USING A DOUBLE GRID FOR THE NUMERICAL STUDY OF THE EFFECT OF A VERTICAL WALL'S GROOVES IN A DIFFERENTIALLY HEATED SQUARE CAVITY ON THE NATURAL CONVECTION OF A CARBON-BASED NANOFLUID

Zakaria LAFDAILI [*1], *Hamid MOUNIR* [1], *Soufiane NOUARI* [2], *Sakina EL-HAMDANI* [2], *Lahoucine BELARCHE* [3]

[1]EMISys Research Team, E3S Research Center, Mohammadia School of Engineers, Mohammed V University in Rabat, Morocco

[2]Laboratory Mechanics, Processes, Energy and Environment (LMPEE)

[3]Laboratory of Energy, Materials and Systems Engineering (LGEMS)

National School of Applied Sciences, University Ibn Zohr, Agadir, Morocco

[*]Corresponding author; E-mail: lafdaili@emi.ac.ma

In this work we study numerically the natural convection of a carbon-based nanofluid (water + C_{60}) in a differentially heated square cavity. One of the vertical walls of the cavity is grooved and maintained at a high temperature. The purpose of this work is to compare the effect of the macrostructural aspect of the grooves and the effect of the addition of fullerene nanoparticles (C_{60}) to pure water on the heat exchange by natural convection in this type of geometry. To better quantify the convective heat exchange numerically near the grooves we used a refined grid with two non-conforming blocks at the hot wall level. The governing equations were discretized by the finite volume method using a power law scheme which offers a good stability characteristic in this type of flow. A numerical code was designed and produced in this context to use numerical simulation as an investigative tool. The results are represented in the form of streamlines and isothermal fields. The variation in the mean Nusselt number of the cold wall to the right of the cavity is calculated as a function of the volume fractions of the nanoparticles ($0 \leq \Phi \leq 0.06$) for different numbers and sizes of the grooves and for different Rayleigh numbers ($10^3 \leq Ra \leq 10^6$).

Keywords: *Numerical study, natural convection, double refined grid, grooves, nanofluid.*

1. Introduction

The intensification of heat exchange by convection is based on several techniques, among which are those focusing on the increase of the total exchange surface by modifying its macrostructural aspect and those aiming to improve the thermophysical properties of the thermal fluids. This study is part of this framework.

The thermophysical properties of the heat transfer fluids used often limit the efficiency of convective heat exchange. In 1873, Maxwell proposed a theoretical model which introduces the metallic particles into heat transfer fluids to improve their thermal conductivities. But the size of these particles, being the order of a micron, posed a problem. They are too large to float for a long time in the pipes, and if we increased the speed of circulation of the fluid to keep them afloat, they would then damage the walls. Advances in nanotechnology have made it possible to suspend nanoscale particles in a base liquid, which has greatly improved its thermal properties and allowed for more intense heat transfer. The exceptional thermal properties of nanofluids, especially carbon-based ones, have been in fact the subject of much investigation over the past two decades [1-8]. In particular, there is a clear increase in heat exchange that no study has yet been able to explain satisfactorily [9]. Carbon-based nanofluids have become a promising technology in the context of heat transfers.

The macrostructural aspect of the enclosures considerably influences the heat transfer by natural convection. This was a reason for a significant amount of both theoretical and experimental works that have studied natural convection in enclosures for different geometric configurations [10-14]. The problem of natural convection in differentially heated cavities is the subject of growing interest, owing to the multiplicity of technological applications that refer to it (solar collectors, building heaters, cooling of electronic circuits and nuclear reactors, …). Also, due to vast amount of available experimental data, it constitutes an ideal case for the validation of numerical codes.

Methods for reliable numerical modeling of a physical problem are based on the faithful representation of the mesh of the computational domain. Industrial geometries are often complex and difficult to mesh, especially with quadrilaterals (in 2D) for orthogonal structured meshes. It is then necessary to go through topological divisions in simple geometries. This is called multiblock grid [15-19].

In this work, we numerically modeled the natural convection of a C_{60} fullerene-based nanofluid in a differentially heated square cavity with a grooved left wall. This can provide a contribution in the field of the design of heat exchangers, in particular, plate heat exchangers which exhibit very good thermal performance for a small footprint.

Thus, to better quantify the convective flow near the grooves and to predict the effect of the volume fraction of the nanoparticles, the Rayleigh number, the number and dimensions of the grooves, we adopted a mesh with two non-conforming blocks refined in wall level grooved. The mesh obtained is then of better quality. However, the blocks are independent while the equations are solved over the entire domain. It is therefore necessary to connect the solutions between the two blocks. A method which is based on an interpolation of the variables located on the interfaces is then implemented.

2. Configuration studied and equations

The configuration studied is shown in Fig. 1, and it consists of a square cavity, filled with water containing different concentrations of the nanoparticles based on fullerene (C_{60}). The cavity is formed by a grooved left vertical wall maintained at a high temperature T_h, a right vertical wall maintained at a low temperature T_c and two horizontal walls considered adiabatic.

The viscous dissipation is negligible, the thermophysical properties of nano-fluids are constant, except for the variation in density which is estimated by the Boussinesq approximation. The thermophysical properties of the pure fluid and of the nanoparticles studied are presented in Tab. 1.

