STUDY ON THE SIMPLIFIED MODEL OF VERTICAL DOUBLE U-PIPE GROUND HEAT EXCHANGER

by

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A simplified semi-analytical model of vertical double U-pipe ground heat exchanger was established. The validity of the established model is examined by contrasting the figured outcomes with experiment data, emulation results of 3-D numerical model and calculation results of infinite line-source model for different inlet boundary conditions and configurations. After one hour, the semi-analytical model's relative error is less than 0.32% under the boundary condition of given inlet fluid temperature. Under the boundary condition of given total heat input rate, the semi-analytical model's relative error after ten hours is less than 0.11%, while the infinite line-source model's relative error is less than 0.60%. The semi-analytical model is in good agreement with experiment and numerical model, and has higher calculation accuracy than infinite line-source model.

Key words: *vertical double U-pipe ground heat exchanger, heat transfer, semi-analytical model, temperature distribution*

Introduction

Ground source heat pump system (GSHPS) consume geothermal energy for building heating and cooling [1, 2], and ground heat exchanger (GHE) is a main component of GSHPS [3]. The GHE is the medium of heat exchange between fluid and soil, and has a great influence on the economy and operation of the GSHPS [4, 5].

The GHE can be divided into two configurations, *i.e.* vertical [6-8] and horizontal [9, 10] configurations. At present, vertical GHE is normally superior to horizontal GHE. Compared with horizontal GHE, vertical GHE has deeper buried depth and does not occupy too much area during installation, and has higher and more stable heat transfer performance [4, 11]. As an crucial research topic, models of vertical double U-pipe GHE (VDUGHE) have been extensively studied [12-16], such as analytical models and numerical models.

Line-source models, cylindrical-source models, equivalent-diameter models, and composite medium models are the four primary types of analytical models. The line-source models consider the buried pipe as a constant line-source in soil and analyse the unsteady heat conduction problem. Zeng *et al.* [17] created a finite line-source model (FLSM) and produced an analytical solution by assuming the buried pipe as a line-source of finite length. Babak *et al.* [18] derived a 1-D analytical expression of heat input rate per length under the condition of constant temperature of borehole wall by utilizing Green's function methods, and verified it through numerical model and experiment data. Erol and Francois [19] proposed a FLSM by investigating heat conduction and convection caused by groundwater seepage in multi-layer porous media, and

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verified the applicability of the model by comparing it with numerical simulation results. To sum up, the line-source models consider the borehole and soil as a whole, and ignore the differences of borehole grout materials, pipes, fluids and soil, which leads to some calculation errors.

The cylindrical-source models simplify borehole as cylindrical heat source in soil, and derive analytical solutions to solve the problem. Nian and Cheng [20] established a new analytical G-function for analysing the problem by considering the influence of borehole thermal capacity. Pan *et al.* [21] established an analytical model of multi-layer cylindrical-source for layered vertical GHE by using integral transformation methods on the basis of Green's function, and verified the validity of the model through model degradation, numerical simulation and experiment data. In conclusion, the cylindrical-source models provide a good method to solve the heat transfer calculation of different materials of borehole, but the borehole thermal capacity is whether ignored or regarded as a whole.

Li *et al*. [22-24] developed an infinite line-source model (ILSM) of composite media. In this paper, the legs of U-pipe are simplified as four infinite line-sources, and the problem is simplified as heat transfer problem of infinite line-sources in composite cylindrical medium, and this model is checked by comparison with experiment result, showing that this model has high precision. Li *et al*. [25] established an analytical full-scale model that consists of FLSM, ILSM and a composite-medium model.

The equivalent diameter models equate the U-pipe as one-fold pipe situated in middle of borehole. Wang *et al.* [26] equated the U-pipe as two adjacent half-pipes, divided the borehole into two symmetrical portions, and established a semi-numerical model of single U-pipe GHE. By contrasting the computed results with experiment results, simulation results, and results from other models, the effectiveness of this model is demonstrated. Wei *et al.* [27] introduced an equivalent diameter method of the composite medium ILSM, created a new short-term analytical model, and verified this model's short-term performance through comparison with existing analytical models and experiment results.

Numerical models include 1-D numerical models [28, 29], 2-D numerical models [30, 31], 3-D numerical models [32, 33] and thermal resistance and capacity (TRC) models [34-37]. The 1-D and 2-D numerical heat transfer models disregard radial or axial heat transfer, and there will be errors in predicting the GHE heat transfer characteristics, so temperature distribution of fluid may not be precisely obtained. The 3-D numerical heat transfer models [38] have high accuracy, but they have some disadvantages such as complex mesh division and long calculation time. Additionally, the traits of TRC models are comparable to those of 1-D numerical model.

In order to simulate VDUGHE and address the drawbacks of the aforementioned models, a simplified semi-analytical model is derived. The semi-analytical model adopts numerical method to analyse fluid heat transmission in the VDUGHE, and adopts the composite cylindrical-source model to analyse grout and soil heat transmission. For various boundary conditions and double U-pipe configurations, the model is matched with experiment result, a 3-D numerical model, and ILSM to justify its validity.

