INVESTIGATION OF AERODYNAMICS EDUCATION BASED ON THE WAVERIDER PARAMETERIZED DESIGN PROCEDURE

by

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Original scientific paper
https://doi.org/10.2298/TSCI220311032H

This paper describes the development and coding of a design tool for waverider parameterization based on the Taylor-Maccoll equation and streamline tracing method. This tool, intended for undergraduates studying in the field of aerodynamics, illustrates the relationship among various parameters in terms of their effect on the aerodynamic performance of waverider models with the same freestream Mach number. The design procedure is evaluated and validated for multiple shock wave profiles and freestream Mach numbers, and is found to be in good agreement with the CFD results. For undergraduates studying aeronautical engineering, the suggested design tool offers an intuitive image of waverider creation and a clear picture of the guiding principles of waveriders. By imposing a set of design criteria, the tool also gives students enough freedom to arrive at their own waverider shape.

Key words: *waverider, aerodynamic, parameterize design*

Introduction

In order to decrease average drag and reap tremendous thrust margins at hypersonic speeds, a special technique which can effectively integrate the engine with the airframe should be used in the process of designing a supersonic or hypersonic vehicle. In this context, waverider have been studied by more and more researchers.

A waverider is a supersonic or hypersonic automobile that has a connected shock wave all alongside its main facet, giving the impression that the vehicle is riding on top of the shock wave, [1] so it is called waverider. The format of the waverider is well-known for turning in suitable aerodynamic performance, and a fantastic lift-to-drag (L/D) ratio is one of the most essential right aerodynamic behaviors. The fundamental principle behind waverider design is that the body looks to be moving along with a shock wave attached to it and that the

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excessive stress at the back of the shock wave does now not leak via the main aspect to the pinnacle floor [2].

As can be seen from the brief introduction: designing a waverider with satisfactory performance, such as a conical waverider or osculating waverider in a hypersonic vehicle, is very important in aerospace engineering [3-5]. Waverider design is a comprehensive problem involving shock wave solutions, streamline tracing [6], and the generation of stream surfaces and these are all important concepts in aerodynamics education and are often encountered by undergraduates in the area of aerospace engineering, the place they research the fundamental concept and methodology for the layout of waveriders [7-9]. So, it's very important to find a vivid and convenient method for undergraduates to know how to establish simple waverider models and understand the basic principles of waverider, which can further stimulate students' interest in exploring aerodynamics.

Generally, there are three types of waveriders, namely caret-wing waveriders, conical waveriders, and osculating waveriders, which are of splendid hobby for high-speed flight automobile sketch [10, 11]. The waverider is often characterized in an inviscid flow and stays tethered to the body's 3-D shock wave. A high L/D ratio for waverider vehicles is achieved by keeping the high-pressure flow that is behind the shock wave from leaking from the lower surface to the upper surface.

In order to avoid the abrupt corners produced by the planar wedge flow, Rasmussen *et al*. [12] devised the conical waverider, which was formed from a circular cone flowfield. Similar waveriders have been produced from a more generic conical flow-field by many researchers. Conical waveriders have the inherent disadvantage that the conical flow-field or the producing cone must be preassigned, which typically restricts the selection of suitable input flow-fields even if the exact solutions for conical shocks can be calculated with ease. The conical waverider used to be proposed through Rasmussen *et al*. [12] and was once derived from a round cone flow-field in order to keep away from the sharp corners generated through the planar wedge waft. Numerous researchers have generated waveriders from a extra ordinary conical flow-field in a comparable manner. Although the genuine options for conical shocks can be effortlessly calculated, the conical waveriders have the inherent drawback that the conical flow-field or the producing cone ought to be preassigned, which often limits the preference of splendid inlet flow-fields.

To solve this problem, Sobieczky proposed a design methodology for osculating cone waveriders, which is an approximation method for describing a 3-D osculating waverider through a series of axisymmetric flows [13, 14]. Thus, a waverider could be generated based on arbitrary shock shapes by discretizing the 3-D flow into a number of 2-D strips. This virtually entails dealing with a sweep floor and assigning the preferred shock wave profile (SWP) at the base plane. The conical shock line is then taken as the part curve placed alongside the SWP to structure a sweep surface. To a positive extent, this makes the osculating cone waverider geometry extra bendy than that of the conical waverider.