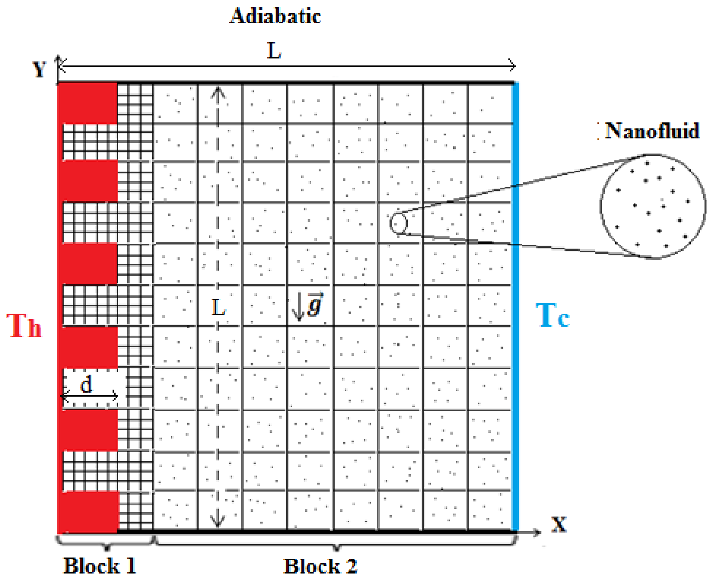

Fig 1. **Geometry, grid and boundary conditions of the studied configuration**

Tab. 1 Thermophysical properties of water and nanoparticles [2]

	Pr	ρ (kg/m^3)	Cp (J/kgK)	k (W/mK)	β (10^{-5}K^{-1})
Pure water	6,2	997,1	4179	0,613	21
C_{60}	–	3500	509	2300	1,72

The dimensionless equations written in the Cartesian coordinate system (X, Y, Z) are as follows:

Equation of continuity:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

Momentum equation in the X direction:

$$U.\frac{\partial U}{\partial X}+V.\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{\mu_{nf}}{\rho_{nf}\alpha_f}.\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

Momentum equation in the Y direction:

$$U.\frac{\partial V}{\partial X}+V.\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{\mu_{nf}}{\rho_{nf}\alpha_f}.\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}.Ra.Pr.\theta \tag{3}$$

Thermal energy equation:

$$U.\frac{\partial \theta}{\partial X}+V.\frac{\partial \theta}{\partial Y}=\frac{\alpha_{nf}}{\alpha_f}.\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

Tab. 2 Dimensionless variables and reference quantities

	Dimensionless quantities	reference quantities
Spatial quantities	$X, \quad Y$	$\frac{x}{L}, \quad \frac{y}{L}$
Velocity	$U, \quad V$	$\frac{uL}{\alpha_f}, \quad \frac{vL}{\alpha_f}$
Temperature	θ	$\frac{(T - T_c)}{(T_h - T_c)}$
Pressure	P	$\frac{p}{\rho_{nf}{\alpha_f}^2}$

with:

ρ_{nf}, density of the nano-fluid defined by:

$$\rho_{nf} = (1 - Q).\rho_f + Q.\rho_P \tag{5}$$

α_{nf}, thermal diffusivity of the nano-fluid defined by:

$$\alpha_{nf} = \frac{K_{nf}}{(\rho C_p)_{nf}} \tag{6}$$

K_{nf}, the effective thermal conductivity of the nano-fluid defined by de Maxwell –Garnetts model [20] :

$$K_{nf} = K_f.\left[\frac{(K_p + 2K_f) - 2Q.(K_f - K_p)}{(K_p + 2K_f) + Q.(K_f - K_p)}\right] \tag{7}$$

μ_{nf} , the dynamic viscosity of the nano-fluid defined by the Brinkman model [21]:

$$\mu_{nf} = \frac{\mu_f}{(1-Q)^{2.5}} \tag{8}$$

The mean Nusselt number Nu_{av} of the hot wall is expressed as follows:

$$Nu_{av} = \frac{1}{L}\int_0^L Nu_{loc} dY \tag{9}$$

where, the local Nusselt numbers are defined as follows:

$$Nu_{loc} = -\frac{K_f}{K_{nf}}.\frac{\partial\theta}{\partial X}\Big]_{X=0} \tag{10}$$

3. Numerical solution and code validation

The dimensionless equations (1-4) were discretized by the finite volume method by adopting a power law scheme and the SIMPLE algorithm for the pressure correction on a mesh with two non-conforming blocks Block 1 and Block 2 refined near the grooved wall (Fig. 2). The system of equations is solved iteratively by the line-by-line scanning method using Thomas's algorithm.