The semi-analytical model is mainly used to predict the short-term and medium-term fluid temperatures of VDUGHE. Prediction of short-term fluid temperatures of GHE is important especially for thermal response test [23]. However, traditional analytical models normally ignore the borehole heat capacity and axial effect, and have large errors to predict the short-term responses of GHE.

Simplified semi-analytical model for VDUGHE

The single U-pipe is divided into two neighboring half-pipes with wall thickness and borehole is divided into two symmetrical portions in the semi-numerical model created by

Wang *et al.* [26]. Likewise, this study simplifies vertical double U-pipe ground heat exchanger (VDUGHE) as four equivalent quarter pipes, which are assumed to have a thickness of 0 m, as shown in fig. 1. The four equivalent quarter pipes include two inlet quarter pipes and two outlet quarter pipes, which correspond to the two inlet pipes and two outlet pipes, respectively.

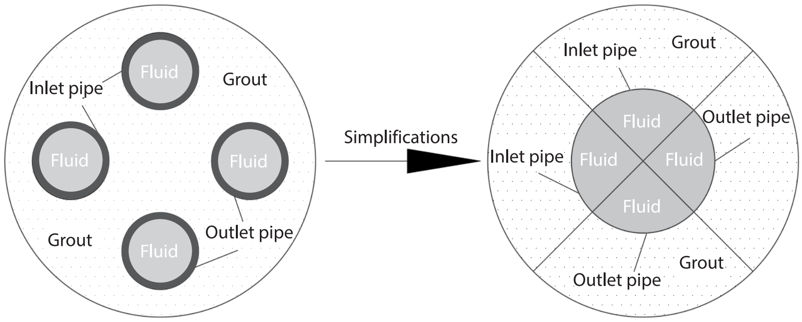


Figure 1. Borehole cross-sections before and after simplifications

Following assumptions are made:
- Soil is homogeneous.
- Groundwater seepage is not considered.
- The VDUGHE is simplified as four equivalent quarter pipes, which are assumed to have a thickness of 0 m.
- There are no mass transfer among the four equivalent quarter pipes, and there exists heat transfer among them through several thermal resistances between any two quarter pipes.
- Assumed that only radial heat transmission exists in grout and soil.
- Physical properties of all materials keep constant.

Some properties of borehole are changed, containing r_{ie} (radius of four equivalent quarter pipes) and $(\rho c)_{ge}$ (equivalent volumetric heat capacity of grout). The equations are:

$$r_{ie} = r_b \exp\left[-2\pi k_g \left(R_b - \frac{1}{8\pi r_i h}\right)\right] \tag{1}$$

$$(\rho c)_{ge} = \frac{r_b^2 - 4r_o^2}{r_b^2 - r_{ie}^2}(\rho c)_g \tag{2}$$

It should be mentioned that R_b (borehole thermal resistance) is unknown, which can be estimated by matching experiment data or simulation data.

Next, we develop a semi-analytical model to analyse heat transmission of VDUGHE, in which the heat transfer equations of fluids in the inlet and outlet pipes can be numerically calculated and the heat transmission equations of grout and soil can be analytically calculated.

For the fluids in the two inlet quarter pipes (*i.e.* fluids in the two inlet pipes), the energy equations:

$$\pi r_i^2 \rho_f c_f \frac{\partial T_{f1}(z,t)}{\partial t} + 2\pi r_i h\left[T_{f1}(z,t) - T_{gi}(z,t)\right] + \frac{T_{f1}(z,t) - T_{f2}(z,t)}{R_{12}} + \\ + \frac{T_{f1}(z,t) - T_{f2}(z,t)}{R_{13}} = -Mc_f \frac{\partial T_{f1}(z,t)}{\partial z}, \ (0 \le z \le H, \ t > 0) \tag{3}$$

where R_{12} is the thermal resistance between fluids in the pipes 1 and 2, and R_{13} is the thermal resistance between fluids in the pipes 1 and 3 [39]. The numbers of the pipes are presented in fig. 6:

$$R_{12}=\frac{\cosh^{-1}\left(\dfrac{x_{12}^{2}}{2r_{\mathrm{o}}^{2}}-1\right)}{2\pi k_{\mathrm{g}}}+\frac{2}{\pi r_{\mathrm{i}}h}+\frac{2}{\pi k_{\mathrm{p}}}\ln\left(\frac{r_{\mathrm{o}}}{r_{\mathrm{i}}}\right) \tag{4}$$

$$R_{13}=\frac{\cosh^{-1}\left(\dfrac{x_{13}^{2}}{2r_{\mathrm{o}}^{2}}-1\right)}{2\pi k_{\mathrm{g}}}+\frac{2}{\pi r_{\mathrm{i}}h}+\frac{2}{\pi k_{\mathrm{p}}}\ln\left(\frac{r_{\mathrm{o}}}{r_{\mathrm{i}}}\right) \tag{5}$$