Over the past many years, engine-airframe integration methodologies for the hypersonic waverider vehicle have been extensively studied by many researchers (Over the previous many years, engine-airframe integration methodologies for the hypersonic waverider automobile have been drastically studied by means of many researchers), and two design methods are formed in the process of hypersonic vehicle designing for a long time [15]. In 1990 this approach, which is based on strip theory, is a shock-based answer that at once determines the flow-field from a given shock wave and approves the direct decision of the inlet flow-field whilst supplying appropriate volumetric and aerodynamic overall performance [16, 17]. The method used to reduce 2-D design problems from 3-D inverse waverider designs is basically an approximation method. The osculating cone approach created by Sobieczky *et al.* [18] is expanded upon by the osculating axisymmetric method. The basic flow-field in each osculating plane is no longer restricted to solely conical flow in this method. Instead, it is a design-compliant axisymmetric flow-field [19-21]. The osculating flow-field approach is an improvement over the osculating cone and osculating axisymmetric methods [22, 23], as it does not require the identical axisymmetric flow-field to be implemented in each osculating plane. Various axisymmetric flow-fields can be selected based on the design specifications. Several innovative procedures with varying shock angles based on osculating techniques have recently been developed by several researchers to boost the design freedom [24, 25]. Furthermore, some When the waverider served as the vehicle's forebody, some researchers looked at forebody-inlet integration; when the waverider served as the vehicle's complete design inspiration, other researchers looked at engine-airframe integration [26-28]. All these studies are only devoted to achieving scientific and technological breakthroughs and verifying existing theories, there is still a lack of research on waverider design teaching methods for undergraduates. So further research in this direction is urgently needed.

The present paper describes an easy-to-use waverider parameterization design method that prescribes the relationship among various parameters of the osculating cone waveriders first proposed by Sobieczky *et al*. [18]. A plan device has been programmed with Python and is supposed for undergraduates in the issue of aerospace engineering. This layout device can generate the favored waverider geometry besides iterative time- and power-consuming simulations, and affords an intuitive effect of waverider era for undergraduate college students in the subject of aerospace engineering. Moreover, this approach provides a convenient way for undergraduates to draw close the standards of waverider deign whilst permitting adequate freedom to format their very own waverider geometries via prescribing an SWP and waft seize curve. The parameterization methodology of the plan process will be temporarily delivered in next section, accompanied through the fundamental coding hierarchy. Validation instances for number waverider geometries are supplied in area three. Finally, the conclusions to this find out about are introduced in area four.

Design methodology

Principle of waveriders

As seen in fig. 1, the caret-wing waverider's lower surface is produced by a planar wedge flow-field. On the planar oblique shock wave, the leading edge is specified as a caret curve, and the lower surface is obtained by tracing the stream surface via the leading edge. In fig.2, the flow capture curve (FCC), which corresponds to the flow capture tube (FCT), demonstrates the conical waverider design approach in general. The FCC for which the intersection with the conical shock surface forms the leading edge of the conical waverider may be arbitrarily defined if we assume that there is a conical flow-field. The leading edge can once more be followed through the conical flow-field to obtain the lower surface.

Figure 3 illustrates the primary steps of the osculating cone waverider design method. The shock surface and the final streamwise station intersect at the SWP at the base plane, which is the initial phase in the process. Due to its possible utility in inlet lip modeling, the SWP is also known as the inlet capture curve. Osculating planes are put up for the second stage in accordance with the SWP curvature and the freestream flow direction. The place of the nearby cone vertex can be recognized from the shock wave's curvature, presuming that every discrete factor alongside the SWP represents a place of conical flow. By linking each conical waft in the osculating planes, the full flow-field may additionally then be recreated Eventually, the decrease floor of the osculating cone waverider is derived through integrating streamlines from the intersection of the shock floor and the FCT.