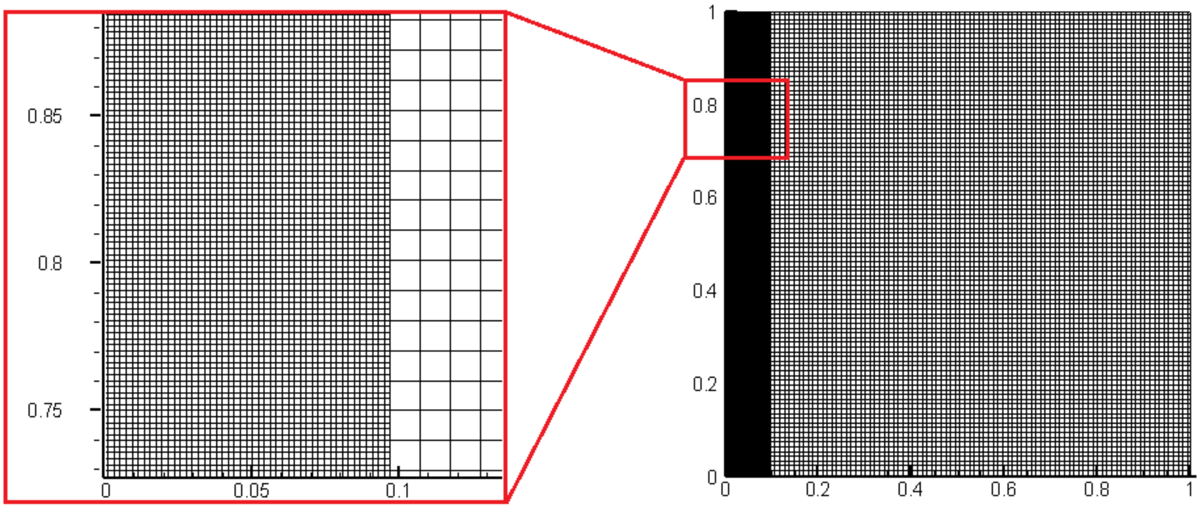

Fig. 2 Refined nonconforming two-block mesh near the left vertical wall

For the grid lines which are continuous at the crossing of a border between two blocks where the nodes of the respective interfaces coincide (Fig. 3), one can write a complete discretization. Thus, the connectivity of these nodes makes it possible to re-establish the structured character of the mesh and thus allow the transfer of information from one block to another.

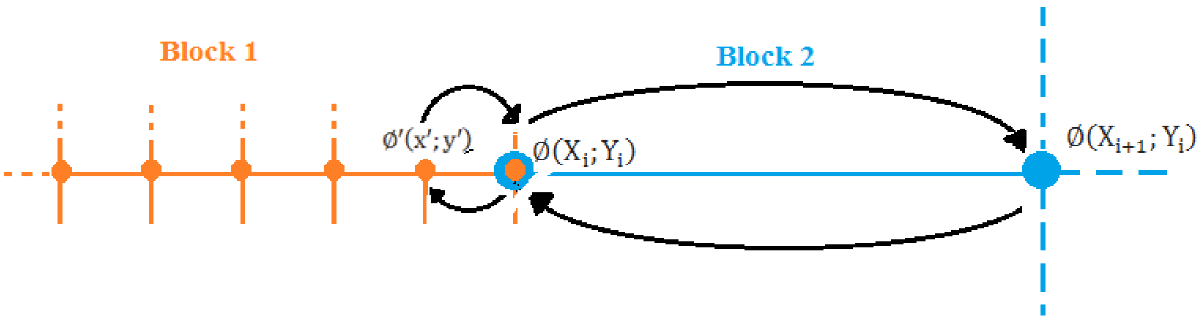

Fig. 3 Connection method for the nodes which coincide at the interface of the two blocks of grid

For the lines of block 1 which do not coincide with other lines of block 2, the proposed solution consists in interpolating the fields ∅ on the nodes located on the interfaces using the nodes of the adjacent block (Fig. 4). This interpolation can be considered as a new boundary condition serving for the discretization of the equations on the nodes located strictly inside the different blocks, Chr. Rome [22].

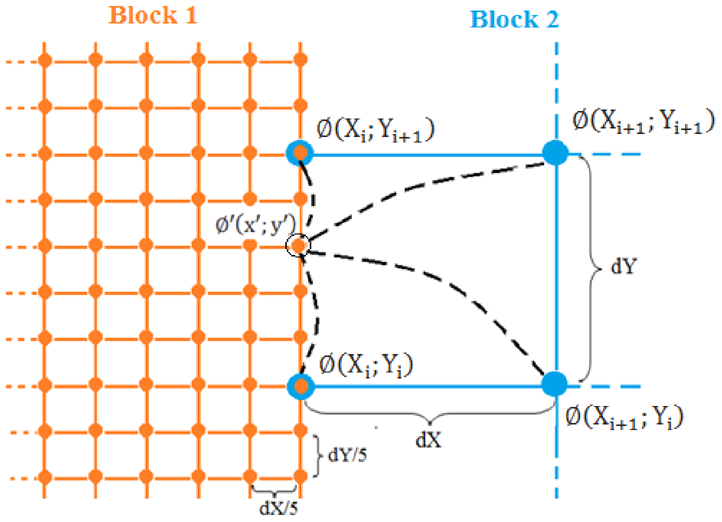

Fig. 4 Connection method for the nodes which do not coincide at the interface of the two blocks of the grid

We consider an interpolation based on polynomials ***P*** of degree 1 with two variables. The canonical basis of these polynomials is given by:

$$\{B_{1\le j\le 4}\} = \{1, x, y, xy\} \tag{11}$$

where :

$$\begin{cases} x = -\dfrac{dX}{2} \\ y = \dfrac{dY}{2} - (Y_{j+1} - Y') \end{cases} \tag{12}$$

The evaluation of a field Ø at point M (x '; y') belonging to a block 1 (Fig. 4), is then expressed using the values of Ø on block 2 by the relation:

$$\begin{aligned} \emptyset(X',Y') = {} & P_1(x,y)\times\emptyset(X_i,Y_j) + P_2(x,y)\times\emptyset(X_{i+1},Y_j) + P_3(x,y)\times\emptyset(X_i,Y_{j+1}) \\ & + P_4(x,y)\times\emptyset(X_{i+1},Y_{j+1}) \end{aligned} \tag{13}$$

where :

$$\begin{cases} P_1(x,y) = \dfrac{1}{4}(1-x-y+xy) \\ P_2(x,y) = \dfrac{1}{4}(1+x-y-xy) \\ P_3(x,y) = \dfrac{1}{4}(1+x+y+xy) \\ P_4(x,y) = \dfrac{1}{4}(1-x+y-xy) \end{cases} \tag{14}$$

To study the influence of the mesh, we calculated the mean Nusselt number for different grids and for different Rayleigh numbers. The results obtained for a square cavity with smooth walls filled with water for Ra= 10^5 are presented in Tab. 3:

Tab. 3 Validation of the grid for various Rayleigh numbers

Grid	Block 1 (50×300)	Bloc 2 (60×60)	Block 1 (50×400)	Bloc 2 (80×80)	Block 1 (50×500)	Block 2 (100×100)	Block 1 (50×600)	Block 2 (120×120)
Nu_{av}	4.608		4.630		4.643		4.647	
Deviation rate of Nu_{av}	-		0, 47%		0,28%		0,08%	

with: $Deviation\ rate\ of\ Nuav = \frac{(Nu_{av}2 - Nu_{av}1)}{Nu_{av}1}\%$

From this table, it appears that:

the grid $\begin{cases} \text{Bloc 1 } (50 \times 500) \\ \text{Bloc 2 } (100 \times 100) \end{cases}$ is sufficiently fine to perform numerical simulations in this type of geometry.

To validate the current numerical method, we compared the values of the mean Nusselt number calculated on the cold wall of a filled cavity (pure water (a); Cu-based nanofluid (b)) with those found by J. Ravnik [23]. The details of the calculation of the mean Nusselt number for values of the Rayleigh number ($10^3 \leq \text{Ra} \leq 10^6$) are presented in Tab. 4.

Tab. 4 Comparison of the values of the mean Nusselt number with the work of J. Ravnik [23]

Ra	Pure water (a) present		Pure water (a) J. Ravnik [23]	
10^3	1.073		1.071	
10^4	2.188		2.078	
10^5	4.576		4.510	
10^6	9.127		9.032	
Ra	water +Cu (b), Q = 0.01, present	water +Cu (b), Q = 0.01, J. Ravnik [23]	water +Cu (b), Q = 0.02, present	water +Cu (b), Q = 0.02, J. Ravnik [23]
10^3	1.341	1.363	1.711	1.758
10^4	2.397	2.237	2.542	2.381
10^5	5.101	4.946	5.503	5.278
10^6	10 .265	10.08	11.207	10.98

It is clear that the results of our code are in good agreement with those proposed by J. Ravnik [23].

4. Results and discussions

Fig. 5 (a, b, c and d) shows the stream lines and the isothermal temperature fields in the nanofluid (Water + 6%C_{60}, Ra = 10^6) inside the square cavities heated by a vertical wall containing grooves whose depth represents 5% of the size of the cavity. The thermal fields are marked by horizontal stratifications inside the cavity and by strong thermal gradients on the active walls, which means that the heat transfer takes place largely by convection. Inside the grooves there are vortices that intensify the heat transfer between the grooved wall and the nanofluid. These grooves also imply an effective increase in the exchange surface of the hot vertical wall, which can lead to an intensification of convective heat exchange.Fig. 5-d which represents the case of a cavity containing 24 grooves shows that the convective flow inside the grooves is weak. This is due to the fine size of the grooves which slow down the flow of the nanofluid which can lead to a decrease in the convective heat transfer.