On the walls of two inlet quarter pipes, the boundary condition:

$$q_{\mathrm{i}}(z,t)=2\pi r_{\mathrm{i}}h\left[T_{\mathrm{f1}}(z,t)-T_{\mathrm{gi}}(z,t)\right],\ \left(0\le z\le H,\ t>0\right) \tag{6}$$

About two inlet quarter pipes, boundary condition:

$$T_{\mathrm{f1}}(z,t)\big|_{z=0}=\begin{cases}\dfrac{Q(t)}{2Mc_{\mathrm{f}}}+T_{\mathrm{f2}}(z,t)\big|_{z=0},\left(t>0,\ \text{for given total heat input rate}\right)\\ T_{\mathrm{in}}(t),\left(t>0,\ \text{for given inlet fluid temperature}\right)\end{cases} \tag{7}$$

Likewise, we can discover the temperature equation and boundary conditions for two outlet quarter pipes:

$$\pi r_{\mathrm{i}}^{2}\rho_{\mathrm{f}}c_{\mathrm{f}}\frac{\partial T_{\mathrm{f2}}(z,t)}{\partial t}+2\pi r_{\mathrm{i}}h\left[T_{\mathrm{f2}}(z,t)-T_{\mathrm{go}}(z,t)\right]+\frac{T_{\mathrm{f2}}(z,t)-T_{\mathrm{f1}}(z,t)}{R_{12}}+$$
$$+\frac{T_{\mathrm{f2}}(z,t)-T_{\mathrm{f1}}(z,t)}{R_{13}}=Mc_{\mathrm{f}}\frac{\partial T_{\mathrm{f2}}(z,t)}{\partial z}\left(0\le z\le H,\ t>0\right) \tag{8}$$

$$q_{\mathrm{o}}(z,t)=2\pi r_{\mathrm{i}}h\left[T_{\mathrm{f2}}(z,t)-T_{\mathrm{go}}(z,t)\right],\ \left(0\le z\le H,\ t>0\right) \tag{9}$$

$$T_{\mathrm{f2}}(z,t)\big|_{z=H}=T_{\mathrm{f1}}(z,t)\big|_{z=H},\ \left(t>0\right) \tag{10}$$

Initially, temperatures of fluids and pipes are:

$$T_{\mathrm{f1}}(z,t)\big|_{t=0}=T_{\mathrm{f2}}(z,t)\big|_{t=0}=T_{\mathrm{gi}}(z,t)\big|_{t=0}=T_{\mathrm{go}}(z,t)\big|_{t=0}=T_{0}(z),\ \left(0\le z\le H\right) \tag{11}$$

The temperatures of grout and soil outside the borehole meet 1-D radial heat transmission equation, which could be analysed by using the composite cylindrical-source model combined with the principle of variable heat flow superposition:

$$T_{\mathrm{gi}}(z,t)\big|_{t=t_n}=T_0(z)+4\sum_{i=1}^{n}\frac{q_{\mathrm{i}}(z,t)\big|_{t=t_i}-q_{\mathrm{i}}(z,t)\big|_{t=t_{i-1}}}{k_{\mathrm{s}}}G(t)\big|_{t=t_n-t_{i-1}} \tag{12}$$

$$T_{\mathrm{go}}(z,t)\big|_{t=t_n}=T_0(z)+4\sum_{i=1}^{n}\frac{q_{\mathrm{o}}(z,t)\big|_{t=t_i}-q_{\mathrm{o}}(z,t)\big|_{t=t_{i-1}}}{k_{\mathrm{s}}}G(t)\big|_{t=t_n-t_{i-1}} \tag{13}$$

where $G(t)$ is G-function of composite cylindrical-source model, φ, ψ, k_0 and γ are intermediate variables, and their expressions are [40]:

$$G(t)=\frac{8k_0^2}{\pi^5\delta^2}\int_0^\infty \frac{1-\exp\left[-\frac{k_g t}{(\rho c)_{ge} r_{ie}^2}\beta^2\right]}{\beta^5\left[\varphi^2+\psi^2\right]}\mathrm{d}\beta \tag{14}$$

$$\begin{aligned}\varphi = Y_1(\beta)\left[Y_0(\beta\delta\gamma)J_1(\beta\delta)-Y_1(\beta\delta\gamma)J_0(\beta\delta)k_0\gamma\right]-\\ -J_1(\beta)\left[Y_0(\beta\delta\gamma)Y_1(\beta\delta)-Y_1(\beta\delta\gamma)Y_0(\beta\delta)k_0\gamma\right]\end{aligned} \tag{15}$$

$$\begin{aligned}\psi = J_1(\beta)\left[J_0(\beta\delta\gamma)Y_1(\beta\delta)-J_1(\beta\delta\gamma)Y_0(\beta\delta)k_0\gamma\right]-\\ Y_1(\beta)\left[J_0(\beta\delta\gamma)J_1(\beta\delta)-J_1(\beta\delta\gamma)J_0(\beta\delta)k_0\gamma\right]\end{aligned} \tag{16}$$

$$k_0=\frac{k_s}{k_g},\ \delta=\frac{r_b}{r_{ie}},\ \gamma=\sqrt{\frac{k_g(\rho c)_s}{k_s(\rho c)_{ge}}} \tag{17}$$

Figure 2 shows the schematic diagram of fluid grid division, in which the fluids in the pipes are divided into several sections along the axial direction. Using the finite difference method of first-order forward difference, eqs. (3) and (8) can be discretized, and they can be solved numerically by combining with the other equations, and the fluid temperature distributions in the VDUGHE can be calculated.