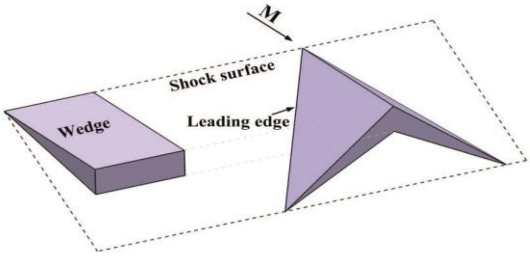

Figure 1. Schematic of the caret-wing waverider

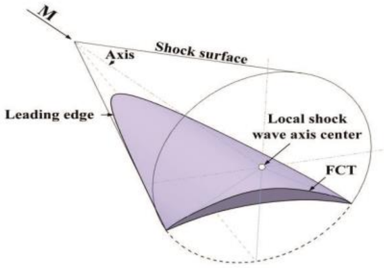

Figure 2. Schematic of the conical waverider

Parameterization method and coding with Python

The waverider parameterization design procedure is programmed with Python in the present work. The parameterization procedure is therefore accomplished by specifying the shock wave end point, E_s, shock wave end angle, A_s°, FCC end point, E_f, and FCC end angle, A_f°, to determine the SWP and FCC at the base plane and then, according to the freestream Mach number and initial shock angle, calculating the conical basic flow-field. Finally, the osculating waverider geometry is derived using the streamline tracing method, as shown in the flowchart in fig. 4.

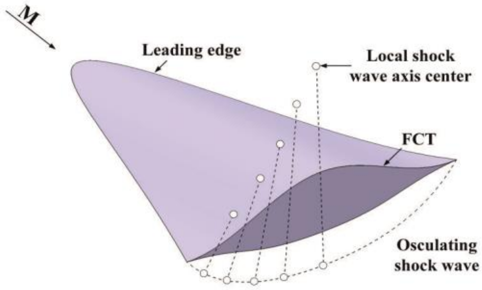

Figure 3. Schematic of the osculating cone waverider

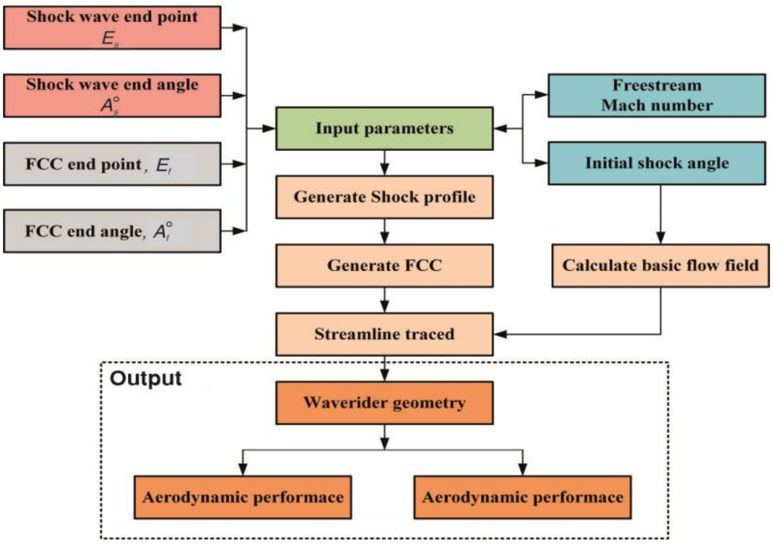

Figure 4. Flowchart of the Python program

The whole Python application can be cut up into 4 dominant steps.

Step 1. Specify the SWP at the base plane

The SWP at the base plane is illustrated by the red solid curve of fig. 5 and is described by a cubic curve equation of the form:

$$f(y, z) = \left[y_S, y_E, z_S, z_E, \tan\left(A_s^\circ\right), \tan\left(0^\circ\right)\right] \tag{1}$$

The start points S_s, end point E_s, and end angle A_s of the SWP are used as the input variables in this equation along with the characteristics of the two feature points. The first-order by-product of the quit point, $\tan(A_s^\circ)$, is used to manage the slope of the SWP at the stop point, and the first-order by-product of the begin point, $\tan(0^\circ)$, is used to manipulate the slope of the SWP at the begin point. Together, *y* and *z* characterize the horizontal and vertical co-ordinates of the function points, respectively. The first-order spinoff of the begin factor is zero in view that the SWP is a curve with symmetry properties. The parameters of the function factors outline the spanwise and vertical co-ordinates of the SWP at the base plane.