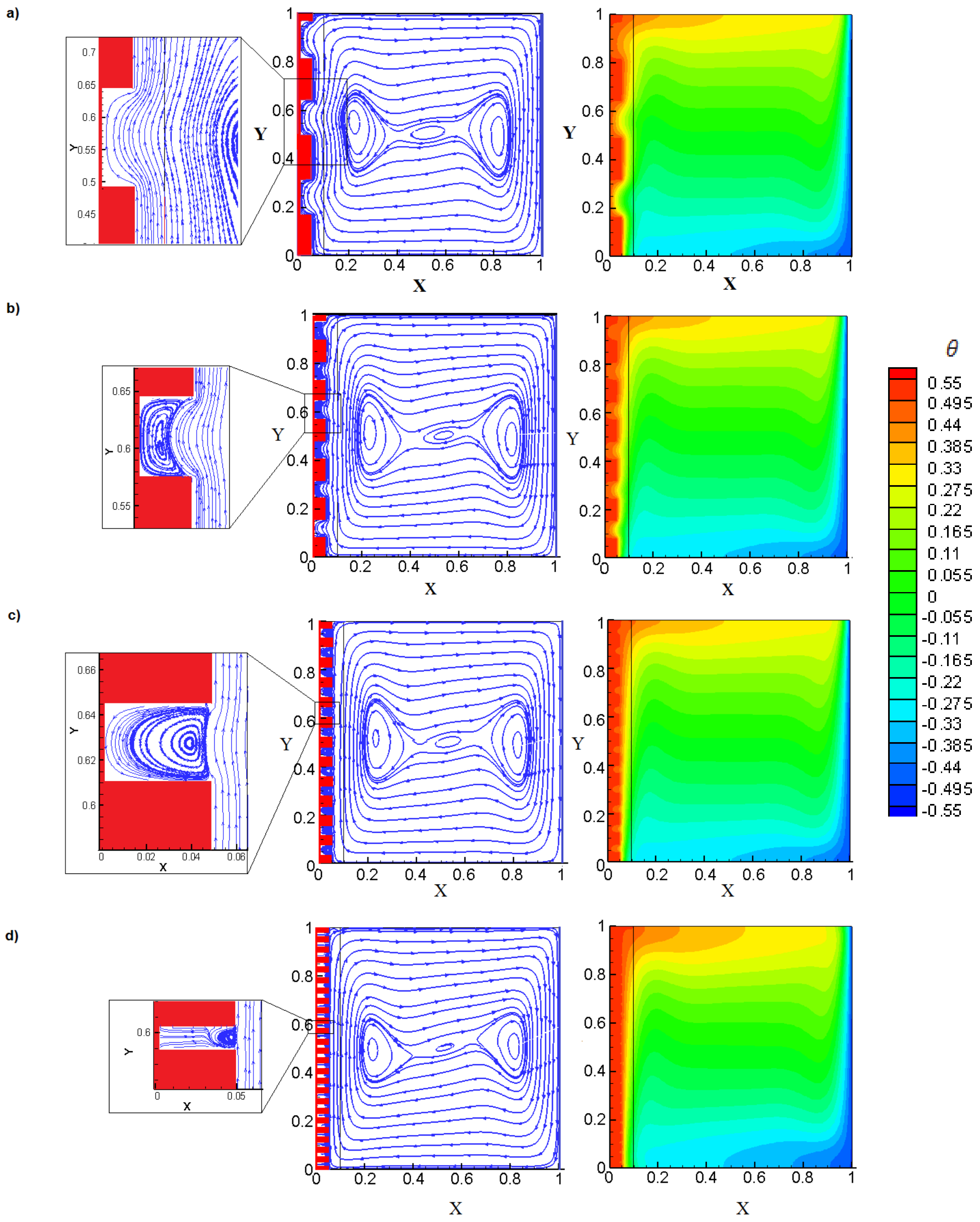

Fig. 5 Streamlines and isothermal contours of nanofluid (Water + 6% C_{60}, Ra = 10^6) in cavities with 3 grooves (a), 6 grooves (b), 12 grooves (c) and 24 grooves (d)

In fig. 6, 7, 8 and 9 we present the variations of the mean Nusselt number for different number of grooves on the hot vertical wall, for the two depths of the grooves (d = 5% L, d = 10% L) successively for the Rayleigh numbers (10^3, 10^4, 10^5 et 10^6).

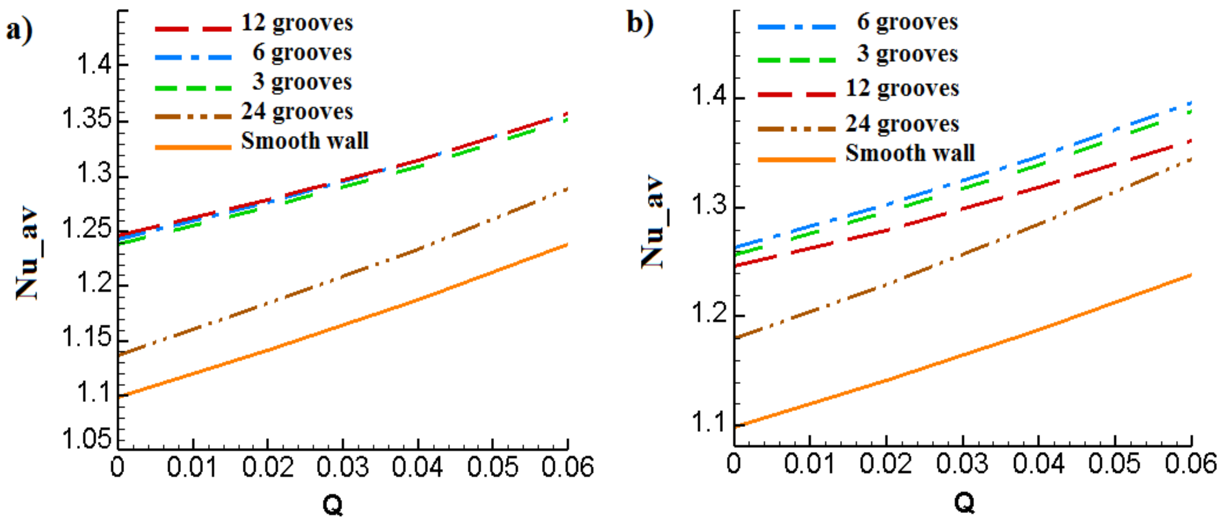

Fig. 6 Variation of the mean Nusselt number as a function of the volume fractions of the nanoparticles for Ra = 10^3 in cavities containing grooves of depth d = 5% L (a), d = 10% L (b)

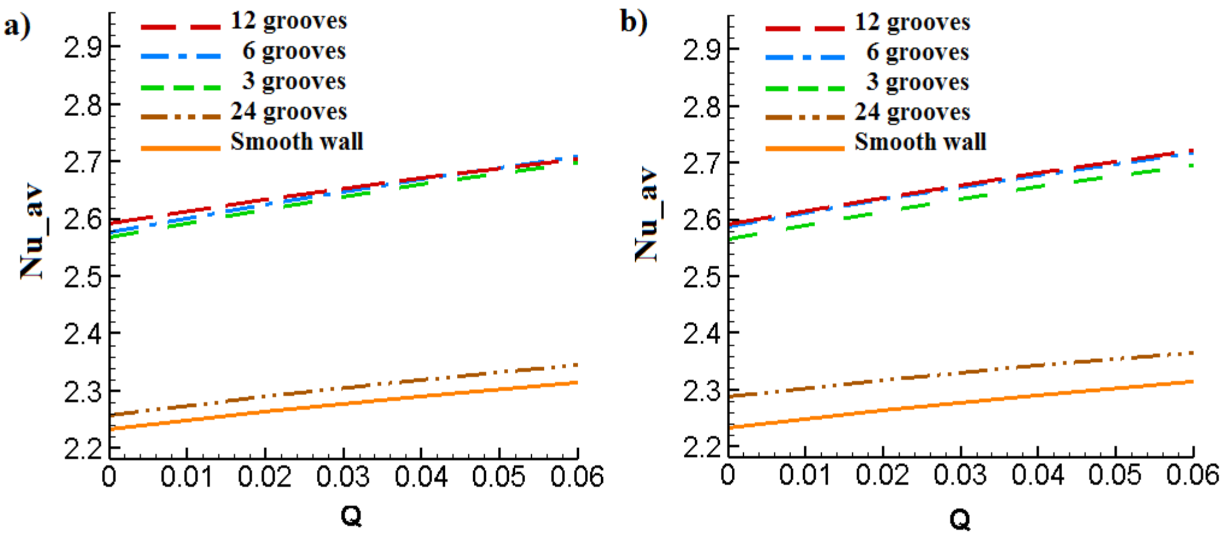

Fig. 7 Variation of the mean Nusselt number as a function of the volume fractions of the nanoparticles for Ra = 10^4 in cavities containing grooves of depth d = 5% L (a), d = 10% L (b)