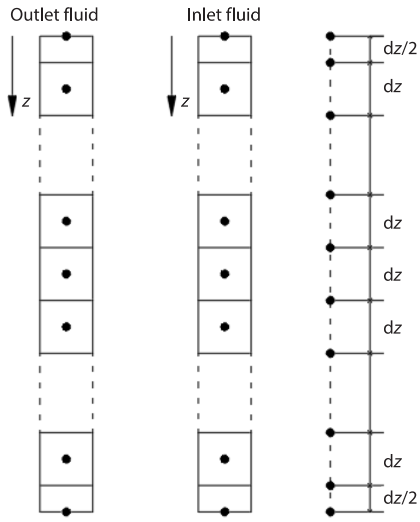


Figure 2. Schematic diagram of fluid grid division (every dot means a fluid node)

Compared with the existing models, the semi-analytical model proposed by this paper not only considers the variation of fluid temperature with depth and heat transfer among the fluids in the inlet and outlet pipes, but also considers heat capacities of fluid and grout, so temperature field figured by this model is probably more precise.

This model is established founded on the semi-numerical model proposed by Wang *et al.* [26], but they are different:

- the objects of the two models are double and single U-pipe GHE, respectively, and the semi-numerical model cannot be applied for VDUGHE,
- the major equations of the two models are different, this model mainly adopts analytical method to analyse the heat transmission process, but the semi-numerical model mainly adopts numerical method, therefore, the equations and grid division of this model are more concise, and
- the verifications of the two models are also different. The novelty of this model is that it has the advantages of convenience and accuracy for simulation of VDUGHE.

The 3-D numerical model

To justify the feasibility of semi-analytical model, a 3-D numerical model of VDUGHE was established based on FLUENT software.

Following assumptions are made:
- Thermal properties of soil, grout, pipes and fluids are assumed to keep constant.
- Because the length of U-bends of VDU is very short, the U-bends are ignored.
- Inlet and outlet boundary conditions of VDUGHE are velocity inlet and pressure outlet boundaries, respectively. Additionally, the fluid-flow in the pipes is simulated using standard k-epsilon turbulence model.
- The inlet boundaries of the model are set by using user-defined functions (UDF): under the boundary condition of given inlet fluid temperature, T_{in}, the outlet fluid temperature, T_{out}, in the inlet pipe is assigned to the inlet temperature of fluid in the outlet pipe; under the boundary condition of given total heat input rate, Q, the outlet temperature of fluid in the inlet pipe is assigned to the inlet temperature of fluid in the outlet pipe, and the inlet temperature of fluid in the inlet pipe is set as a function of Q and T_{out}.
- Adiabatic boundary conditions are assumed at the top and bottom of soil, grout and pipes.

Because of the symmetry of VDUGHE, the 3-D numerical model only simulates a half of the region. Figure 3 shows the grids inside and outside the borehole of numerical model.

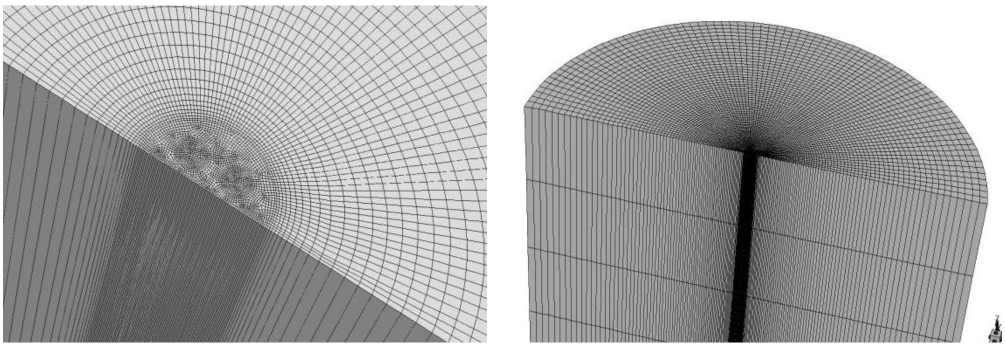


Figure 3. Grid diagram of numerical model

Results and discussion

To confirm the precision of this model, the calculated results of this model are contrasted with experiment data, 3-D numerical model and ILSM. Besides, the 3-D numerical model is also verified by contrast with ILSM.

Comparison with experiment data for given T_{in}

Chen [41] conducted a field test on a VDUGHE in Chongqing, and tab. 1 showed the VDUGHE's parameters. The T_{in} and T_{out} of VDUGHE vary with time, shown in fig. 4. The calculation results of this model established in this paper are contrasted with experiment result for given T_{in}.

