RETRIEVAL OF SOLITON SOLUTIONS OF (1+1)-DIMENSIONAL NON-LINEAR TELEGRAPH EQUATION

by

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In this work, we aim to determine the possible soliton solutions and examine the behaviors of the (1+1)-dimensional non-linear Telegraph equation (NTE) which is used to model signal processing for the propagation of transmission of the electric impulses and also wave theory process by using the extended Kudryashov method. We started by finding the non-linear ordinary differential form of the (1+1)-NTE with the aid of a suitable wave transformation. Then, the extended Kudryashov method approach has been demonstrated and implemented to the obtained non-linear ordinary differential form. As a result, a polynomial expression has been achieved and converted to a linear algebraic equation system. Soliton solutions of the investigated equation are produced by solving the system and choosing the appropriate solution sets. Finally, graphical depictions, gained results and necessary comments are given.

Key words: *electrical transmission, soliton solution, wave transform, electric impulse*

Introduction

Non-linear evolution equations play a very active role in mathematical modeling of the complex problems arise in many fields such as physics [1, 2], biology [3], optics [4, 5], fluid mechanics [6], and so on. To determine the exact solutions of these kind of equations has become attractive for the researchers. In the last decade, many significant solution methods have emerged as a result of the devoted work of these researchers. Methods such as tanh-function method [7, 8], the $(G/G^{'})$ -expansion function method [9, 10], integral method [11], Hirota's direct method [12] and so on. For more work, see [13-33].

In this work, we applied the extended Kudryashov method, which is an effective method to solve the NTE. There are many studies in which the proposed method has been applied in the literature. Hassan *et al.* [13] applied the method to the (2+1)-D Painleve integrable Burgers equations and the (2+1)-D Korteweg-de Vries-Burgers equation. Zayed *et al.* [14] determined the optical solitons and other solutions to fiber Bragg gratings with dispersive reflectivity having Kerr law of non-linear refractive index via the method. Borai et al. emerged the solutions of the Kundu-Eckhaus equation with the help of the extended Kudryashov method in [15].

In this article, we consider the (1+1)-D non-linear Telegraph equation that is given in the following form [11]:

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$$B_{tt} - B_{xx} + B_t + aB + bB^3 = 0 \tag{1}$$

where $B = B(x,t)$ is a real valued function, B_{xx} is the second order dispersion term, a and b are the real values. Equation (1) is referred to as 1-D the second-order Telegraph equation with constant coefficients or 1-D the second-order hyperbolic Telegraph equation with constant coefficients. Equation (1) is not only used for signal analysis for transmission and propagation of electrical signals but also it is used to model a mixture between diffusion and wave propagation of finite velocity to standard heat or mass transport equation. Therefore, the eq. (1) has common usage areas such as transmission lines, heat transfer, random walks, biological population dispersal, chemical kinetics, *etc.*

In the literature, there are many significant works on NTE. For instance, Mirzazadeh *et al.* [11] investigated the exact solution of NTE using by the first integral method. Hossain *et al.* [16] applied modified simple equation method to the NTE to obtain the soliton solutions. In the work of Zayed *et al.* [17], they examined the exact solutions of the TE with the help of generalized Kudryashov method. At the work of Rizvi *et al.* [18], they determined the soliton solutions of the NTE in electrical transmission line using Hirota bilinear method.

Wave transformation of the investigated problem and having NODE form

To determine the NODE of the eq. (1), let us define the wave transformation as:

$$B(x,t) = B(\zeta), \quad \zeta = kx - \omega t \tag{2}$$

where $B(\zeta)$, k, ω present the wave amplitude, frequency and wave number, respectively. Besides, k, ω are non-zero real numbers. When we insert the eq. (2) and derivatives into eq. (1), we determine:

$$aB(\zeta) + bB(\zeta)^3 - \omega B'(\zeta) + (\omega^2 - k^2)B''(\zeta) = 0 \tag{3}$$

as the NODE form of eq. (1). If one can consider the terms $B''(\zeta)$,, $B(\zeta)^3$ in eq. (3) and apply the balance rule, compute the m as $m = 1$, which is known as balance number.

