a_8 \alpha + a_{10} $$

Eq. (4) calculates the effect size based on the least squares, the independent effects of the three variables, the cross effects between them, and the quadratic effects of which are picked as the legitimate parameters (effect size > 2). In the formula, $P$-value is the significance level, and the values above 2 are significant at $P=0.01$. As seen from Fig. 8, the $\alpha^2$, $T^2$ and $\alpha T$ in the formula can be ignored.

$$ Effect \ size = -\log_{10}(P) $$

![Fig. 8 Parameter effects screening by Least Squares Method](image)

The viscosity prediction model is then given as Eq. (5) using the stepwise regression method, fully taking into account the higher $R^2$ (coefficient of determination), lower RMSE (root mean square error), lower AICc (Akaike information criterion), and lower BIC (Bayesian information criterion), as
seen in Tab. 3. This equation consists of 6 variable terms with a goodness of fit up to 0.96. And it is clear that $\phi$ significantly affects the changes in viscosity.

$$\eta_{nf} \times 10^3 = 125.85\varphi + 2.67T + 29.09\alpha - 8.20\varphi^2 + 0.51\varphi T - 171.89\varphi\alpha - 192.10$$ (5)

Where, $\eta_{nf}$ is the nanofluid viscosity, $\eta_f$ is the base fluid viscosity, $\varphi$ is the volume fraction, $T$ is the temperature and $\alpha$ is the particle sphericity.

Fig. 9 shows that the simulation value and the model's predicted value are very well matched, with a relative error between them of less than 9%. It indicates that molecular dynamics methods can be used well to model the dynamic viscosity of nanofluids, but the process is complex; whereas the predictive model is fast and accurate under the same conditions. Fig. 10 shows that the Cook's distance is less than 0.3 and that there are no out-of-the-ordinary data points that significantly alter the regression equation, which reflects the fact that the absence of outliers in the sample data can have a misleading effect on the accuracy of the model predictions. The viscosity prediction equation is hence reliable and precise. A good predictive model can give better and faster values for the viscosity of $\text{AL}_2\text{O}_3$-$\text{H}_2\text{O}$ nanofluids, simply by entering the values of the parameters under different conditions. And it is easier to find and analyse the pattern of the effect of different parameters on the viscosity, as in section 4.2.

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### Table 3 Convergence table of the parameters of the stepwise regression method

<table>
<thead>
<tr>
<th>Variables</th>
<th>$R^2_{\text{max}}$</th>
<th>RMSE$_{\text{min}}$</th>
<th>AICc$_{\text{min}}$</th>
<th>BIC$_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.024</td>
<td>0.209</td>
<td>-12.567</td>
<td>-6.891</td>
</tr>
<tr>
<td>2</td>
<td>0.851</td>
<td>0.082</td>
<td>-117.390</td>
<td>-109.990</td>
</tr>
<tr>
<td>3</td>
<td>0.925</td>
<td>0.059</td>
<td>-153.840</td>
<td>-144.810</td>
</tr>
<tr>
<td>4</td>
<td>0.943</td>
<td>0.052</td>
<td>-167.620</td>
<td>-157.040</td>
</tr>
<tr>
<td>5</td>
<td>0.953</td>
<td>0.048</td>
<td>-175.140</td>
<td>-163.130</td>
</tr>
<tr>
<td>6</td>
<td>0.961</td>
<td>0.044</td>
<td>-183.720</td>
<td>-170.380</td>
</tr>
<tr>
<td>7</td>
<td>0.962</td>
<td>0.044</td>
<td>-182.330</td>
<td>-167.770</td>
</tr>
</tbody>
</table>

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![Fig. 9 Comparison of predicted and simulated values](image1.png)

![Fig. 10 Model parameters Cook's distance effects](image2.png)
4. Results and discussion

4.1 Comparison of viscosity models

Some models have been used to estimate the viscosity of nanofluids, with the concentration of the nanofluid serving as the sole input parameter. Here, three traditional nanofluid viscosity models—discussed in Tab. 4 and illustrated in Fig. 11—are utilized to compare with the model given in this research.

