A NOVEL COUPLING DISCRETIZATION METHOD FOR MODELLING MULTI-PHASE HEAT EXCHANGERS

by

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This paper presents a novel coupling discretization method for modelling multiphase heat exchangers. In the method, the moving boundary method is adopted as the solver to solve each of the finite volume control volume divided by the method of finite volume method. When all finite volume control volume are solved, the finite volume control volume boundary values are updated based on the relationships of finite volume control volume. The solving procedure is initiated when the starting values of hot source fluid and cold source fluid outlet of the heat exchanger are given by the user and terminated when these values no longer change anymore. The experimental results of a plate heat exchanger with R245fa and Therminol 66 as cold source fluid and hot source fluid are adopted to validate the proposed model. Simulation results of 11 operating conditions show that the maximum deviation is within $\pm 4\%$ compared to the measured values. The model presented in this paper is appropriate for heat exchangers under operating conditions either with or without fluid phase change, such as the evaporator and condenser in the ORC system.

Key words: multi-phase heat exchanger, finite volume method, coupling discretization, moving boundary method, ORC

Introduction

The ORC is a currently rapid development technology and is already adopted in the recovery of diesel engines waste heat energy [1], solar energy, geothermal energy and industrial waste heat energy [2]. The typical ORC system is shown in fig. 1, in which six major components exist in the ORC system. Where, the heat exchangers are treated as the most important components in the ORC system since their performance has a huge impact on the compactness and cost of the ORC system. Therefore, the modelling and solution of the heat exchanger are treated as the key sub-procedures to evaluate the performance, compactness and cost of the ORC system. In the heat exchanger design books, the heat exchangers are modelled by the log mean temperature difference (LMTD) and ε -NTU [3], with which only the external boundary parameters or the geometry parameters of the heat exchanger can be calculated. Moreover, the phase change of the fluids in the heat exchanger is neglected in these methods, thus restricting the application in the ORC system.

In order to analyze the performance of plate heat exchangers with multi-fluid, multistream and multi-pass configurations, Qiao *et al.* [4] provided an finite volume method (FVM) based model to solve the problem. Patino *et al.* [5] and Chowdhury *et al.* [6] proposed an FVM based model to model the multi-phase heat exchangers. Yohanis *et al.* [7] proposed an

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enthalpy-based method to solve the heat transfer problem in which one or two fluids phase change happens. For the purpose of developing a general heat exchanger model applying to both super and sub-critical fluids in the heat recovery system. Zegenhagen et al. [8] developed an FVM based heat exchanger model to solve the problem. Gullapalli et al. [9] developed a steady-state ORC system simulationol to predict the performance of the ORC system. The method reported by Bell et al. [10] is a fast and robust algorithm to predict the steady-state performance of the heat exchanger. The experimental results of the 11 kWe prototype ORC and the off-design ORC model are presented by Lecompte et al. [11]. In their paper, the off-design ORC model is validated by the experimental results. In the off-design ORC

model, a hybrid heat exchanger model which combines the FVM and moving boundary method (MBM) is employed. Sarfraz *et al.* [12] presents a detailed segment-by-segment model of a fin-tube heat exchanger that calculates the conduction between all the adjacent tube segments through the fins. Taler *et al.* [13] provides a new heat exchanger model based on FVM. The gas temperature across a tube row is calculated by integral. Huang *et al.* [14] compares three approximation-assisted heat exchanger models for the steady-state simulation of vapor compression system. Bone *et al.* [15] presents a methodology to develop accurate and computationally efficient on- and off-design models of heat exchangers that exhibit complex non-linear behaviours. Jahn *et al.* [16] describes a quasi 1-D segment by segment heat exchanger model that can be used for the design and evaluation process. Hagen *et al.* [17] presents a generic counter current heat exchanger model for cycle optimization process. Chu *et al.* [18] proposes a moving-boundary and finite-volume coupling method to solve the heat exchangers whether fluid phase on both sides changes or not.

In the present work, a novel coupling discretization method for modelling heat exchangers is proposed. In the method, the heat exchanger is first discretised based on FVM and then MBM is applied to each of the finite volume control volume (FVCV) divided by FVM. The phase change control volume (CV) determination procedure in FVM is omitted in the proposed coupling discretization method, which decreases the complexity of the FVM.