A brief description of the proposed method and application to the Telegraph equation

Based on the EKM [11, 33], the solution of eq. (3) is suggested:

$$B(\zeta) = a_0 + \sum_{k=1}^{m} \sum_{i+j=k} a_{ij} \beta^i(\zeta) \phi^j(\zeta) + \sum_{k=1}^{m} \sum_{i+j=k} b_{ij} \beta^{-i}(\zeta) \phi^{-j}(\zeta) \tag{4}$$

where a_0, a_{ij}, b_{ij} are real constants, m is the balance number, the functions $\beta(\zeta)$ and $\phi(\zeta)$ feed the following formulas:

$$\begin{aligned} \frac{\mathrm{d}\beta(\zeta)}{\mathrm{d}\zeta} &= R_2 \beta^2(\zeta) - R_1 \beta(\zeta) \\ \frac{\mathrm{d}\phi(\zeta)}{\mathrm{d}\zeta} &= S_2 \phi^2(\zeta) + S_1 \phi(\zeta) + S_0 \end{aligned} \tag{5}$$

where R_2, S_2 should not be zero together and eq. (5) has the following solutions:

$$\phi(\zeta)=\begin{cases}-\dfrac{S_1}{2S_2}-\dfrac{\sqrt{\delta}}{2S_2}\tanh\left(\dfrac{\sqrt{\delta}}{2}\zeta\right), \delta>0\\ -\dfrac{S_1}{2S_2}-\dfrac{\sqrt{\delta}}{2S_2}\coth\left(\dfrac{\sqrt{\delta}}{2}\zeta\right), \delta>0\\ -\dfrac{S_1}{2S_2}+\dfrac{\sqrt{-\delta}}{2S_2}\tan\left(\dfrac{\sqrt{-\delta}}{2}\zeta\right), \delta<0\\ -\dfrac{S_1}{2S_2}+-\dfrac{\sqrt{-\delta}}{2S_2}\cot\left(\dfrac{\sqrt{-\delta}}{2}\zeta\right), \delta<0\\ -\dfrac{S_1}{2S_2}-\dfrac{1}{S_2\zeta}, \delta=0.\end{cases}\qquad \beta(\zeta)=\begin{cases}\dfrac{R_1}{R_2+R_1e^{R_1(\zeta+\zeta_0)}}, & R_1\neq 0\\ -\dfrac{1}{R_2(\zeta+\zeta_0)}, & R_1=0\end{cases} \tag{6}$$

Here R_1, R_2, S_1, S_2 are real values, $\delta=S_1^2-4S_0S_2$. If we take eq. (4) and consider the balance number as $m=1$, the suggestion of eq. (4) degenerates into following form:

$$B(\zeta)=a_0+a_{10}\beta(\zeta)+a_{01}\phi(\zeta)+\frac{b_{10}}{\beta(\zeta)}+\frac{b_{01}}{\phi(\zeta)} \tag{7}$$

For simplicity, we will accept that $A_0=a_0$, $A_1=a_{10}$, $A_2=a_{01}$, $B_1=b_{10}$, and $B_2=b_{01}$.

When we substitute eq. (7) in eq. (3), taking into account eq. (5), and equalize the coefficients of the obtained equations to zero, we get the following system:

$$Coeff.\ \beta^0\phi^0: -\omega A_2S_0+\omega B_2R_2+\omega B_2S_2+bA_0^3+aA_0-A_2S_0S_1k^2+A_2S_0S_1\omega^2+B_1R_1R_2k^2-$$
$$-B_1R_1R_2\omega^2-B_2S_1S_2k^2+B_2S_1S_2\omega^2+6bA_0A_1B_1+6bA_0A_2B_2=0$$

$$Coeff.\ \beta^1\phi^0: A_1[(-k^2+\omega^2)R_1^2+R_1\omega+(3A_0^2+3A_1B_1+6A_2B_2)b+a]=0$$

$$Coeff.\ \beta^0\phi^1: [(-k^2+\omega^2)S_1^2-S_1\omega+2S_2\omega^2S_0+(3A_0^2+6A_1B_1+3A_2B_2)b-2S_2k^2S_0+a]A_2=0$$