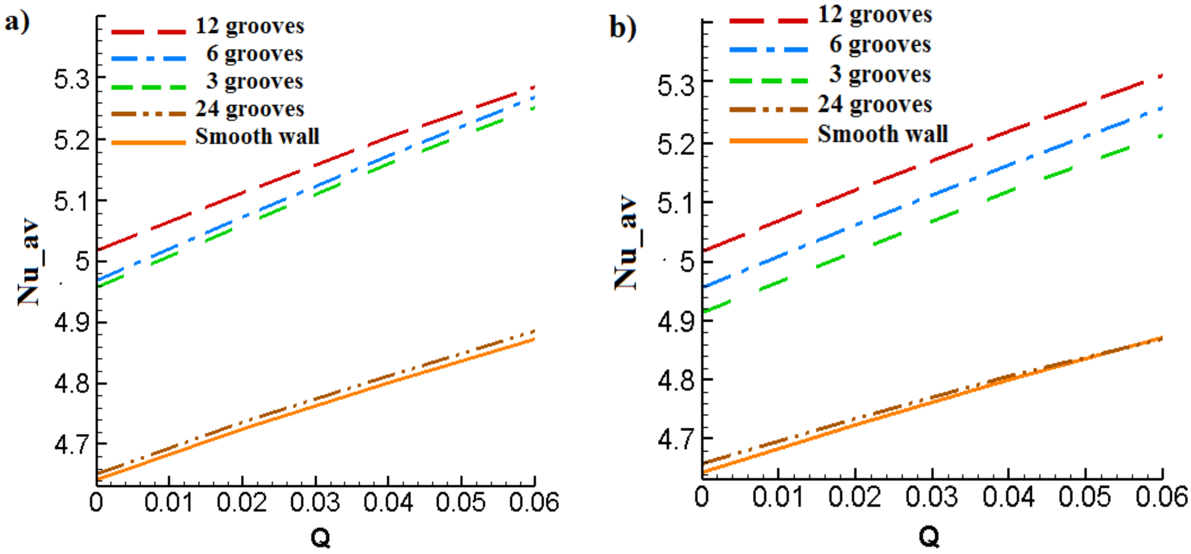

Fig. 8 Variation of the mean Nusselt number as a function of the volume fractions of the nanoparticles for Ra = 10^5 in cavities containing grooves of depth d = 5% L (a), d = 10% L (b)

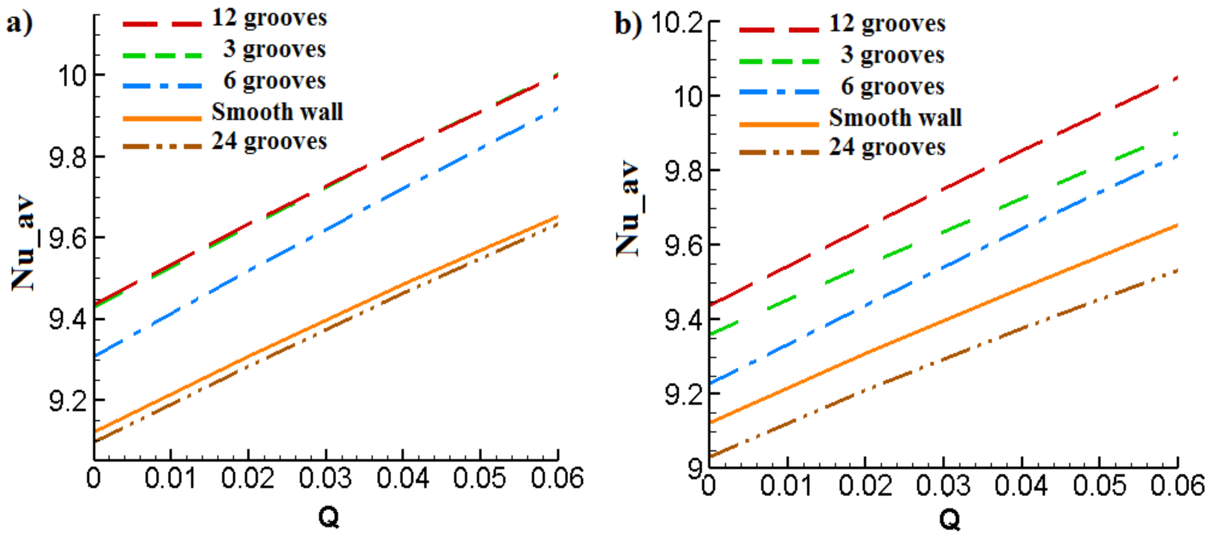

Fig. 9 Variation of the mean Nusselt number as a function of the volume fractions of the nanoparticles for Ra = 10^6 in cavities containing grooves of depth d = 5% L (a), d = 10% L (b)

The results show that the average Nusselt number is greater the larger the volume fraction of the C_{60}-based nanoparticles. This means that this type of nanoparticle considerably improves the thermophysical properties of the fluid, which intensifies the convective heat exchange in this type of flow. Thus, the use of C_{60}-based fluids can significantly improve the design of heat exchange systems for better space optimization.