Experiment data of T_{in} are regarded as input data of this model to calculate T_{out}, and then the figured T_{out} are contrasted with experiment data, shown in fig. 4. It is obvious that the change trend of T_{out} calculated by this model is the same with that of experimental T_{out}. The difference between the calculated results and experiment data is significant in the first few hours, however, as time goes on, the calculated results agree well with experiment result. After 10 hours, the average absolute error of the semi-analytical model is 0.12 °C, demonstrating that the precision of this model is high.

Table 1. Parameters of the VDUGHE

Parameter	Value
VDUGHE length, H	100 m
Borehole radius, r_b	0.065 m
Inner radius of pipes, r_i	0.013 m
Outer radius of pipes, r_o	0.016 m
Half shank spacing, x_c	0.0325 m
Distance between the centers of pipes 1 and 2, x_{12}	0.046 m
Distance between the centers of pipes 1 and 3, x_{13}	0.065 m
Thermal conductivity of pipes, k_p	1.80 W/mK
Thermal conductivity of grout, k_g	2.20 W/mK
Thermal conductivity of soil, k_s	2.594 W/mK
Volumetric heat capacity of pipes, $(\rho c)_p$	1.90 MJ/m³K
Volumetric heat capacity of grout, $(\rho c)_g$	1.98 MJ/m³K
Volumetric heat capacity of soil, $(\rho c)_s$	3.36 MJ/m³K
Volumetric heat capacity of fluid, $(\rho c)_f$	4.17 MJ/m³K
Mass-flow rate in each pipe, M	0.34 kg/s

The matching between this model and experiment data is not very good, but it is also not bad: the average absolute error of this model after 10 hours is only 0.12 °C. The main reasons for the inconsistency between them are:

– there are only a few available experiment data of T_{in}, and the temperature fluctuates up and down, leading to some errors of input data of the semi-analytical model and
– the heat output rate of VDUGHE is relatively small, and the fluid temperatures vary slightly with time, which would lead to larger influence of measurement error.

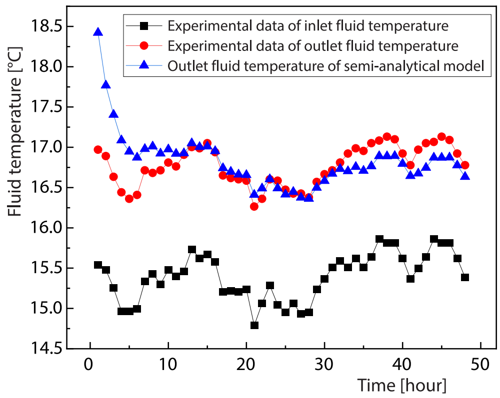


Figure 4. Comparisons of T_{out} figured by the established model with experiment data

Besides, the fluctuation of experimental T_{in} is normal, which is probably caused by heat output rate fluctuation and time-varying atmospheric environment and so on. Because T_{out} is influenced by T_{in}, T_{out} would also fluctuate up and down.

Validation of 3-D numerical model

Numerical model was verified by using ILSM, where the parameters of this model are also presented in tab. 1. The ILSM probably has large errors for short term, but it is accurate enough for long term, and therefore, can be devoted to justify the numerical model, and the contrast between the 3-D numerical model and ILSM is shown in fig. 5. Relative difference between the two models is up to 16% in short time, and decreases rapidly with the increase of simulation time, and the relative difference is less than 0.81% after one hour. The two models agree

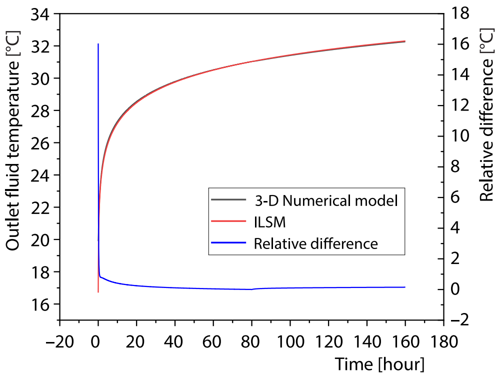


Figure 5. Comparisons of T_{out} figured by the 3-D numerical model with ILSM

well with each other after 10 hours. Therefore, we think that the 3-D numerical model is feasible to simulate VDUGHE.

Comparison between semi-analytical model and 3-D numerical model for given T_{in}

For given T_{in} of 30 °C, the calculation results of established model are contrasted with the simulation results of numerical model for three configurations shown in fig. 6. The basic parameters of proposed model are shown in tab. 1, but the given T_{in} is 30 °C, and the half shank spacings are different.