Table 4 A comparison table between the four models

<table>
<thead>
<tr>
<th>No.</th>
<th>Equation</th>
<th>Subject</th>
<th>Range of volume fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Einstein[36] : $\frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\phi$</td>
<td>spherical shape</td>
<td>$\phi &lt; 0.2%$</td>
</tr>
<tr>
<td>2</td>
<td>Brinkman[37] : $\frac{\mu_{nf}}{\mu_{bf}} = \frac{1}{(1 - \phi)^{2/5}}$</td>
<td>spherical shape</td>
<td>$\phi &lt; 0.2%$</td>
</tr>
<tr>
<td>3</td>
<td>Batchelor[38] : $\frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\phi + 6.5\phi^2$</td>
<td>spherical shape</td>
<td>$\phi &lt; 0.2%$</td>
</tr>
<tr>
<td>4</td>
<td>Self-defined viscosity model:</td>
<td>arbitrary shape</td>
<td>$\phi \leq 6.28%$</td>
</tr>
<tr>
<td></td>
<td>$\frac{\eta_{nf} \times 10^3}{\eta_{f}} = 125.85\phi + 2.67T + 29.09\alpha$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-8.20\phi^2 + 0.51\phi T - 171.89\alpha$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-192.10$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to the table and graph above, Einstein, Brinkman, and Batchelor models have only one independent variable of volume fraction, which is less than 0.2 percent with a good prediction. As a result, none of them can make an accurate prediction unless other effective parameters such as temperature and particle shape are included. The viscosity model proposed in this paper has a high level of prediction accuracy and agreement with experimental values, with a maximum relative error of less than 10%.

4.2 Sensitivity analysis of viscosity prediction model

1) Effect of the volume concentration on the viscosity

Furthermore, a sensitivity analysis of the forecasting model is performed. Error! Reference source not found. shows the relative viscosity increase at different temperatures. The results show that
the viscosity ratio has a non-linear increasing tendency, and the volume fraction of 6.5% reaches its highest value of 0.9886 at 300 K before starting to decline. Additionally, the tipping point lags as the temperature rises. For a given volume fraction, it can be shown that the ratio increases more as the temperature rises; this trend is more obvious for volume fractions above 2%. For example, at 360K, the viscosity ratio increases by more than 37% with the volume fraction increasing from 0.02 percent to 8%, and the increase in viscosity ratio is nearly 20% regardless of temperature change while the volume fraction is less than 2%. The nanofluid becomes more viscous as the number of particles grows, increasing both the gravitational attraction between nearby particles and the frictional resistance that must be overcome when flowing.

For a given volume fraction, it can be shown that the viscosity ratio increases more as the temperature rises; this trend is more obvious for volume fractions above 2%. For example, at 360K, the viscosity ratio increases by more than 37% with the volume fraction increasing from 0.02 percent to 8%, and the increase in viscosity ratio is nearly 20% regardless of temperature change while the volume fraction is less than 2%. The nanofluid becomes more viscous as the number of particles grows, increasing both the gravitational attraction between nearby particles and the frictional resistance that must be overcome when flowing.

Fig. 12 Variation of viscosity with volume fraction under spherical particles

2) Effect of the temperature on the viscosity

Fig. 12 and Fig.13 both demonstrate that temperature has a significant impact on the nanofluid’s viscosity. Figure 13 shows that for spherical Al$_2$O$_3$-H$_2$O nanofluid, an increase in temperature causes a corresponding rise in the viscosity ratio; the ratio grows to its highest value at 360 K with a volume fraction of 6.28%. Additionally, the viscosity ratio increases more quickly at the same temperature the larger the volume percentage. For instance, at 320K, the growth rate at the volume fraction of 6.28 percent is higher than that at the volume fraction of 1.24% by more than 30%. In addition, the viscosity ratio of change slows down at the same temperature and is more pronounced at low temperatures.