Step 2. Specify the FCC at the base plane

The FCC at the base plane is demonstrated by the black solid curve of fig. 5, and is also described by a cubic curve equation. However, the two feature points are replaced by points S_f and E_f of the FCC. Note that the end point of the FCC (E_f) is attached to the end point of the SWP (E_s) to maintain the waverider characteristics of the generated geometry. The start point of the FCC (S_f) can only be changed in the z direction and must be located between O and E_s: if the z-co-ordinate of S_f is greater than the z-coordinate of the local shock wave axis center, O, the waverider geometry cannot be generated. As a result, the z-co-ordinate of S_f is defined as follows, where the coefficient C ranges from 0.2-0.9:

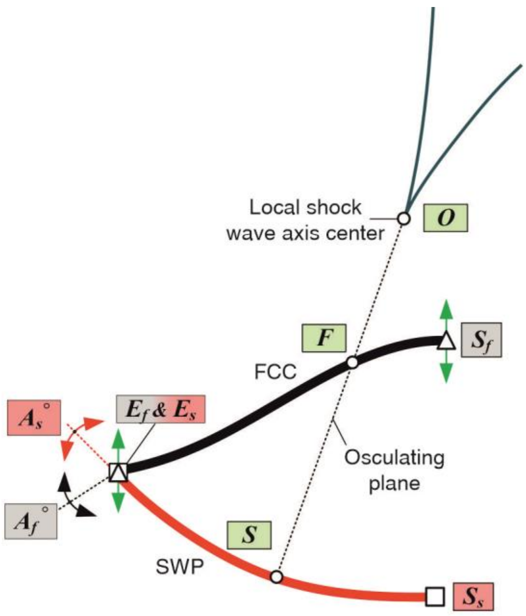

Figure 5. Design principle for osculating cone waverider

$$z_{S_f} = z_{E_s} + C\left(z_{O_s} - z_{E_s}\right)$$

Step 3. Calculate the conical basic flow-field and generate the osculating cone waverider

The conical simple flow-field is an axisymmetric float round a round cone at zero incidence, fig. 6. The flow-field can be calculated by solving the Taylor-Maccoll equation with the given freestream Mach number and initial shock angle $\beta°$. As shown in fig. 5, for any FCC point *U*, the points *O* and *S* are the local shock wave axis center and the SWP, respectively. Based on the relationship among the three curves, the compression surface of the waverider can be generated by the streamline tracing method in the basic flow-field, as illustrated in fig. 6. The streamline between the FCC point *U* and the compression surface point *L* is the profile of the osculating cone waverider in the local osculating plane. These steps are repeated for each osculating plane to obtain all the streamlines. Finally, the streamlines form the compression surface of the osculating cone waverider.

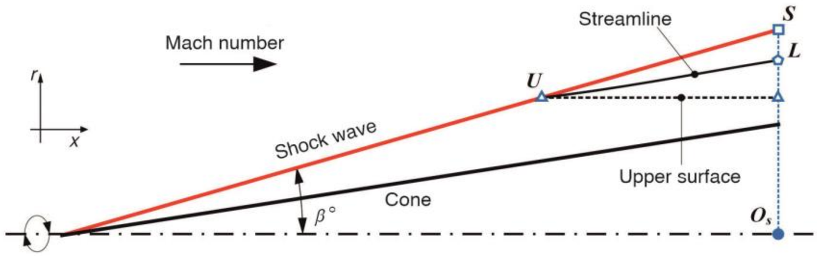

Figure 6. Schematic of the conical basic flowfield

Validation cases

The waverider parameterization design procedure is evaluated and validated by comparing the prescribed input parameters to the L/D ratio and volume factor (VF) of osculating cone waveriders given by CFD results. The CFD calculations used the industrial software program ANSYS ICEM for computational grid technology and ANSYS FLUENT for simulation.

Waverider geometries

Three osculating cone waverider cases with different input parameters are considered, tab. 1. In each case, four parameters are modified:
- the end angle of the SWP A_s°,
- the end angle of the FCC A_f°,
- the *z*-coordinate of the SWP and FCC end point $Z_{E_s\&E_f}$, and
- the coefficient C.

The differences among the three cases, influenced by the different input parameters, are illustrated in fig. 7 in terms of the surface shape of the three-osculating cone waveriders. Within the scaled unit length in the crosswise direction, 100 points on the FCC are used to discretize and reconstruct the compressed surfaces of the waveriders, as shown in fig. 8. These surfaces are then used to construct waveriders for structured grid generation and inviscid steady CFD simulations. In addition, L/D and VF of each waverider case are obtained by the parameterization method. As illustrated in fig. 8, the L/D values of waverider Cases 1-9 are 6.03, 6.00, 5.95, 5.93, 5.90, 6.01, 6.06, 6.20, and 6.28, respectively. The VF values of waverider Cases 1-9 is 2.45, 2.48, 2.53, 2.75, 2.96, 2.64, 2.10, 2.24, and 3.09, respectively. The aerodynamic performance of the waverider cases output by the parameterization method is useful in enabling undergraduates to understand the principles of supersonic flow.