Methodology

For the heat exchanger model based on the coupling discretization method, the heat exchanger is first divided into multiple CV according to FVM and then a single CV is furthermore dealing with MBM. For the purpose of avoiding confusion, the CV divided by FVM are referred to as FVCV and the CV divided by MBM are referred to as MBCV in the following parts of the paper. The flow chart of the heat exchanger model based on the coupling discretization method is shown in fig. 2. It can be seen from fig. 2 that four steps are needed to constitute the solver of the heat exchanger model namely the determination of the initial values of the geometry parameters of the heat exchanger, setting initial values of FVCV which includes boundary values. It should be pointed out that the flow chart shown in fig. 2 is very compact and the phase

change CV identification procedures are simplified in the original FVM with MBM being able to deal with phase change of the fluid in FVCV. Once all of the FVCV are solved with MBM, the FVCV boundary values can be updated and then the convergence of the solution can be reached. The processes will be repeated until the convergence criteria can be satisfied. The solving process based on the assumptions is shown:

- The heat transfer from the evaporator to the environment is negligible.
- The fouling of evaporator is negligible.
- The pressure drop of hot side fluid (HSF) and cold side fluid (CSF) flows through the heat exchanger is negligible.
- The flow of HSF and CSF in the heat exchanger is simplified as 1-D flow.

Constitution and relationship of the CV

The mesh of the MBM and FVM is the combination of multiple CV, where a possible CV is shown in fig. 3. It can be seen from fig. 3 that the CV includes inlet, outlet and center nodes. The fluid thermophysical parameters are needed by the algorithm, such as p, T, h, s, X, stored in the nodes for the purpose of facilitating the program writing. The thermophysical parameters of the center nodes of HSF and CSF

Start Initial geometry parameters Initial FVCV i = 0Colve FVCV using MBM i > m? Ves Update FVCV boundary values No $\Delta h^{co} | < \varepsilon$ and $|\Delta h^{ho}| < \varepsilon$?





Figure 3. Single CV

are calculated by linear interpolation of the inlet and outlet nodes, as shown in eqs. (1) and (2), respectively. Similarly, the wall temperature is calculated by linear interpolation of the center temperature of HSF and CSF, as shown in eq. (3). The relationship of the heat transfer rate between HSF and CSF should satisfy eq. (4) for the steady-state of the heat exchanger. The right boundary of CV i and the left boundary of CV i + 1 are in the same position in the actual heat exchanger. Therefore, the thermophysical parameters of the nodes on the boundary should be of the same values, as shown in eqs. (5) and (6). It is worth noting that the linear interpolation adopted in eqs. (1)-(3) may be inappropriate when using relatively small number of FVCV to approximate the large heat exchanger:

$$\chi_i^{\rm h,c} = \frac{\chi_i^{\rm h,i} + \chi_i^{\rm h,o}}{2} \tag{1}$$

$$\chi_{i}^{c,c} = \frac{\chi_{i}^{c,i} + \chi_{i}^{c,o}}{2}$$
(2)

$$T_{i}^{w} = \frac{T_{i}^{h,c} + T_{i}^{c,c}}{2}$$
(3)

$$\dot{Q}_{i}^{h} = \dot{Q}_{i}^{c} \tag{4}$$

$$\chi_{i+1}^{h,o} = \chi_i^{h,i} \tag{5}$$

$$\chi_{i+1}^{c,i} = \chi_i^{c,o} \tag{6}$$

The outlet specific enthalpy of HSF and CSF can be calculated if the inlet parameters of fluids and the heat transfer rate of the CV \dot{Q} are known, as shown in eqs. (7) and (8). The other thermophysical parameters of the fluids of the CV can be updated using the function provided by the open source thermal physical property library CoolProp [19] with p and h as the input parameters. The wall temperature of the CV is a key parameter for the calculation of the convective heat transfer coefficients. Thus, if we know the wall temperature of the CV, the convective heat transfer coefficient of the plate heat exchanger, respectively. Furthermore, the heat transfer area of HSF and CSF are shown in eqs. (9) and (10), respectively:

$$h_i^{\rm h,o} = h_i^{\rm h,i} - \frac{\dot{Q}_i^{\rm h}}{\dot{m}_i^{\rm h}}$$
(7)

$$h_{i}^{c,o} = h_{i}^{c,i} + \frac{\dot{Q}_{i}^{c}}{\dot{m}_{i}^{c}}$$

$$\tag{8}$$

$$\mathcal{A}_{i}^{h} = \frac{\dot{\mathcal{Q}}_{i}^{h}}{\alpha_{i}^{h} \left(T_{i}^{h,c} - T_{i}^{w}\right)}$$
(9)

$$A_{i}^{c} = \frac{Q_{i}^{c}}{\alpha_{i}^{c} \left(T_{i}^{w} - T_{i}^{c,c}\right)}$$
(10)



The geometrical parameters are crucial to ensure the rest of the algorithm can be correctly executed. The geometrical parameters of plate heat exchanger that can be obtained from technical manual provided by the manufacturer are shown in fig. 4.

With these given parameters, the other important parameters, such as heat transfer area, hydraulic diameters and so on, can be calculated by the method reported in [3]. The detailed description of the calculation procedures is given in *Appendix A*.

Initial FVCV boundary values

The mesh of the FVCV for a counter-flow heat exchanger divided by FVM is shown in fig.