The Brownian motion of the nanoparticles is improved by higher temperature, which also lessens the attraction between the particles and reduces the viscosity of the nanofluid. However, the relative viscosity of the nanofluid will increase with increasing temperature if the rate of reduction is larger than the rate at which the viscosity of the base fluid decreases with increasing temperature. Therefore the concentration of nanoparticles must be increased or the temperature of the nanofluid must be raised in order to effectively increase the viscosity of the base fluid (viscosity ratio over 1).
3) Effect of the particle sphericity on the viscosity

Irregularities in the shape of the nanoparticles can exacerbate their interaction in the base fluid. As seen in Fig. 14, the viscosity ratio decreases as the nanoparticles get closer to having a spherical form, showing that spherical particles have less mutual adsorption and shear forces. The lowest viscosity ratio is found in spherical particle nanofluids, which do not surpass 1 even at volume fractions of 6.28%. In spherical particle nanofluids, the rate of change of the viscosity ratio decreases with increasing volume fraction, and at the same sphericity, the higher the volume fraction, the higher the viscosity ratio. For example, at a volume fraction of 6.28%, the sphericity of 0.3 has a growth rate of 72%. Additionally, the viscosity ratio begins to decrease at a slower rate with a volume fraction of more than 2.48%, and this trend continues to decline as the volume fraction rises.

Fig. 13 Variation of viscosity ratio with temperature under spherical particles

Fig. 14 Variation of viscosity ratio with particle sphericity at 300K

5. Conclusions

This study uses molecular dynamics to examine how the viscosity of an Al₂O₃-H₂O nanofluid changes as a function of parameters, including temperature (290K to 360K), volume fraction (1.24% to 6.28%), and particle sphericity (0.3 to 1, a convergent variable that integrates particle shape and size). And to properly supply its viscosity features, a viscosity prediction model is nicely built. The following are some of the major conclusions:

1) With a prediction accuracy of over 90%, a ternary second-order viscosity prediction model of an Al₂O₃-H₂O nanofluid is proposed, incorporating factors for volume fraction, temperature, and particle sphericity.

2) The viscosity of Al₂O₃-H₂O the nanofluid can be effectively increased by adding nanoparticles to the base fluid. The volume fraction, which has non-negligible first- and second-order effects, is the key element in the formula for estimating viscosity. The relative viscosity of the nanofluid increases with
increasing volume fraction, although the increasing trend is rapid below 2% concentration and slows down and even decreases above 6.5%. Low temperatures make this effect particularly noticeable.

3) The viscosity of Al₂O₃-H₂O nanofluids is significantly influenced by temperature, and temperature and volume fraction interact more strongly. Viscosity of the base solution is more likely to grow when particle concentration in the base solution increases at high temperatures.

4) The regularity of the particles added to the base liquid has an inverse relationship with the increase in the viscosity of the nano-fluid. When low concentration nanoparticles are added to the base fluid, the more irregular the particles, the more strong the interaction effect, and the greater the viscosity enhancement to the nanofluids are.

**Nomenclature:**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Temperature, K</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Volume percent, vol%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Particle sphericity</td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>Shear viscosity, $Pa \cdot s$</td>
</tr>
<tr>
<td>$\mu_{bf}$</td>
<td>Base fluid viscosity, $Pa \cdot s$</td>
</tr>
<tr>
<td>$L$</td>
<td>Particle length, Å</td>
</tr>
<tr>
<td>$W$</td>
<td>Particle width, Å</td>
</tr>
<tr>
<td>$R^2$</td>
<td>Determination coefficient</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root mean square error</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Fluid viscosity, $Pa \cdot s$</td>
</tr>
<tr>
<td>$\eta_f$</td>
<td>Base fluid viscosity, $Pa \cdot s$</td>
</tr>
<tr>
<td>$\eta_{nf}$</td>
<td>Nanofluid viscosity, $Pa \cdot s$</td>
</tr>
<tr>
<td>$\eta_{nf}/\eta_f$</td>
<td>Relative viscosity</td>
</tr>
<tr>
<td>$V_F$</td>
<td>Particle volume, m³</td>
</tr>
<tr>
<td>$S_P$</td>
<td>Particle surface area, m²</td>
</tr>
<tr>
<td>$H$</td>
<td>Particle height, Å</td>
</tr>
<tr>
<td>$AIC_C$</td>
<td>Akaike information criterion</td>
</tr>
<tr>
<td>$BIC$</td>
<td>Bayesian information criterion</td>
</tr>
</tbody>
</table>

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References:


[18] Vakilinejad, A., et al., Experimental Investigation And Modeling Of The Viscosity Of Some Water-Based Nanofluids


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