Table 1. Input parameters of three osculating cone waverider cases

Parameter	A_s°	A_f°	$Z_{E_s\&E_f}$	C
Case 1	–45	5	6	0.5
Case 2	–45	15	6	0.5
Case 3	–45	30	6	0.5
Case 4	–45	30	6	0.6
Case 3	–45	30	6	0.7
Case 6	–40	5	6	0.5
Case 7	–35	5	6	0.5
Case 8	–30	5	5	0.5
Case 9	–30	5	4	0.5

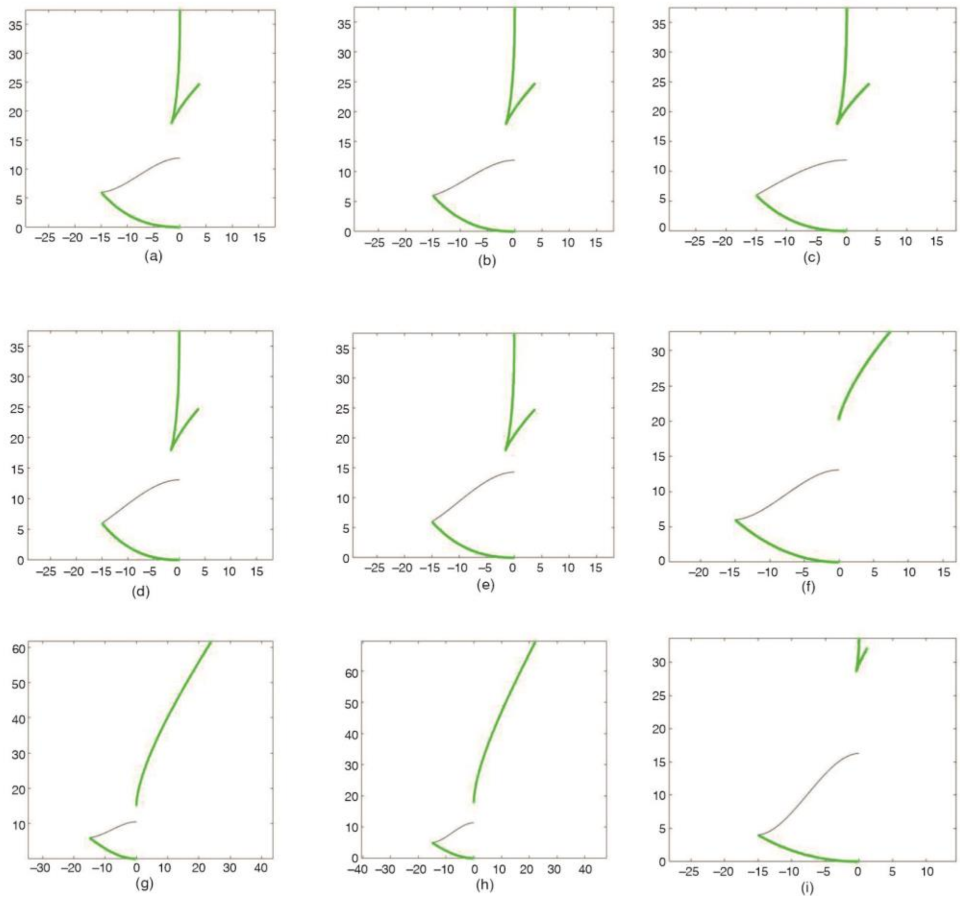

Figure 7. Osculating cone waverider cases with different input parameters; Case 1 (a)-Case 9 (i)