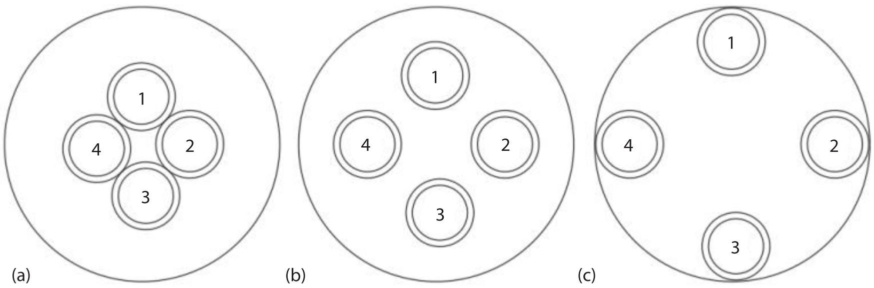
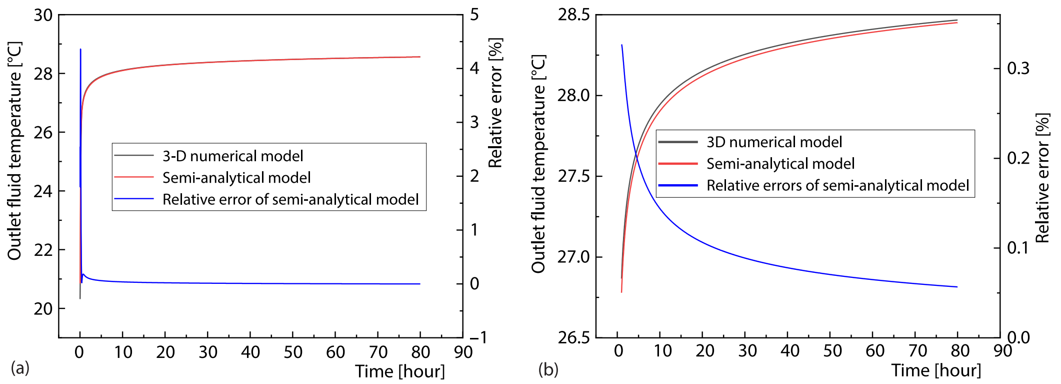


Figure 6. Three configurations of VDUGHE: (a) x_c = 0.0255 m, (b) x_c = 0.0325 m, and (c) x_c = 0.048 m

For the semi-analytical model, borehole thermal resistance is judged by matching the calculated result with T_{out} of the 3-D numerical model at the time of 80 hours. Figure 7 shows the comparisons of T_{out} between proposed model and 3-D numerical model for different half shank spacings. Since the temperature distribution trend of the three kinds of half shank spacings in the first hour are the same, the comparison between the two models can be more intuitively reflected by ignoring the data of the first hour. Therefore, the comparison in the first hour is ignored in figs. 7(b) and 7(c). For the three different configurations, results of two models match well, and the relative errors of this model are small on the whole. Figures 7(a)-7(c) are comparison diagrams for x_c is 0.0255 m, 0.0325 m, and 0.048 m, respectively. The borehole thermal resistances R_b are 0.0672 mk/W, 0.0498 mk/W, and 0.0314 mk/W, respectively. It is clear that the relative errors are large in the short period of time, and that the maximum relative errors for the three configurations are 4.30%, 3.25%, and 1.89%, respectively. With the increase of simulation time, the relative errors of the proposed model decrease and tend to zero, and the relative errors of this model after 1 hours for the three configurations are less than 0.16%, 0.32%, and 0.09%, respectively. The result shows that for given T_{in}, the semi-analytical model of VDUGHE proposed in this paper has large error in the analysis of fluid temperature field in the short term, but has high accuracy in the analysis of medium and long-term fluid temperature field.

Comparison of semi-analytical model, 3-D numerical model and ILSM for given Q

By assuming the borehole as a line-source with constant Q, Magraner *et al.* [16] established an ILSM. In this paper, the ILSM is cited and contrasted with the 3-D numerical model and semi-analytical model. Table 2 shows the parameters of ILSM.



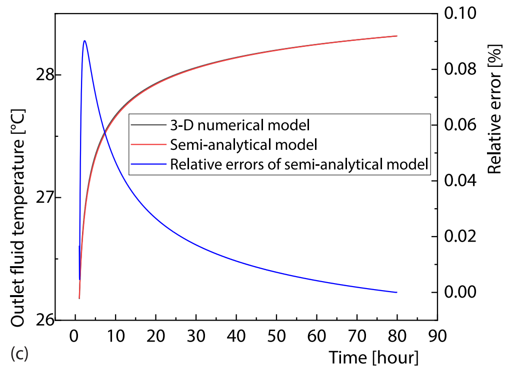


Figure 7. Comparison diagrams of T_{out} between proposed model and 3-D numerical models for different half shank spacings: (a) $x_c = 0.0255$ m, (b) $x_c = 0.0325$ m, and (c) $x_c = 0.048$ m

Table 2. Parameters of ILSM

Parameter	Value
VDUGHE length, H	100 m
Borehole radius, r_b	0.065 m
Total heat input rate per depth, Q_z	60 W/m
Fluid mass-flow rate, M	0.34 kg/s
Thermal conductivity of soil, k_s	2.594 W/mK
Volumetric heat capacity of soil, $(\rho c)_s$	3.36 MJ/m^{3}K
Euler constant, γ_i	0.5772

For given Q of 6000 W, the calculation results of this model and ILSM are contrasted with the simulation results of 3-D numerical model, and tab. 1 shows the basic parameters of this model, except for given Q of 6000 W and the difference of half shank spacings. The comparison diagrams of the three models under different half shank spacings are made, respectively. At the same time, the relative errors and absolute errors of the proposed model and ILSM are figured by contrast with 3-D numerical model. It should be mentioned that borehole thermal resistances of this model and ILSM are obtained by matching T_{out} of numerical model at the time of 80 hours.