5. As can be seen in fig. 5, the CSF flows from CV 0 to CV m - 1, and the HSF flows from CV m - 1 to CV 0. The energy of the HSF is transferred to the CSF through the wall between the two fluids, resulting in the temperature decrease and increase of HSF and CSF along with the flow direction. The boundary values of the heat exchanger, such as HSF and CSF outlet parameters, are the basis for starting the iteration processes of FVM. However, these boundary values



Figure 4. Main geometrical parameters of single plate appear in plate heat exchanger

are unknown parameters for solving and cannot be obtained in advance. Therefore, empirical values are given instead. The empirical values of the inlet and outlet specific enthalpy of HSF and CSF are generated by the linear interpolation of the boundary values of the heat exchanger, as shown in eqs. (11) and (12), respectively. When the boundary values of the FVCV are known, the other parameters needed to start the FVM iteration process can be calculated using eqs. (1)-(3), respectively:



Figure 5. Mesh of heat exchanger

$$h_{i}^{h,o} = h^{h,o} + i \frac{h^{h,i} - h^{h,o}}{n}$$

$$h_{i}^{h,i} = h^{h,o} + (i+1) \frac{h^{h,i} - h^{h,o}}{n}$$
(11)

$$h_{i}^{c,i} = h^{c,i} + i \frac{h^{c,o} - h^{c,i}}{n}$$

$$h_{i}^{c,o} = h^{c,i} + (i+1) \frac{h^{c,o} - h^{c,i}}{n}$$
(12)

The MBM-based FVCV solving procedures

After the procedures shown in section Initial FVCV boundary vales, all of the FVCV have the initial boundary values. However, eq. (4) may not be satisfied by the FVCV since the enthalpy of HSF and CSF is generated by linear approximation. Therefore, energy balance procedures must be taken to ensure FVCV satisfy the eq. (4). In addition, fluid phase transition may occur in any FVCV, making the energy balance procedures complexer. To solve the problem properly, the MBM is adopted for the energy balance procedures of FVCV. The MBM shown in this paper is very similar to the one shown in [10], except the required heat transfer area calculation procedures. In other words, the LMTD algorithm is replaced by the iteration of the true wall temperature of the MBCV. The flow chart of the MBM is shown in fig. 6. It can be seen in fig. 6 that there exists five steps in the MBM. Firstly, the external pinching analysis is to obtain the upper boundary of the input parameters of the Brent meth-



Figure 6. Flow chart of the MBM based FVCV solving procedures

od, in other words, the theoretical maximum heat transfer rate of the FVCV. Secondly, FVCV are divided into MBCV for processing the phase change. Thirdly, internal pinching analysis is to update the theoretical maximum heat transfer rate of the FVCV to ensure the correct execution of the Brent method. Fourthly, the actual heat transfer rate of the FVCV is obtained using Brent's method [20]. The range of the input parameter of Brent method \dot{Q} is shown in eq. (13), where \dot{Q}_{max} is the minimum value of the results of the internal and external pinch analysis of the FVCV. The convergence criteria of the Brent method, which is the heat transfer area ratio of the calculated to the actual, is shown in eq. (14):

$$0 < \dot{Q} < \dot{Q}_{\max} \tag{13}$$

$$\left| r(\dot{Q}) \right| < \varepsilon \tag{14}$$

The MBCV division

Due to the heat transfer rate, boundary values and bubble and dew state of HSF and CSF of the FVCV are known parameters, it is very easy to split FVCV into moving boundary control volume (MBCV) where both side fluids are in the definite phase state. Example of the division of the FVCV is shown in fig. 7. As shown in fig. 7, two MBCV are obtained from MBCV division procedures since only bubble node exists in CSF and three MBCV exists because of the existence of dew node of HSF and CSF. On the condition that both bubble node and dew node exist in HSF and CSF, a maximum of five MBCV may be obtained from the MBCV division procedures. However, the condition rarely exists in the one FVCV because of the small heat transfer area of FVCV compared to the overall heat exchanger. The group of phase change node and heat balance node constitute the virtual boundary and the FVCV is divided into MBCV by the virtual boundary in reality. On the condition that HSF or CSF satisfies eq. (15) or eq. (16), then the virtual MBCV boundary exists. The heat balance nodes exist in the virtual MBCV boundary can be updated using eqs. (7) and (8) according to HSF and CSF.



$$p^{h} < p^{h,crit}$$

$$h^{h,o} < h^{h,x} < h^{h,i}$$
(15)

$$p^{c} < p^{c,crit}$$

$$h^{c,i} < h^{c,x} < h^{c,o}$$
(16)