$$Coeff.\ \beta^{-1}\phi^0: B_1[(-k^2+\omega^2)R_1^2-R_1\omega+(3A_0^2+3A_1B_1+6A_2B_2)b+a]=0$$

$$Coeff.\ \beta^0\phi^{-1}: B_2[(-k^2+\omega^2)S_1^2+S_1\omega+2S_2\omega^2S_0+(3A_0^2+6A_1B_1+3A_2B_2)b-2S_2k^2S_0+a]=0$$

$$Coeff.\ \beta^1\phi^1: 6bA_0A_1A_2=0$$

$$Coeff.\ \beta^1\phi^{-1}: 6bA_0A_1B_2=0$$

$$Coeff.\ \beta^{-1}\phi^1: 6bA_0A_2B_1=0$$

$$Coeff.\ \beta^{-1}\phi^{-1}: 6bA_0B_1B_2=0$$

$$Coeff.\ \beta^2\phi^0: A_1(3k^2R_1R_2-3\omega^2R_1R_2+3bA_0A_1-\omega R_2)=0$$

$$Coeff.\ \beta^0\phi^2: A_2(-3k^2S_1S_2+3\omega^2S_1S_2+3bA_0A_2-\omega S_2)=0$$

$$Coeff.\ \beta^3(\zeta)\phi^0(\zeta):\left[bA_1^2+(-2k^2+2\omega^2)R_2^2\right]A_1=0$$

$$Coeff.\ \beta^0\phi^3:\left[bA_2^2+(-2k^2+2\omega^2)S_2^2\right]A_2=0$$

$$Coeff.\ \beta^{-3}\phi^0:bB_1^3=0$$

$$Coeff.\ \beta^0\phi^{-3}:B_2\left[bB_2^2+(-2k^2+2\omega^2)S_0^2\right]=0$$

$$Coeff.\ \beta^0\phi^{-2}:B_2(-3k^2S_0S_1+-3\omega^2S_0S_1+3bA_0B_2+\omega S_0)=0$$

$$Coeff.\ \beta^{-2}\phi^0:3bA_0B_1^2=0$$

$$Coeff.\ \beta^1\phi^2:3bA_1A_2^2=0$$

$$Coeff.\ \beta^1\phi^{-2}:3bA_1B_2^2=0$$

$$Coeff.\ \beta^{-1}\phi^2:3bA_2^2B_1=0$$

$$Coeff.\ \beta^{-1}\phi^{-2}:3bB_1B_2^2=0$$

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$$Coeff.\ \beta^{-2}\phi^{-1}:3bB_1^2B_2=0 \tag{8}$$

After solving the previous system, we achieve the following sets and related solution functions:

Case I. Let $R_1\neq 0,\ \delta>0$. For the following set:

$$\text{Set}_1=\left\{\begin{matrix}\omega=\omega, & k=\dfrac{\sqrt{9a^2-2a}\,\omega}{3a}, & R_1=-\dfrac{3a}{2\omega}, & R_2=\dfrac{3\sqrt{-ab}\,A_1}{2\omega} \\ S_1=S_1, & S_2=S_2,\ A_0=0, & A_1=A_1,\ A_2=0, & B_1=0,\ B_2=0\end{matrix}\right\} \tag{9}$$

the solution of eq. (1) related to eq. (9) is:

$$B_1(x,t)=\frac{A_1a}{a\mathrm{e}^{-\frac{\sqrt{9a^2-2a}\,x}{2}+\frac{3ta}{2}}-\sqrt{-ab}\,A_1} \tag{10}$$

Case II. Let $R_1 = 0, \delta > 0$. For the set:

$$\text{Set}_2 = \left\{ \begin{array}{l} \omega = \dfrac{3\sqrt{9a^2 - 2a}\,k}{9a-2}, \quad R_2 = R_2, \quad S_1 = 0, \quad S_2 = -\dfrac{a(9a-2)}{16k^2 S_0} \\ A_0 = -\dfrac{\sqrt{9a^2-2a}}{2\sqrt{(-9a+2)b}}, \quad A_1 = 0, \quad A_2 = \dfrac{\sqrt{(-9a+2)b}\,a}{8bS_0 k}, \quad B_1 = 0, \quad B_2 = 0 \end{array} \right\} \tag{11}$$