Figure 8 shows the comparisons of the three models for different half shank spacings and fig. 8 also ignores the contrast of the first 1 hour. It is clear that this model is in line with the 3-D numerical model in the whole simulation time, while ILSM has a large error in a short time.

Figure 8(a) shows the contrast of the three models when the half shank spacing is 0.0255 m. The errors of the semi-analytical model and ILSM are large in the short period of time, and with the increase of simulation time, the relative errors and the absolute errors decrease and tend to zero. The relative errors of this model and ILSM after 10 hours are less than 0.067% and 0.351%, respectively. The absolute errors of this model and ILSM after 10 hours are less than 0.0189 °C and 0.0996 °C, respectively. It is clear that the relative errors and the absolute errors of this model are smaller than ILSM, indicating that this model has greater advantages than ILSM.

Figures 8(b) and 8(c) shows the contrast of three models when the half shank spacings are 0.0325 m and 0.048 m, respectively. Similarly, it is clear that the errors of this model

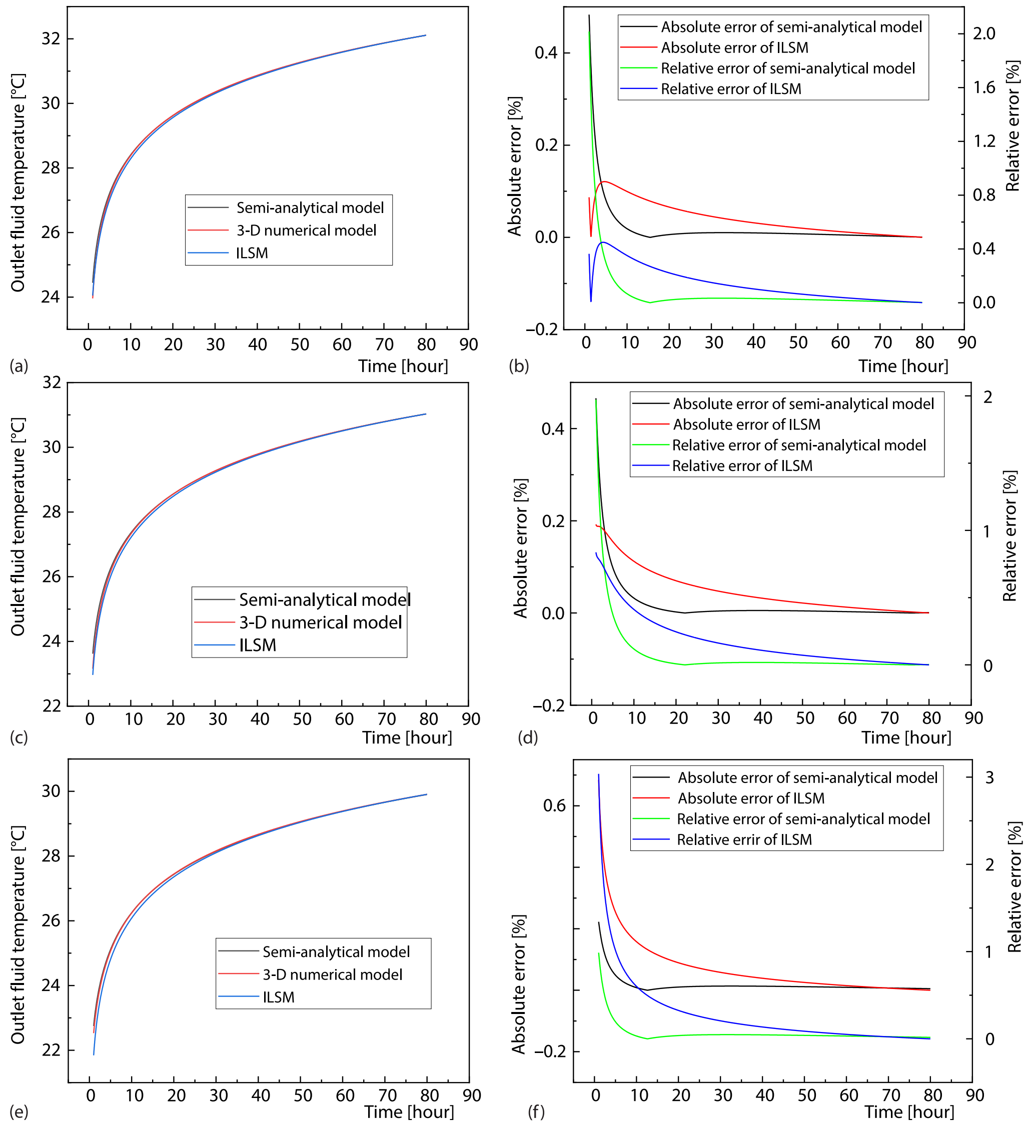


Figure 8. Comparisons of the three models for different half shank spacings; (a) x_c = 0.0255 m, (b) x_c = 0.0325 m, and (c) *xc* = 0.048 m