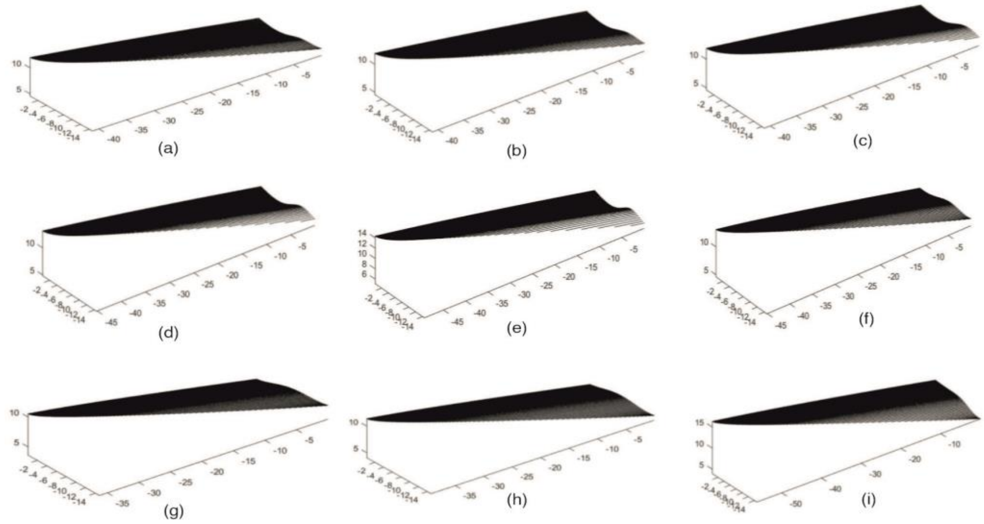

Figure 8. Compressed surface of three osculating cone waverider cases; Case 1 (a)-Case 9 (i)

Waverider model

In this section, we clear up a sensible case via the use of the proposed integration method. We chose three instances out of the nine instances and three geometric fashions of waverider are created, fig. 9, based totally on the parameterization diagram manner illustrated in fig. 9. The design point is H = 26 km, M_0 = 6.0.

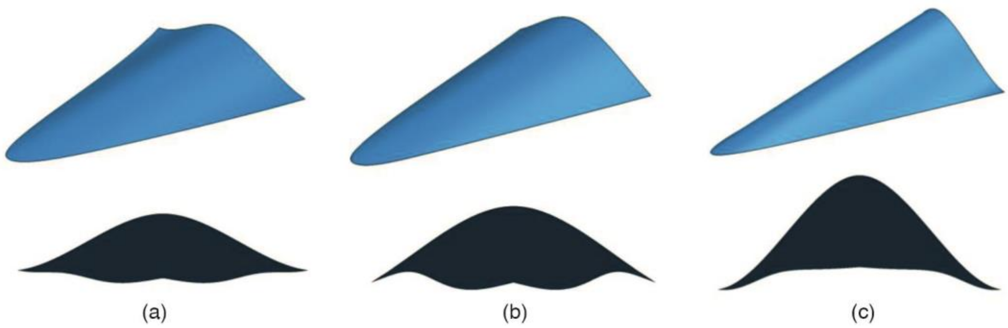

Figure 9. Geometric model of the waverider cases; (a) Case 1, (b) Case 5, and (c) Case 9

To validate the impact of the stress, manipulate method, inviscid glide fields of the waverider instances had been calculated. Finite-volume discretization was once used to resolve glide equations, Roe's scheme primarily based on the flux-difference splitting scheme interpolation was once adopted to discretize inviscid fluxes vector. The iterative technique and its options can be viewed to converge if the residuals attain minimal values, that is, values much less than four orders of magnitude of their unique value. To decorate the effectively of numerical simulation, half mannequin illustrated in fig. 10 used to be chosen for mesh generation. The whole mesh quantity is three million.

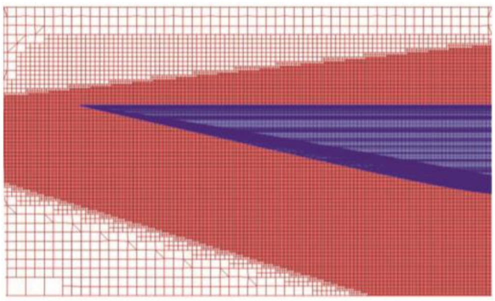

Figure 10. Schematic format of grid employed in computational take a look at of waverider