the solution of eq.(1) corresponding to eq.(11) is considered:

$$B_{2,1}(x,t) = -\frac{k\sqrt{\dfrac{a(9a-2)}{k^2}}\tanh\left[\dfrac{\sqrt{\dfrac{a(9a-2)}{k^2}}\,k\left(9xa - 3\sqrt{9a^2-2a}\,t - 2x\right)}{36a-8}\right] + \sqrt{9a^2-2a}}{2\sqrt{(-9a+2)b}} \tag{12}$$

or

$$B_{2,2}(x,t) = -\frac{k\sqrt{\dfrac{a(9a-2)}{k^2}}\coth\left[\dfrac{\sqrt{\dfrac{a(9a-2)}{k^2}}\,k\left(9xa - 3\sqrt{9a^2-2a}\,t - 2x\right)}{36a-8}\right] + \sqrt{9a^2-2a}}{2\sqrt{(-9a+2)b}} \tag{13}$$

Case III. Let $R_1 \neq 0, \delta < 0$. For:

$$\text{Set}_3 = \left\{ \begin{array}{l} \omega = \dfrac{3\sqrt{9a^2-2a}\,k}{9a-2}, \quad R_1 = \dfrac{\sqrt{9a^2-2a}}{2k}, \quad R_2 = \dfrac{\sqrt{(-9a+2)b}\,A_1}{2k}, \quad S_1 = S_1 \\ S_2 = S_2, \quad A_0 = \dfrac{\sqrt{(-9a+2)b}\sqrt{9a^2-2a}}{(9a-2)b}, \quad A_1 = A_1, \quad A_2 = 0, \quad B_1 = 0, \quad B_2 = 0 \end{array} \right\} \tag{14}$$

the solution of eq. (1) according to eq. (14) is taken:

$$B_3(x,t) = \frac{a\sqrt{(-9a+2)b}\,\mathrm{e}^{-\frac{(9a-2)(3at - x\sqrt{9a^2-2a})}{18a-4}}}{b\left[\sqrt{(-9a+2)b}\,A_1 + \sqrt{9a^2-2a}\,\mathrm{e}^{-\frac{(9a-2)(3at - x\sqrt{9a^2-2a})}{18a-4}}\right]} \tag{15}$$

Case IV. Let $R_1 \neq 0, \delta = 0$. For the following set:

$$\text{Set}_4 = \left\{ \begin{array}{l} \omega = \omega, \quad k = \dfrac{\sqrt{9a^2-2a}\,\omega}{3a}, \quad R_1 = \dfrac{3a}{2\omega}, \quad R_2 = R_2 \\ S_2 = S_2, \quad A_0 = \dfrac{\sqrt{-ba}}{b}, \quad A_1 = \dfrac{2\omega R_2}{3\sqrt{-ba}}, \quad A_2 = 0, \quad B_1 = 0, \quad B_2 = 0 \end{array} \right\} \tag{16}$$

the solution of eq. (1) corresponding to eq. (16) is given:

$$B_4(x,t) = -\frac{3a^2 \mathrm{e}^{\frac{\sqrt{9a^2-2a}\,x}{2}-\frac{3ta}{2}}}{\sqrt{-ba}\left(3a\mathrm{e}^{\frac{\sqrt{9a^2-2a}\,x}{2}-\frac{3ta}{2}}+2\omega R_2\right)} \tag{17}$$

Case V. Let $R_1 = 0, \delta < 0.$ For:

$$\mathrm{Set}_5 = \left\{ \begin{array}{l} k = \dfrac{\sqrt{9a^2-2a\,\omega}}{3a}, \quad \omega=\omega, \quad R_2 = R_2, \quad S_1 = S_1, \quad S_2 = \dfrac{4\omega^2 S_1^2 - 9a^2}{16\omega^2 S_0} \\ A_0 = \dfrac{(2S_1\omega+3a)\sqrt{-ab}}{6ab}, \quad A_1 = 0, \quad A_2 = 0, \quad B_1 = 0, \quad B_2 = -\dfrac{2S_0\omega}{3\sqrt{-ab}} \end{array} \right\} \tag{18}$$

the solution of eq. (1) related to eq. (18) is given:

$$B_5(x,t) = -\frac{\left\{\omega\sqrt{-\dfrac{a^2}{\omega^2}}\cot\left[\dfrac{\sqrt{-\dfrac{a^2}{\omega^2}}\,\omega\left(\sqrt{9a^2-2a}\,x-3ta\right)}{4a}\right]+a\right\}(2S_1\omega+3a)}{6\sqrt{-ab}\,\omega\left\{\cot\left[\dfrac{\sqrt{-\dfrac{a^2}{\omega^2}}\,\omega\left(\sqrt{9a^2-2a}\,x-3ta\right)}{4a}\right]\sqrt{-\dfrac{a^2}{\omega^2}}+\dfrac{2S_1}{3}\right\}} \tag{19}$$

Result and discussion

This section contains the graphical presentations of the results which are obtained in the article.

We characterized fig. 1 which is obtained by choosing $B_1(x,t)$ in eq. (10), Set_1 in eq. (9) and selecting the particular values as $a = -1.65, b = 1, \omega = -1.2,$ $S_0 = 1, S_1 = 3, S_2 = 1, A_1 = 1.$ Figure 1(a) shows the 3-D chart of $B_1(x,t)$, fig. 1(b) shows the 2-D charts of $B_1(x,t)$ for $t = 1,2,3,4,5.$ Figure 1(a) describes a kink soliton while is a well-known soliton type and fig. 1(b) represents the wave at $t = 1,2,3,4,5.$ From fig. 1(b), we can see that wave is a leftward traveling wave.

Figure 2 is determined by selecting $B_{2,1}(x,t)$ in eq. (12), Set_2 in eq. (11) and with the specific values $a = 1.65, b = -2.6, \omega = -1,$ $S_2 = 1.5, R_2 = -1.$ Figure 2(a) depicts the 3-D chart of $B_{2,1}(x,t)$, fig. 2(b) shows the 2-D charts of $B_{2,1}(x,t)$ for $t = 1,2,3,4,5.$ Figure 2(a) illustrates a kink soliton shape, and fig. 2(b) have the waves which have leftward traveling wave character.

Figure 3 is plotted by choosing $B_3(x,t)$ in eq. (15), Set_3 in eq. (14) and for the values $a = 1.65, b = -2.6, \omega = -1,$ $S_2 = 1.5, R_2 = -1.$ Figure 3(a) describes the 3-D chart of $B_3(x,t)$, fig. 3(b) shows the 2-D charts of $B_3(x,t)$ for $t = 1,2,3,4,5.$ Figure 3(a) illustrates a kink soliton shape, and fig. 3(b) have the waves which have rightward traveling wave character.

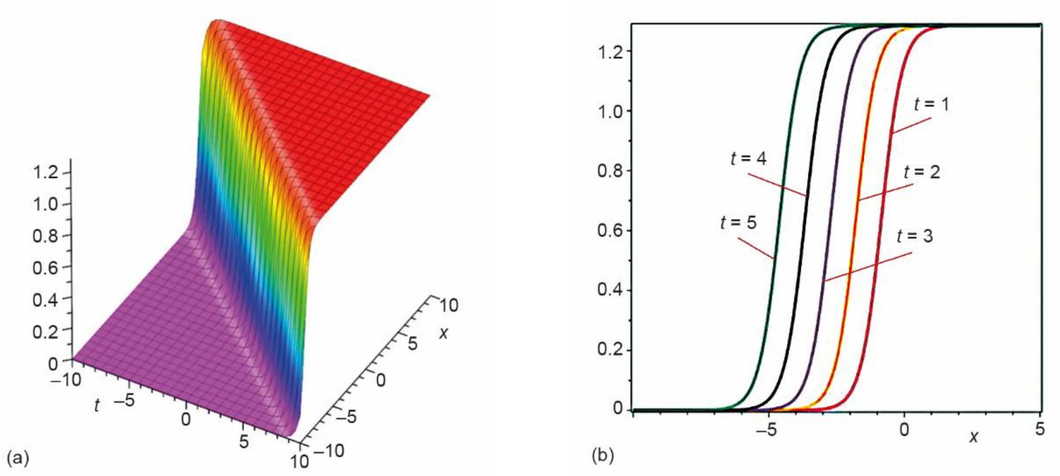

Figure 1. Representation of $B_1(x, t)$ in eq. (10) with Set_1 in eq. (9) and a = –1.65, b = 1, ω = –1.2, S_0 = 1, S_1 = 3, S_2 = 1, A_1 = 1; (a) 3-D graph of $B_1(x, t)$ and (b) 2-D graph of $B_1(x, t)$