and ILSM are large in the short period of time, and with the increase of simulation time, the relative errors and the absolute errors decrease and tend to zero. When the half shank spacing is 0.0325 m, the relatively errors of the proposed model and ILSM after 10 hours are less than 0.11% and 0.41%, respectively. When the half shank spacing is 0.048 m, the relatively errors of the proposed model and ILSM after 10 hours are less than 0.032% and 0.602%, respectively. It is clear that the relative errors and the absolute errors of this model are smaller than ILSM, which further shows the superiority of semi-analytical model established in this paper.

The ILSM has large errors in short-time, which is because of ignoring the borehole heat capacity and the thermal interference among the pipes, but this model considers the influences of these factors, resulting in much higher accuracy in short time.

Conclusion

For VDUGHE, this paper makes some simplifications the VDUGHE is simplified as four equivalent quarter pipes with a thickness of 0 m, which include two inlet quarter pipes and two outlet quarter pipes. Next, we propose a semi-analytical model to analyse the heat transmission of the VDUGHE, in which heat transfer equations of fluids in the four equivalent quarter pipes are numerically calculated and the heat transfer equations of the grout and soil are analytically calculated grounded on composite cylindrical-source model. For the sake of verifying the accuracy of the model to simulate VDUGHE with different half shank spacings, the calculation results of this model are contrasted with experiment data and simulation results of a 3-D numerical model under the boundary condition of given T_{in}, and this model is contrasted with 3-D numerical model and ILSM under the boundary condition of given Q. The main conclusions are.

- Under the boundary condition of given T_{in}, this model is in good agreement with the experiment data. And this model agrees well with numerical model for different half shank spacings. Compared with numerical model, the relative errors of this model are less than 0.32% after 1 hours.
- Under the boundary condition of given Q, this model is in good agreement with 3-D numerical model for different half shank spacings. The errors of this model are much lower than those of ILSM especially during short time. The relative errors of this model are less than 0.11% after 10 hours, while the relative errors of ILSM are less than 0.60% after 10 hours.
- The semi-analytical model proposed in this paper can precisely analyse fluid temperature field of VDUGHE under the two boundary conditions. The novelty of the semi-analytical model is:
  - this model considers borehole heat capacity and axial effect, and therefore, has higher accuracy than analytical models, besides, it can be applied for not only given T_{in} but also given Q, and analytical models normally are applied for given Q and
  - grid division of the established model is very simple, and only dozens of grids are needed, therefore, compared with numerical models, the semi-analytical model is more convenient and needs much less computation time.

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Nomenclature

c_f – specific heat capacity of fluid
$G(t)$ – G-function of composite cylindrical-source model
H – VDUGHE length
h – convective heat transfer coefficient in each pipe
J_0, J_1 – zero-order and first-order Bessel functions of the first kind
k_0 – intermediate variable
k_g – thermal conductivity of grout
k_p – thermal conductivity of pipes
k_s – thermal conductivity of soil
M – mass-flow rate of fluid in each pipe
Q – total heat input rate of VDUGHE
Q_z – total heat input rate per depth
$q_i(z,t)$ – heat flow per depth on the wall of any equivalent inlet pipe
$q_o(z,t)$ – heat flow per depth on the wall of any equivalent outlet pipe
R_{12} – thermal resistance between fluids in pipes 1 and 2
R_{13} – thermal resistance between fluids in pipes 1 and 3
R_b – borehole thermal resistance
r_b – borehole radius
r_i – inner radius of pipes
r_{ie} – radius of four equivalent quarter pipes
r_o – outer radius of pipes
$T_0(z)$ – initial temperature
$T_{f1}(z,t)$ – fluid temperature in inlet pipes
$T_{f2}(z,t)$ – fluid temperature in outlet pipes
$T_{gi}(z,t)$ – wall temperature of two equivalent inlet pipes
$T_{go}(z,t)$ – wall temperature of two equivalent outlet pipes
T_{in}, T_{out} – inlet and outlet fluid temperatures
t – time
t_i, t_n – time at the i^{th} and n^{th} time steps
x_{12} – distance between the centers of pipes 1 and 2
x_{13} – distance between the centers of pipes 1 and 3
x_c – half shank spacing
Y_0, Y_1 – zero-order and first-order Bessel functions of the second kind
z – depth

Greek letters

β – integration variable
γ – intermediate variable
γ_i – Euler constant
δ – intermediate variable
ρ_f – density of fluid
$(\rho c)_f$ – volumetric heat capacity of fluid
$(\rho c)_g$ – volumetric heat capacity of grout
$(\rho c)_{ge}$ – equivalent volumetric heat capacity of grout
$(\rho c)_p$ – volumetric heat capacity of pipes
$(\rho c)_s$ – volumetric heat capacity of soil
φ – intermediate variable
ψ – intermediate variable

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