External pinching

The direction of energy transfer, which only goes from high temperature to low temperature, is limited by the second law of thermodynamics. Therefore, the temperature of HSF should always be greater than or at least equal to CSF. As mentioned in section *The MMBbased FVCV solving procedures*, the boundary parameters of FVCV are generated by linear approximation, thus the temperature of HSF and CSF may not satisfy the Second law of thermodynamics and the pinching analysis needs to be conducted to eliminate these situations. The external pinching analysis sets the outlet temperature of HSF and CSF to theoretical minimum and maximum value that can be reached, as shown in eqs. (17) and (18). With the help of eqs (17) and (18), the heat transfer rate of HSF and CSF can be calculated using eqs. (19) and (20). Furthermore, take the results of eqs. (17) and (18) as the parameters of eq. (21), the theoretical maximum heat transfer rate of the FVCV can be obtained:

$$T_0^{\rm h,o} = T_0^{\rm c,i}$$
 (17)

$$T_{m-1}^{c,o} = T_{m-1}^{h,i}$$
(18)

$$\dot{Q}_{\text{ext}}^{\text{h}} = \dot{m}^{\text{h}} \left(h_{\text{m-l}}^{\text{h,i}} - h_{0}^{\text{h,o}} \right) \tag{19}$$

$$\dot{Q}_{\text{ext}}^{c} = \dot{m}^{c} \left(h_{\text{m-1}}^{c,o} - h_{0}^{c,i} \right)$$
(20)

$$\dot{Q}_{\text{ext}} = \min\left(\dot{Q}_{\text{ext}}^{\text{h}}, \dot{Q}_{\text{ext}}^{\text{c}}\right) \tag{21}$$

Internal pinching

The Second law of thermodynamics should not only be satisfied with the boundary temperature of FVCV, but also the boundary temperature of MBCV. Therefore, the internal pinching analysis, whose purpose is to remove the impossible temperature distribution of the internal MBCV, should be executed. The possible position of internal pinching of HSF and CSF is the phase change point, thus the internal pinching analysis simply resets the temperature of the phase change node and the heat balance node to reasonable values. Bubble point is the reasonable internal pinching point of CSF exists, update of the heat transfer rate of CSF is needed, as shown in eq. (23). Similarly, Dew point is the reasonable internal pinching point that appears in HSF, as shown in eq. (24). If the internal pinching point of HSF exists, the heat transfer rate of HSF is needed to be updated using eq. (25).

$$T_{\rm i}^{\rm h,i} = T_{\rm i}^{\rm c,bp} \tag{22}$$

$$\dot{Q}_{int}^{c} = \dot{m}^{c} \left(h_{i}^{c,bp} - h_{0}^{c,i} \right) + \dot{m}^{h} \left(h_{m-1}^{h,i} - h_{i}^{h,i} \right)$$
(23)

$$T_{\rm i}^{\rm c,o} = T_{\rm i}^{\rm h,dp} \tag{24}$$

$$\dot{Q}_{\rm int}^{\rm h} = \dot{m}^{\rm c} \left(h_{\rm i}^{\rm c,o} - h_{\rm 0}^{\rm c,i} \right) + \dot{m}^{\rm h} \left(h_{\rm m-1}^{\rm h,i} - h_{\rm i}^{\rm h,dp} \right)$$
(25)

Theoretical maximum heat transfer rate of the FVCV

The theoretical maximum heat transfer rate is an input parameter of Brent's method, as shown in eq. (26). In eq. (26), \dot{Q}_{ext} is defined by eq. (21) and \dot{Q}_{int}^{h} and \dot{Q}_{int}^{c} are defined by eqs. (23) and (25), respectively:

$$\dot{Q}_{\text{max}} = \min\left(\dot{Q}_{\text{ext}}, \dot{Q}_{\text{int}}^{\text{h}}, \dot{Q}_{\text{int}}^{\text{c}}\right) \tag{26}$$

Required heat transfer area for MBCV

The updated wall temperature of MBCV can be calculated by eq. (27), the result of the combination of eqs. (4), (9) and (10). The unknown parameter R, which is defined as the heat

transfer area ratio of HSF to CSF of the heat exchanger, is shown in eq. (28). Note that R is an inherent parameter of the heat exchanger and can be obtained from the geometry parameters of the heat exchanger. One more thing should be noted is the omittance of the thermal resistance of the wall in eq. (27). Afterwards, the iteration of the wall temperature of the MBCV can be started until eq. (29) is satisfied by the MBCV. When the wall temperature of the MBCV is updated, eqs. (9) and (10) can be adopted to update the required heat transfer area for HSF and CSF, respectively. When the required heat transfer area of MBCV is updated, a summation process is needed to calculate the required heat transfer area of FVCV, as shown in eqs. (30) and (31):

$$T_j^{w,u} = \frac{R\alpha_j^{h}T_j^{h,c} + \alpha_j^{c}T_j^{c,c}}{R\alpha_j^{h} + \alpha_j^{c}}$$
(27)

$$R = \frac{A^{\rm h}}{A^{\rm c}} \tag{28}$$

$$\left|T^{w} - T^{w,u}\right| < 0.1$$
 (29)

$$A_{i}^{h,r} = \sum_{j=0}^{n-1} A_{j}^{h}$$
(30)

$$A_t^{c,r} = \sum_{j=0}^{n-1} A_j^c$$
(31)