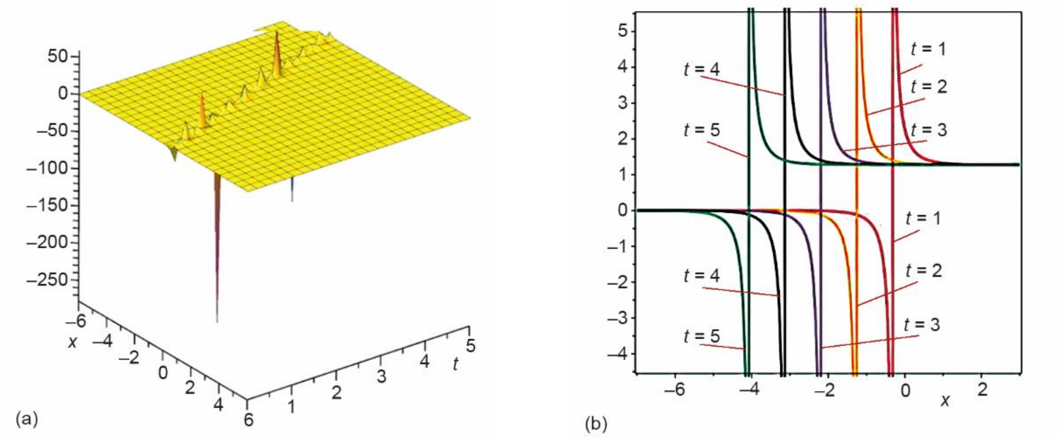

Figure 2. Illustration of $B_{2.1}(x, t)$ in eq. (12) with Set_2 in eq. (11) and a = 1.65, b = –2.6, ω = –1, S_2 = 1.5, R_2 = –1; (a) 3-D graph of $B_{2,1}(x, t)$ and (b) 2-D graph of $B_{2,1}(x, t)$

Although figs. 1-3 graphics reflect the kink soliton character as a general image, they physically have a different meaning in all three graphics. Figure 1 indicates that wave moves to the leftward depend on time according to x-direction, while fig. 3 indicates that wave moves to the rightward. In a sense, the wave in fig. 1 moves to left and the wave in fig. 3 moves to right. Also, if we accept the level where the vertical amplitude of the soliton is zero as the neutral level, in fig. 1 the entire soliton is above the neutral level. At fig. 3, the entire soliton is below the neutral level. The soliton in fig. 2, as in fig. 3, moves to the left de-

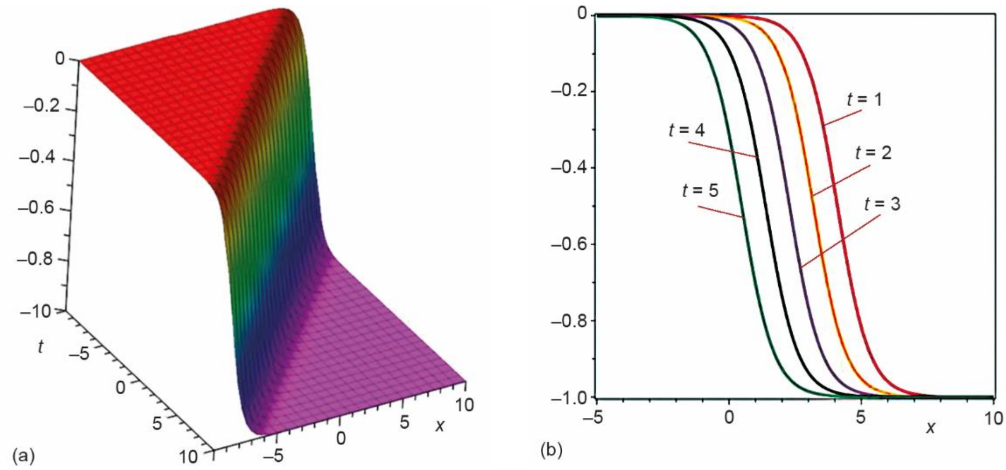

Figure 3. Representation of $B_3(x, t)$ in eq. (15) with Set_3 in eq. (14) and $a = -1$, $b = 1$, $k = 1$, $R_2 = 0.5$, $S_1 = 1$; (a) 3-D graph of $B_3(x, t)$ and (b) 2-D graph of $B_3(x, t)$