The macrostructural aspect of the grooves (their number and dimensions) also influences natural convection in this type of geometry. Fig. 6, 7, 8 and 9 show that the average Nusselt number strongly depends on these two parameters (number of grooves and their dimensions), in fact, for a high number of grooves on the heated wall, the heat exchange surface increases which promotes the convective exchange, but the viscosity of the nanofluid and the fine dimensions of the grooves slow down the flow. In Fig. 10 we can clearly see this observation. Indeed, the average Nusselt number decreases markedly in the case where the hot wall contains 24 grooves.

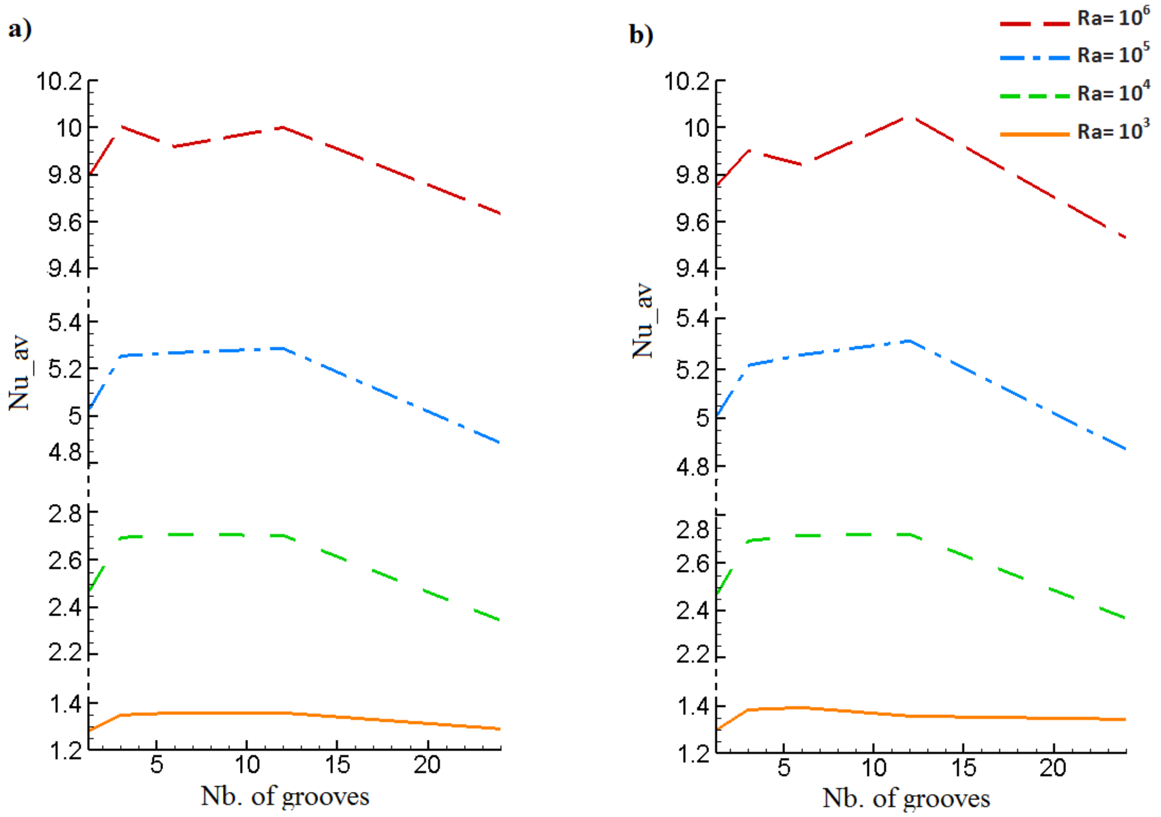

Fig. 10 Variation of the mean Nusselt number as a function of the number of grooves on the heated wall for different Rayleigh numbers and for the two groove depths, d = 5% L (a) and d=10% L (b).

5. Conclusion

The Numerical simulations which are carried out for different Rayleigh numbers (10^3, 10^4, 10^5 and 10^6), for different volume fractions of nanoparticles based on fullerene C_{60} ($0 \leq \Phi \leq 0.06$) and for different dimensions of the grooves in the heated wall of the square cavity have shown that:

- The use of fullerene-based nanoparticles suspended in water provides a significant improvement in thermal performance to the solution. This is due to their high thermal conductivities and their low densities.
- The Nusselt number increases regularly for increasing volume fractions.
- The effect of the volume fraction of fullerene-based nanoparticles on the value of the Nusselt number is greater the greater the Rayleigh number.
- The number and depth of grooves on a wall increase the heat exchange surface and promote the formation of vortices which intensify heat transfer by natural convection. but the fine dimensions of the grooves there slow down the flow of nano-fluid, which can lead to a decrease in convective heat transfer.

The results obtained clearly show that the use of C_{60} fullerene-based nanofluids and the macrostructural aspect of the grooves on the active walls of the cavities considerably influence the heat transfer in this type of geometry.