The residual function using in Brent method for the convergence estimate is shown in eq. (32), where the ratio of the hot side required to the actual heat transfer area of the heat exchanger is adopted:

$$r(\dot{Q}) = 1 - \frac{A_{\rm i}^{\rm h.r.}}{A_{\rm i}^{\rm h}} \tag{32}$$

The FVCV boundary value update procedures

The energy balance of each of the FVCV is guaranteed by the procedures of section *The MMB-based FVCV solving procedures*. However, eqs. (5) or (6) may not be satisfied by the FVCV, thus the convergence of the heat exchanger is not guaranteed. Therefore, the inlet parameters of HSF and CSF of the FVCV need to be updated according to eqs. (5) and (6). The convergence of the heat exchanger is guaranteed if the outlet enthalpy of HSF and CSF satisfy eq. (33):

$$\Delta h^{c,o} = \left| h^{c,o} - h^{c,o,u} \right| < \varepsilon$$

$$\Delta h^{h,o} = \left| h^{h,o} - h^{h,o,u} \right| < \varepsilon$$
(33)

Convective heat transfer coefficient of the plate heat exchanger

The single phase heat transfer coefficient correlations of Martin [3] are adopted in this paper as shown in eq, (34). The two-phase heat transfer coefficient correlations of Huang [21] are adopted in this paper as shown in eq. (35). The details of these correlations are further reported in *Appendix B.1* and *Appendix B.2* for simplicity:

$$\mathrm{Nu}_{\mathrm{sp}} = \frac{\alpha_{\mathrm{sp}} D_{\mathrm{h}}}{k} = c_q \operatorname{Pr}^{(1/3)} \left(\frac{\eta}{\eta_{\mathrm{w}}}\right)^{(1/6)} [2\mathrm{Hg}\sin 2\phi]q$$
(34)

$$\mathrm{Nu}_{\mathrm{tp}} = \frac{\alpha_{\mathrm{tp}} D_{\mathrm{h}}}{k_{\ell}} = 1.87 \cdot 10^{-3} \left[\frac{q'' d_0}{k_{\ell} T_{\mathrm{sat}}} \right]^{0.56} \left[\frac{\Delta h_{\mathrm{fg}} d_0}{\gamma_{\ell}^2} \right]^{0.31} \mathrm{Pr}^{0.33}$$
(35)

Validation

In order to evaluate the accuracy of the heat exchanger model reported in this paper, the 11 experimental steady-state operating conditions shown in tab. 1 of the plate heat exchanger are adopted to validate the heat exchanger model.

Geometry and fluids inlet parameters of the plate heat exchanger

The geometry parameters of the heat exchanger are shown in tab. 1 and the application range of T66 and R245fa is shown in tab. 2. The inlet parameters of HSF and CSF of the plate heat exchanger which are obtained from tab. A.2 from [11] are shown in tab. 1 for the purpose of simplicity.

Final and the second se								
Parameter	Unit	Value						
Model	[-]	SWEP B200T SC-M						
N _p	[-]	150						
$B_{\rm p}$	[m]	0.243						
L _p	[m]	0.525						
â	[m]	$0.94 \cdot 10^{-3}$						
Λ	[m]	$0.3 \cdot 10^{-3}$						
Temperature range	[°C]	-196~225						
Maximal pressures	[MPa]	4.5 at 135 °C and 3.6 at 225 °C						
Material	[-]	Stainless steel						
Weight	[kg]	69.8						

Table 1. Geometry parameters of the plate heat exchanger

Table 2. Application range of flu	ids in heat exchangers
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Parameter	T_{\min} [°C]	T_{\max} [°C]	p_{\max} [MPa]	$d_{\rm max}$ [kgm ⁻³]	$p_{\rm crit}$ [MPa]	$T_{\rm crit}$ [°C]
T66	-7	345	_	_	-	-
R245fa	-102.1	166.85	200.0	1648.8	3.651	154.01

Effect of the FV mesh size on the heat transfer rate

An analysis of the FV mesh size effect on the heat transfer rate is carried out and the results are shown in fig. 8. It can be seen from fig. 8 that the heat transfer rate of the 11 cases of the plate heat exchanger is declining along with the FV mesh size increase. However, when the FV mesh size is larger than 10, the variation of the heat transfer rate is very little among the 11 cases. Therefore, the FV mesh size is set to 10 in the following sections of the paper.





heat exchanger

Heat transfer rate validation of the plate

results are shown in fig. 9. Note that the x- and

In this paper, the heat transfer rate of the plate heat exchanger is adopted to validate the accuracy of the heat exchanger model and the





Case study

y-axis are the simulation and experimental heat transfer rate of the plate heat exchanger, respectively. As can be seen from fig. 9, the heat trans-

fer rate deviates less than $\pm 4\%$ from the equal line. Therefore, the model proposed in this paper is able to predict the heat exchanger performances with enough accuracy.