Comparison with CFD results

This section compares the CFD results with those obtained using the parameterization method. The inviscid static pressure ratios at the design plane of the three waverider cases under the input incoming flow condition are shown in figs. 11(a)-11(c). The red line and red dashed line denote the preassigned SWP. Based on the osculating approach, the flow-field is discretized into osculating planes with the same shock angles for a shock wave provided by eq. (1). The Taylor-Maccoll equation of conical flows, which is incorporated into the parameterization approach as explained in section *Parameterization method and coding with Python*, is used to derive the local streamline as well as the 2-D flow behind the shock in each osculating plane. The 3-D shock surface's generating geometry is established azimuthally by the integration of the streamlines, as seen in fig. 5. These theoretical results are shown on the left of figs. 11(a)-11(c). Starting from the derived generating geometries, three inviscid CFD calculations give the numerical flow-field, as shown on the right of fig. 11. In principle, at the base plane, there is no significant deviation between the theoretical and CFD results. The SWP of the three waverider cases agree well with the parameterization results. Furthermore, there is nearly no high pressure air leaking from the decrease floor to the top surface. This shows the three waverider instances with distinctive enter parameters can trip on the preassigned SWP below the particular flight conditions, and the preassigned shock wave stays connected to the main part for the complete size of the waverider. Therefore, the parameterization approach proposed in this paper is a sensible way of fixing the inverse waverider problem.

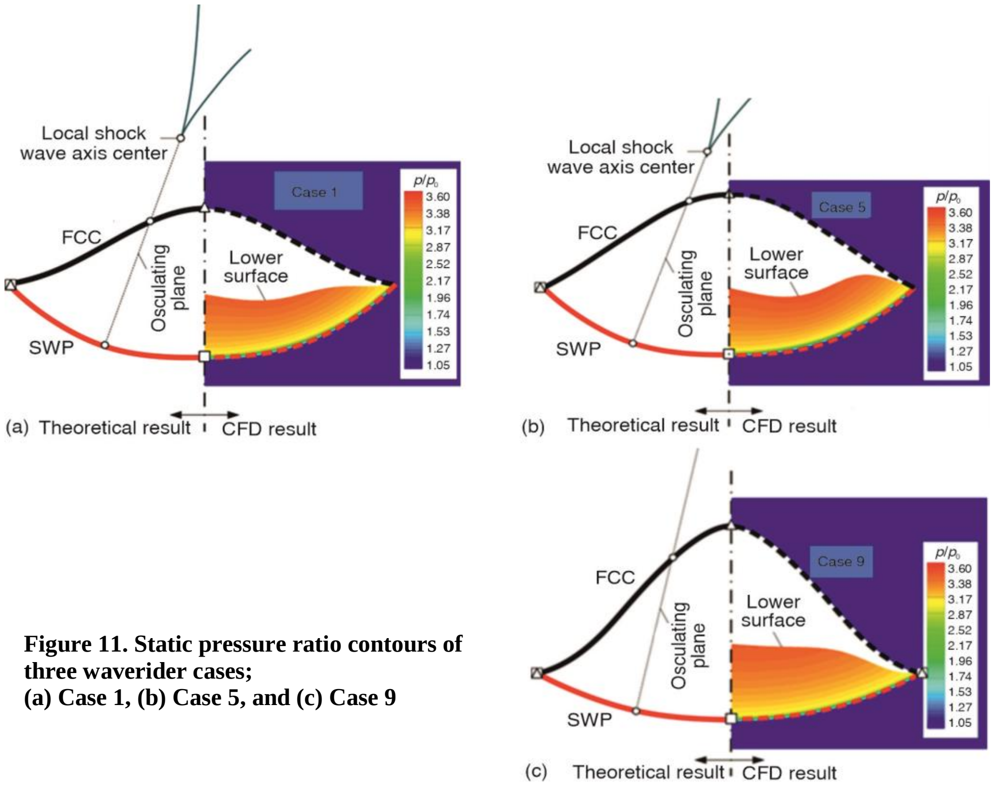

Figure 11. Static pressure ratio contours of three waverider cases; (a) Case 1, (b) Case 5, and (c) Case 9

The detailed aerodynamic performance of the three waverider cases, especially L/D, is important in verifying the accuracy of the parameterization method. Table 2 presents the L/D values of the three waveriders obtained by the parameterization method and the numerical results. In all three cases, the deviation between the L/D results obtained by CFD and the parameterization method is less than 0.5%. This verifies that the accuracy of the parameterization method is maintained at a high level and is not affected by the input parameters.