pending on time, although it is formed below the neutral level. So the direction of the soliton is to the left. In this sense, as an orientation, figs. 2 and 3 representations are to be called the kink soliton (a wave moving to the right or to the right depending on time), fig. 1 can be called an anti-kink soliton (a wave that turns left or moves to the left depending on time).

Figure 4 is obtained by taking $B_1(x,t)$ in eq. (10) with Set_1 in eq. (9) and $a = -1.65, b = 1, \omega = 1.2,\ S_0 = 1, S_1 = 3, S_2 = 1, A_1 = -0.25.$ Figure 4(a) shows a singular soliton shape and fig. 4(b) represents the waves at the values of $t = 1, 2, 3, 4, 5.$ The wave has a leftward traveling wave character. In the fig. 4 representation, we see that the singular solution character behaves differently to the left and right of the place where the singularity occurs. While approaching the point where singularity occurs from the left, the amplitude of the soliton suddenly decreases from the neutral level to an infinitely small negative value, and on the right it increases from a positive value $[1 < B_1(x,t) < 2]$ to an infinitely large value. This behavior naturally does not mean that the amplitude of the wave takes an infinitely small (large) value, but that the amplitude of the wave ceases to be measurable at this point.

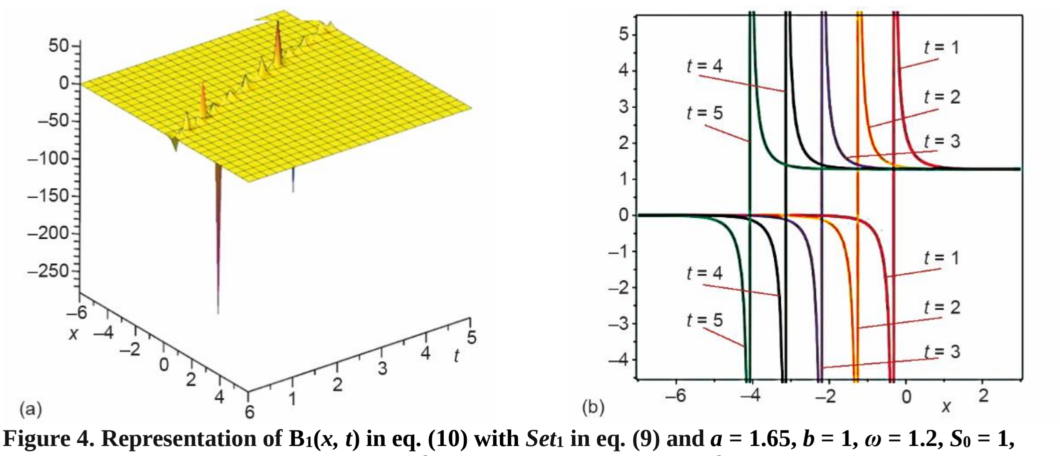

Figure 4. Representation of $B_1(x, t)$ in eq. (10) with Set_1 in eq. (9) and $a = 1.65$, $b = 1$, $\omega = 1.2$, $S_0 = 1$, $S_1 = 3$, $S_2 = 1$, $A_1 = -0.25$; (a) 3-D graph of $B_1(x, t)$ and (b) 2-D graph of $B_1(x, t)$

Conclusion

In this paper, we investigated non-linear Telegraph equation that has an important place in the modelling of electrical transmission. Although, many studies have been done on this equation, the extended Kudryashov method is applied to the form of the non-linear Telegraph equation for the first time. Kink and singular soliton solutions are produced. Graphical representations have been given with 3-D and 2-D to show the physical illustrations of some obtained solutions.

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