For the purpose of deeply understanding the computing procedures of the novel coupling discretization method, the detailed simulation results of Case 1 and brief discussions are illustrated in the following parts of the paper. The similar results can be obtained in the rest of the cases, thus not shown in this paper anymore.

Simulation results

The temperature and quality distribution of FVCV and MBCV are shown in fig. 10. The temperature and quality distribution of FVCV are shown in the lower sub-figure of fig. 10, while the temperature and quality distribution of the phase change FVCV are more detailed shown in the upper sub-figure of fig. 10. As can be seen from fig. 10, CSF phase change happens in FVCV 3 and 8 during the heat transfer process of HSF and CSF, in which the phase



Figure 10. Temperature and quality distribution of FVCV and MBCV

state of CSF varies from sub-cooled to two-phase in FVCV 3 and from two-phase to super-heated in FVCV 8. Since no phase change occurs in the other FVCV, only one MBCV exists in the MBM based on FVCV solving procedures. In this condition, the MBCV will be equivalent to FVCV and thus not repeated shown in fig. 10. The temperature of HSF and CSF in the FVCV has a non-linear distribution due to the different heat transfer coefficient and temperature difference of the FVCV. The temperature slope of CSF in sub-cooled region is larger than that of the super-heated region because of the large temperature difference of CSF in sub-cooled region. On the contrary, the slope of HSF temperature has been slightly smoothed since the large specific heat of HSF. At last, the pinch point of Case 1 appears on the bubble point of CSF.

The convective heat transfer coefficient distribution of FVCV is shown in fig. 11. It can be seen from fig. 11 that the convective heat transfer coefficient of HSF is almost the same in all of the FVCV, with only slight change occurring at FVCV 0, 8b, and 9. However, the convective heat transfer coefficient of CSF changes significantly in the phase change FVCV and

the two-phase FVCV. The change of the former is mainly due to the phase change in FVCV x, so that the FVCV x is represented as MBCV xa and xb in fig. 11, where MBCV xa and xb represent the sub-liquid part and two-phase part of the FVCV 3, respectively. Nevertheless, the change of the latter is mainly due to the bubbles occurring in the two-phase region and the evaporation heat transfer. It can be obviously seen from fig. 11 that the convective heat transfer coefficient of CSF in two-phase region is significantly larger than that of the single phase region due to the strong disturbance caused by the formation and detachment of bubbles.



Figure 11. Convective heat transfer coefficient distribution of FVCV and MBCV

The wall temperature distribution of FVCV is shown in fig. 12. The wall temperature rises sharply when CSF flows in sub-liquid region and super-heated region because of the relatively large heat transfer temperature difference between HSF and CSF. However, the wall temperature gradually levels out along with CSF beginning to evaporate due to the decrease of the heat transfer temperature difference between HSF and CSF. Moreover, only slight rise of the wall temperature is observed when CSF flows in the two-phase region due to relatively small change of the temperature of HSF.



Figure 12. Wall temperature distribution of FVCV and MBCV



Figure 13. Heat transfer area distribution of FVCV and MBCV

The heat transfer area distribution of the FVCV is shown in fig. 13. As can be seen from fig. 11, the heat transfer area of each FVCV is equal under the precondition of the FVM. Note that the heat transfer area of FVCV 3 and 8 is split into two parts by the MBM based on FVCV solving procedures and is actually equal to the sum of the two parts. The heat transfer area of FVCV 3a and 3b represents the sub-liquid and two-phase part of FVCV 3, respectively. Accordingly, the heat transfer area of FVCV 8a and 8b represents the two-phase and super-heat-ed part of FVCV 8, respectively.

Conclusion

The performance of the ORC evaporator and condenser significantly affects cycle efficiency. Therefore, modelling of heat exchangers is important for the development of the ORC system simulationals. In this paper, we proposed a novel coupling discretization method for modelling heat exchangers with fluid phase change. The proposed algorithm is proved to be effective to the ORC heat exchangers solving and can be easily integrated with the ORC system simulationals. In addition, the accuracy of the algorithm is verified by the 11 steady-state experimental operating conditions and the results show that the algorithm can meet the engineering requirements. The detailed results of Case 1 are shown in section 6 for the purpose of deeply understanding the computing procedures of the novel coupling discretization method. Our future work will be focused on the integration of the algorithm with the ORC system simulationals and verify the performance of the ORC system simulationals.