Nomenclature

Latin Symbols:

Cp	Specific heat at constant pressure (J/Kg.K)
g	Gravitational acceleration (m/s^2)
L	Cavity height (m)
K	Fluid thermal conductivity ($\mathrm{W.m^{-1}.K^{-1}}$)
P	Pressure (Pa)
T	Temperature (K)
u, v, w	Velocity components (m/s)
x, y, z	Cartesian coordinates in block 2 of grid (m)
x', y', z'	Cartesian coordinates in block 1 of grid (m)

Dimensionless quantities

Nu	Nusselt Number
Pr	Prandtl Number
Ra	Rayleigh Number
U, V	Nondimensional velocities
X, Y	Nondimensional coordinates
θ	Non-dimensional temperature

Greek symbols:

β	Volumetric coefficient of thermal expansion (K^{-1})
μ	Dynamic viscosity (N.s/m^2)
α	Thermal diffusivity (m^2/s)
ρ	Density (Kg/m^3)
υ	Kinematic viscosity (m^2/s)
Q	Particle volume fraction

Subscripts:

av	Average
c	Cold
f	Fluid
h	Hot
nf	Nanofluid
P	Nanoparticle
ref	Reference

References

[1] Zangooee, M. R., *et al*., Hydrothermal analysis of MHD nanofluid (TiO_2-GO) flow between two radiative stretchable rotating disks using AGM, *Case Studies in Thermal Engineering, 14* (2019), 100460

[2] Lafdaili, Z., *et al*., Numerical study of the turbulent natural convection of nanofluids in a partially heated cubic cavity, *Thermal Science, 25* (2021), 4, pp. 57-57

[3] Gunnasegaran, P., *et al*., Numerical study of fluid dynamic and heat transfer in a compact heat exchanger using nanofluids, *International Scholarly Research Network ISRN Mechanical Engineering, 11* (2012), Article ID 585496

[4] Jasim, Q. K., *et al*., Improving thermal performance using Al_2O_3-water nanofluid in a double pipe heat exchanger filling with porous medium, *Thermal Science, 24* (2020), pp. 4267-4275

[5] Che Sidik, N. A., *et al*., A review on the use of carbon nanotubes nanofluid for energy harvesting system, *International Journal of Heat and Mass Transfer, 111* (2017), pp. 782-794

[6] Park, S. S., *et al*., A study on the characteristics of carbon nanofluid for heat transfer enhancement of heat pipe, *Renewable Energy, 65* (2014), pp. 123-129

[7] Sheikholeslam, M., *et al*., MHD effects on nanofluid with energy and hydrothermal behavior between two collateral plates: Application of new semi analytical technique, *Thermal Science, 21* (2017), 5, pp. 2081-2093

[8] Ramzan, M., *et al*., Unsteady MHD carbon nanotubes suspended nanofluid flow with thermal stratification and nonlinear thermal radiation, *Alexandria Engineering Journal, 59* (2020), 3, pp. 1557-1566

[9] Choi, S. U. S., Enhancing thermal conductivity of fluids with nanoparticules, developments and applications of non –newtonian flows, *FED-Vol.231/MD-Vol.*66, (1995), pp. 99-105

[10] Batchelor, G. K, *et al*., Heat transfer by free convection across a closed cavity between vertical boundaries at different temperatures, *Quaterly of Applied Mathematics, 12* (1954), pp. 209-233

[11] Lafdaili, Z., *et al*., Numerical study of turbulent natural convection of nanofluids in differentially heated rectangular cavities, *Thermal Science, 25*, (2021), 1, pp. 579 - 589

[12] Tiwari, R. K., *et al.*, Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids, *International Journal of Heat and Mass Transfer*, *50* (2007), pp. 2002–2018

[13] Ho, C. J., *et al*., Numerical simulation of natural convection of nanofluid in a square enclosure: Effects due to uncertainties of viscosity and thermal conductivity, *International Journal of Heat and Mass Transfer, 51* (2008), pp. 4506–4516

[14] Belarche, L., Three-dimensional digital simulation of natural convection in a discreetly heated cavity subjected to different thermal boundary conditions, Ph. D. thesis, *University of IBN ZOHR*, Morocco, 2014 (in French)

[15] Achdou, Y., *et al*., A new cement ot glue non-conforming grids with Robin infertaces conditions: the finite volume case, *Numer. Math., 92* (2002), pp. 593-620

[16] Angot, Ph., Contribution to the study of heat transfers in complex systems: application to electronic components, Ph. D. thesis, *University of Bordeaux 1*, France, 1989 (in French)

[17] Arbogast, T., Yotov, I., A non-mortar mixed nite element method for elliptic problems on non-matching multiblock grids, *Comput. Methods Appl. Mech. Engn., 149* (1997), pp. 255-265

[18] Cai, W.-H., *et al.*, User-intervened structured meshing methods and applications for complex flow fields based on multiblock partitioning, *Journal of Computational Design and Engineering, 8* (2021), 1, pp. 225–238

[19] Monteiro, E., *et al*., Finite volume method analysis of heat transfer in multiblock grid during solidification, in: *Heat Transfer - Mathematical Modelling, Numerical Methods and Information Technology* (Ed. A. Belmiloudi), IntechOpen, 2011, pp. 129-150

[20] Maxwell, J. C., *A Treatise on Electricity and Magnetism*, Clarendon Press, U.K., 1891

[21] Brinkman, H. C., The viscosity of concentrated suspensions and solutions, *J. Chem. Phys., 20* (1952), pp.571-581

[22] Rome, Ch., A method of connecting nonconforming meshes for solving Navier- Stokes equations, Ph. D. thesis, University of Bordeaux 1, France, 2006 (in French)

[23] Ravnik, J., *et al*., Analysis of three-dimensional natural convection of nanofluids by BEM, *Eng. Anal. Bound. Elem., 34* (2010), 12, pp. 1018–1030

Received: 01.11.2021.
Revised: 23.06.2022.
Accepted: 06.07.2022.