Table 2. The L/D comparison of three osculating cone waverider cases

Type	L/D (theoretical)	L/D (CFD)	Deviation
Case 1	6.0348	6.0112	0.39%
Case 2	5.9045	5.8977	0.11%
Case 3	6.2839	6.2549	0.46%

Future work

In the present work, a parameterization method has been developed for the generation of hypersonic vehicles in aerodynamics education using the theory of osculating waveriders. However, the assumption of this one kind of aircraft is as an alternative strict for sensible applications. Therefore, in future work, emphasis will be positioned on the improvement of an aerodynamic graph device for all types of waverider generation, protecting caret-wing, conical-cone. Besides, one of the most interest topic in the aerospace technologies is the design of hypersonic flight vehicles. The critical point to realize hypersonic flight is the effectively integrated concept for the airframe and propulsion, especially integrating the hypersonic inlet into the forebody of the airframe. In the external flow of hypersonic vehicle, the waverider had been investigated extensively. It could be divided into three types include the caret waverider, conical waverider and osculating waverider. The caret waverider is first proposed by non-weiler, which is derived from an inviscid 2-D wedge flow with planar initial shock wave. Based on this concept, Ferguson proposed a method to integrate the air breathing propulsion system and airframe. The conical waverider is proposed by Rasmussen which is used the axisymmetric conical flow as the basic flow field to derive waverider. As the basic flow field change from plane to axial symmetry, the L/D ratio and volume are increased. Since then, numerous researchers investigated the integrated method based on the conical waverider. Nevertheless, the conical waveriders possess an essential drawback: the initial shock wave or the shock wave generator must be preassigned, which limits the choice of appropriate airframe. To solve this problem, Sobieczky *et al*. [18] proposed osculating waverider concept, which is an approximation method to describe a 3-D osculating waverder designs by a series of axisymmetric flow. Thus, the waverider could be generated based on arbitrary shock shapes by discretizing the 3-D flow into a number of 2-D strips. Using this method, Takashima proposed an integrated method to couple the waverider and 2-D hypersonic inlet.

As to the air breathing propulsion system, the inlet is the core element to influence the impulse. Historically, numerous researches have been conducted on 2-D hypersonic inlets. Due to simple structure and geometry, the integrated of 2-D inlets with hypersonic airframe have been extensive investigated. However, another promising inlet with a higher level of compression efficiency has been introduced to the air breathing propulsion community, namely, 3-D inward-turning inlet. A typical inward-turning inlet is first applied by Molder and named Busemann inlet. It possesses the best isentropic compression efficiency. However, the compressed path is too long, which makes the large viscosity loss and influences the start ability of inlet. To overcome this drawback, the truncated Busemann inlet is introduced. You *et al* proposed the design concept of internal waverider inlet, which is the application of waverider concept in the internal flow. This inlet has the practicality and efficiency in total pressure recovery, external drag and integrated with external waverider configuration. Owing to the outstanding aerodynamic performance of inward turning inlet and waverider, the integrated methods of the two components had been conducted in recent years.

Conclusion

This paper has presented a parameterization method for waverider generation intended for undergraduate students in the field of aerodynamics. Students can prescribe the SWP and FCC as the input parameters, and then derive their waverider geometries through the Python code based on the Taylor-Maccoll equation and streamline tracing method. The basic methodology of the tool and the code has been explained in detail. Three validation waverider cases were considered to illustrate the capability and accuracy of the parameterization method for different input parameters. At a moderate inflow of Mach 6, the SWP given by the theoretical result was found to be in very good agreement with the CFD result. The L/D ratio of the two results had a maximum deviation of only 0.46%, which demonstrates the accuracy of the parameterization method used in waverider design. In general, this plan device affords an intuitive affect of waverider era for undergraduate students in the area of aerodynamics. It additionally gives a clear photograph of the underlying concepts of supersonic glide and shock wave theory. Furthermore, the device gives college students with adequate freedom to graph their personal waverider geometry by means of definitely prescribing the SWP and FCC. Future work will focus on a parameterization method covering all kinds of waveriders.

Acknowledgement

The authors would like to acknowledge the support of the Science and Technology Innovation Committee Foundation of Shenzhen, Grant No. JCYJ20200109141403840.

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Paper submitted: March 11, 2022
Paper revised: January 12, 2023
Paper accepted: January 13, 2023