Appendix A

Geometry parameters of plate heat exchanger

The area of one plate of the plate heat exchanger is shown:

$$A_p = \Phi A_0 \tag{A1}$$

where A_0 is the plane surface area which is shown in eq. (A2) and Φ – the calculated in eq. (A3). The unknown parameter X in equation is defined in eq. (A4). The hydraulic diameter of the plate heat exchanger is defined in eq. (A5):

$$A_0 \approx B_{\rm p} L_{\rm p} \tag{A2}$$

$$\Phi(X) \approx \frac{1}{6} \left(1 + \sqrt{1 + X^2} + 4\sqrt{1 + \frac{X^2}{2}} \right)$$
(A3)

$$X = \frac{2\pi\hat{a}}{\Lambda} \tag{A4}$$

$$D_{\rm h} = \frac{4\hat{a}}{\Phi} \tag{A5}$$

The total heat transfer area of the plate heat exchanger can be simply calculated:

$$A^{\rm h} = A^{\rm c} = A_{\rm p} \left(N_{\rm p} - 2 \right) \tag{A6}$$

The hot side and cold side total cross-section area of the gap of the two plates is calculated in eqs. (A7) and (A8), respectively:

$$A_{\rm gap}^{\rm h} = 2\hat{a}B_{\rm p}N_{\rm gap}^{\rm h} \tag{A7}$$

$$A_{\rm gap}^{\rm c} = 2\hat{a}B_{\rm p}N_{\rm gap}^{\rm c} \tag{A8}$$

Appendix B

Single phase and two-phase evaporation heat transfer coefficient of plate heat exchanger

B.1. Single phase heat transfer.

For the single phase heat transfer in plate heat exchanger, the Nusselt number shown in eq. (A9) is adopted in this paper. The value of the constants cq and q is set to 0.122 and 0.374, respectively. The unknown parameter Hagen number is calculated in eq. (A10). In eq. (A10), Reynolds number and ζ are obtained from eqs. (A11) and (A13), respectively. The mass flux *G* in eq. (A12) is obtained from eq. (A12) and the unknown parameters ζ_0 and ζ_1 are acquired by combining eq. (A14) or (A15) with (A16). Note that eq. (A14) is accepted if Re < 2000, otherwise eq. (A15) will be accepted:

$$Nu_{sp} = \frac{\alpha_{sp}D_{h}}{k} = c_{q} \operatorname{Pr}^{(1/3)} \left(\frac{\eta}{\eta_{w}}\right)^{(1/6)} \left[2\operatorname{Hg}\sin 2\varphi\right]^{q}$$
(A9)

$$Hg = \frac{Re^2 \zeta}{2}$$
(A10)

$$\operatorname{Re} = \frac{GD_{h}}{\mu}$$
(A11)

$$G = \frac{\dot{m}}{A_{\rm gap}} \tag{A12}$$

$$\frac{1}{\sqrt{\zeta}} = \frac{\cos\varphi}{\sqrt{b}\tan\varphi + c\sin\varphi + \frac{\zeta_0}{\cos\varphi}} + \frac{1 - \cos\varphi}{\sqrt{\zeta_1}}$$
(A13)

$$\zeta_0 = \frac{64}{\text{Re}}$$
(A14)
 $\zeta_{1,0} = \frac{597}{\text{Re}} + 3.85$

$$\zeta_{0} = \frac{1}{\left[1.8\ln(\text{Re}) - 1.5\right]^{2}}$$

$$\zeta_{1,0} = \frac{39}{\text{Re}^{0.289}}$$
(A15)

$$\zeta_1 = a\zeta_{1,0} \tag{A16}$$

B.2. Two-phase evaporation heat transfer. For the two-phase evaporation heat transfer in plate heat exchanger, the Nusselt number shown in eq. (A17) is adopted in this paper. In eq. (A17), T_{sat} is the saturation temperature of the fluid, d_0 is the bubble departure diameter as shown in eq. (A18), and γ is the thermal diffusivity as shown in eq. (A19).

$$Nu_{tp} = \frac{\alpha_{tp} D_{h}}{k_{\ell}} = 1.87 \cdot 10^{-3} \left[\frac{q'' d_{0}}{k_{\ell} T_{sat}} \right]^{0.56} \left[\frac{\Delta h_{tg} d_{0}}{\gamma_{\ell}^{2}} \right]^{0.31} Pr^{0.33}$$
(A17)

$$d_0 = 0.0146\theta \left[\frac{2\sigma}{g(\rho_\ell - \rho_\nu)}\right]^{0.5}$$
(A18)

$$\gamma_l = \frac{k}{\rho c_p} \tag{A19}$$

Appendix C

Inlet and outlet parameters of fluids of the 11 steady-state operating conditions, see tab. 3.

Table 3. Inlet and outlet parameters of fluids of the 11 steady-state operating conditions

Parameter	1	2	3	4	5	6	7	8	9	10	11
HSF	T66										
$\dot{m}^{ m h}$	1.568	1.565	1.566	3.005	3.003	3.003	3.005	1.494	1.496	1.505	1.504
$T^{\mathrm{h,i}}$	120.0	120.0	120.0	119.9	119.9	120.0	119.9	109.9	110.0	109.9	110.0
$H^{\mathrm{h,i}}$	174.1	174.1	174.1	173.9	173.9	174.1	173.9	155.0	155.2	155.0	155.2
$p^{\mathrm{h,i}}$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
X ^{h,i}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
T ^{h,o,ER}	89.7	90.3	90.3	103.3	103.2	103.1	103.1	83.1	83.1	83.0	83.3
$H^{\mathrm{h,o,ER}}$	117.9	118.9	118.9	142.7	142.5	142.4	142.4	106.1	106.1	105.9	106.5
$T^{\mathrm{h,o,MR}}$	90.2	90.7	90.8	102.9	102.9	102.9	102.9	84.4	84.4	84.6	84.6
$H^{\rm h,o,MR}$	118.8	119.6	119.8	142.1	142.2	142.1	142.0	108.3	108.5	108.7	108.7
$p^{\mathrm{h,o}}$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
X ^{h,o}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
CSF						R245fa					
<i>ṁ</i> °	0.352	0.350	0.353	0.383	0.383	0.387	0.385	0.290	0.292	0.291	0.292
$T^{c,i}$	33.5	35.9	36.5	38.1	38.5	38.5	38.5	35.7	35.7	35.7	35.5
H ^{c,i}	244.5	247.7	248.5	250.7	251.3	251.3	251.3	247.4	247.4	247.4	247.2
$p^{c,i}$	1.171	1.167	1.178	1.266	1.266	1.270	1.271	0.972	0.972	0.973	0.975
X ^{c,i}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
$T^{c,o,ER}$	116.3	116.6	115.7	119.4	119.5	119.4	119.4	108.1	107.8	107.8	107.9
$H^{c,o,ER}$	498.3	498.7	497.4	500.3	500.4	500.2	500.2	492.2	491.8	491.8	491.9
$T^{c,o,MR}$	109.6	110.4	108.9	118.9	118.9	118.9	118.8	102.7	103.1	103.2	103.1
$H^{c,o,MR}$	489.8	490.9	488.9	499.2	499.2	499.1	499.0	485.5	486.0	486.2	485.9
p ^{c,o}	1.171	1.167	1.178	1.266	1.266	1.270	1.271	0.972	0.972	0.973	0.975
Xc,o	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Nomenclature

- amplitude of the corrugation, [m] â
- heat transfer area, [m²] A
- A_0 plane projection of the plate surface, [m²]
- actual plate surface, [m²]
- width of the corrugation pattern, [m]
- $B_{\rm p}$ plate width, [m]
- factor in modified Leveque analogy, [-] C_q
- hydraulic diameter, [m] $D_{\rm h}$
- d_0 bubble departure diameter, [m]
- Hg Hagen number, [1–]
- enthalpy, [kJkg⁻¹] h
- k - thermal conductivity, [Wm⁻²K⁻¹]
- L - length between two crossing points, [m]
- $L_{\rm p}$ plate length, [m]
- m total number of FVCV
- \dot{m} mass-flow rate, [kgs⁻¹]
- $N_{\rm p}$ number of the plate, [–]
- Nu Nusselt number, [-]
- Pr Prandtl number, [–]
- pressure, [Pa]
- Ò - heat transfer rate, [kW]
- q- exponent in modified Leveque analogy, [-]
- a'' heat flux, [Wm²]
- Re Reynolds number
- S - entropy, [kJkg⁻¹K⁻¹]
- Т - temperature, [C]
- X quality [–] or wave number, [–]

Greek symbols

- convective heat transfer coefficient, [Wm²K⁻¹] α
- thermal diffusivity, [m²s⁻¹] V
- effectiveness of heat exchanger or residual ε
- viscosity, [Pas] η
- wavelength, [m] Λ
- density, [kgm⁻³] D
- Φ - surface enhancement factor, [-]
- inclination angle of the corrugation φ
- fluid state parameter, which stands for T, p, χ H, S, or X

Subscripts

ext - external

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- latent heat of evaporation fg - FVCV index i int internal - saturation liquid l max - maximum min – minimum sat - saturation sp - single phase - two-phase tp - at wall temperature W **Superscripts** bp - bubble point - cold, center, calculate с crit - critical point dp - dew point - geometry g - hot h - inlet 1 - ideal id
- outlet 0
- required
- r - shell
- s - tube
- t u
- update
- W – wall
- bp or dp Х

Acronyms

- CV- control volume
- CSF - cold side fluid
- ER - experimental results
- FVM - finite volume method
- FVCV finite volume control volume
- hot side fluid HSF
- LMTD log mean temperature difference
- MBCV moving bounding control volume
- MBM moving bounding method
- MR - modeling results
- NTU - number of transfer units
- R heat transfer area ratio
- WHR - waste heat recovery

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